A New Light Synthesis Optimization Program. Test on Simulated Algol Data

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(Received 20 October, 1988 – accepted 21 February, 1989)

ABSTRACT

Simulation of light curve effects in Algol systems that are near contact or that may evolve into contact requires a different model than the tri-axial ellipsoid used in many current studies. A new light synthesis optimization program, based on the Roche model, satisfies the model requirements and determines system parameters with excellent accuracy, given a sufficiently large set of observations.

A widely used criterion for stopping iterative solutions is inadequate. A working criterion is proposed as a replacement.

I. Introduction

Extensive analysis of Algol-type light curves has been performed with a tri-axial ellipsoid model (Etzel 1981, Etzel and Olson 1985). This model is demonstrably adequate when the lobe-filling inner radius is less than 0.35 (Etzel and Olson 1985). However, this model breaks down for closer component spacing.

The widely-used Wilson-Devinney (1971, hereafter WD) program, based on the Roche model, also has been applied to Algol systems (Wilson, et. al. 1972, Giuricin, Mardirossian, and Mezzetti 1983 and references therein). The Roche model has the advantage that its parameters apply no matter what the component spacing, including contact. Radii for equivalent spherical volumes are separately calculable, for comparison with tri-axial ellipsoid results. Easily available computer power now permits routine use of Roche model algorithms.

This author has completed a new light synthesis optimization program based on the Roche model and has begun application to contact binary systems. It was of interest to test its operation on simulated data for a system like Algol.

II. Program Description

A light synthesis simulation of an eclipsing binary, based on the Roche model, requires assigned values for 14 parameters: $\underline{q}$, Ω_1, Ω_2, $\underline{i}$, $\underline{T}_1$, $\underline{T}_2$, $\underline{A}_1$, $\underline{A}_2$, $\underline{b}_1$, $\underline{b}_2$, $\underline{u}_1$, $\underline{u}_2$, $\underline{L}_3$, and $\underline{L}_{ref}$, where $\underline{q}$ = mass ratio, Ω_1,

Space Science Reviews **50** (1989), 269–278.

Ω_2 are the photospheric Roche potentials, $\underline{i}$ = orbital inclination, $\underline{A}_1$, $\underline{A}_2$ are bolometric albedos, $\underline{b}_1$, $\underline{b}_2$ are gravity brightening exponents, $\underline{u}_1$, $\underline{u}_2$ are limb darkening coefficients, $\underline{L}_3$ is fractional system light due to a third component, and $\underline{L}_{ref}$ is a light reference value which normally equals 1.0. The first four parameters are geometric and the next eight are physical. Note that $\underline{T}_1$ and $\underline{T}_2$ refer to the polar temperatures of the respective components.

Most optimization programs (e.g. WD) use differential corrections. The required light derivative with respect to a given parameter, at a given orbital phase, is determined by differencing the calculated system light at two closely spaced values of that parameter and dividing by the parameter increment. If the number of parameters subject to optimization is appreciable, many calculated light values are necessary to evaluate the derivatives, and the entire process must be repeated for each iteration. If the parameter increment is small the light values are nearly equal and effects of computational noise in their difference become amplified in dividing by the small increment. If the parameter increment is large the calculated derivative may be inaccurate.

These considerations have led to development of an optimization procedure based on the simplex technique (Kallrath and Linnell 1987, Linnell and Kallrath 1987). This technique requires no light derivatives. However the simplex technique provides no convenient means to calculate a covariance matrix, so it is possible only to estimate probable errors of optimized system parameters. The simplex technique also lacks good means to calculate correlations among parameters. These additional considerations have prompted a re-examination of the differentials corrections approach.

The light at a given orbital phase is given, schematically, by

$$\ell = \int_{\theta}\int_{\phi} \left[I_1 \cos\gamma \frac{r^2}{\cos\beta} \sin\theta \right] d\phi d\theta + \int_{\theta}\int_{\phi} \left[I_2 \cos\gamma \frac{r^2}{\cos\beta} \sin\theta \right] d\phi d\theta$$
$$- \int_{\theta,ecl}\int_{\phi,ecl} \left[I_{1,2} \cos\gamma \frac{r^2}{\cos\beta} \sin\theta \right] d\phi \, d\theta, \qquad (1)$$

where the integer subscripts designate the components. The reason for the term "schematically" in the last sentence is this: $\underline{I}_1$ and $\underline{I}_2$ are in physical units, so $\underline{\ell}$ is initially in physical units and must subsequently be normalized. See a separate publication (Linnell 1989) for details. Consider the derivative of $\underline{\ell}$ with respect to one of the physical parameters, $\underline{T}_1$, say. It can be shown by Leibnitz's theorem (Linnell 1989) that

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$$\frac{\partial \ell}{\partial T_1} = \int_\Theta \int_\phi \left[\frac{\partial I_1}{\partial T_1} \cos\gamma \frac{r^2}{\cos\beta} \sin\Theta\right] d\phi d\Theta$$
$$- \int_{\Theta,ecl} \int_{\phi,ecl} \left[\frac{\partial I_{1,2}}{\partial T_1} \cos\gamma \frac{r^2}{\cos\beta} \sin\Theta\right] d\phi d\Theta, \tag{2}$$

Now

$$I_1 = I_1\ (\tau=0)\ [1 - u_1 - u_2 + u_1 \cos\gamma + u_2 \cos^2\gamma]\ , \tag{3}$$

and

$$I_1\ (\tau=0) = \frac{6F(T_o)}{\pi(6 - 2u_1 - 3u_2)}\ , \tag{4}$$

and

$$F(T_o) = \frac{c_1}{\lambda^5} \frac{1}{\exp(c_2/\lambda T_o)-1}\ , \tag{5}$$

on the black body approximation. $\underline{T}_o$ relates to $\underline{T}_{eff}$ on the Eddington approximation by $\underline{T}_{eff} = 1.2324\ \underline{T}_o$. $d\underline{F}(T_o)/d\underline{T}_o$ follows directly from eq. (5). Thus it is possible to calculate $\partial\underline{I}_1/\partial\underline{T}_1$ analytically. Then $\partial\underline{\ell}/\partial\underline{p}$, where $\underline{p}$ is any of the physical parameters, can be calculated via eq. (2) during the same process that calculates $\underline{\ell}$ via eq. (1). No differencing of two light values is necessary.

This procedure does not work for the geometric variables because the limits of integration are functions of those variables. An improvement over the standard practice still is possible.

Stirling's formula for the derivative of a function y(x), based on finite diagonal differences is

$$\frac{dy}{dx} = \frac{1}{h}\left[\frac{\Delta y_{-1} + \Delta y_0}{2} + \mu\, \Delta^2 y_{-1} + \frac{3\mu^2-1}{3!}\frac{\Delta^3 y_{-2}+\Delta^3 q_{-1}}{2} + \ldots\right]\ , \tag{6}$$

where $\mu = \frac{x - x_0}{h}$.

Choose the spacings of geometric parameters so that third and higher differences are small enough to be neglected. For a particular geometric parameter, say $\underline{i}$, and at a particular orbital longitude ψ, the difference table for our problem is

i_{sm}	$\ell_{\psi,ism}$		
		$\Delta\ell_{-1}$	
i_o	$\ell_{\psi,io}$		$\Delta^2\ell_{-1}$
		$\Delta\ell_0$	
$i_{\ell g}$	$\ell_{\psi,i\ell g}$		

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Since this formulation is second order in the differences, spacing for a given parameter can be somewhat larger than for the first order approximation in current use.

The light synthesis program (Linnell 1984) calculates system light at a specified set of fiducial orbital longitudes. Light values for specific observational phases are calculated by interpolation among the fiducial values. Calculation of $\partial \underline{\ell}/\partial \underline{i}$ at the fiducial orbital longitudes then requires three files of light values $\underline{\ell}_{ism}$, $\underline{\ell}_{io}$, $\underline{\ell}_{i\ell g}$. As $\underline{i}$ changes with successive iterations, only μ, from eq. (6), changes. Provided $\underline{i}_{sm} \leq \underline{i} \leq \underline{i}_{\ell g}$, the three tables of light values for $\underline{i}$ need not be recalculated. Similar comments apply to the other geometric parameters. Since one central reference light calculation is common to all geometric parameters, a total of nine files of light values permits calculation of fiducial longitude light derivatives for all geometric parameters. A light derivative at a particular observational phase follows by interpolation.

A summary of important features of the light synthesis program package is useful at this point. (1) The program assigns individual first-order and second-order limb darkening coefficients to each photospheric mesh point by 3D interpolation (wavelength, local temperature, local log gravity) in an external data file. Thus, limb darkening coefficients normally are not adjustment parameters. The particular external data file can be changed by declaration in the program execution file. The limb darkening coefficients used in the study are by Wade and Rucinski (1985), based on Kurucz atmospheres. (2) The normal intensity at each mesh point can be calculated either by the black body law or by 3D interpolation in an external model atmosphere data file. As with limb darkening, the particular model atmosphere data file can be changed by declaration in the program execution file. (3) Irradiation at a given mesh point follows by integration over the limb-darkened visible disk of the companion at that mesh period. It fully allows for penumbral region effects. (4) The system light at an arbitrary orbital phase follows by interpolation among the fiducial points. This permits the program to handle an indefinitely large number of individual observations. (5) The program automatically accommodates to a contact, semi-detached, or detached configuration by setting logical constraints on permissible Roche photospheric potentials.

The differential corrections program, called DIFCORR, links to the output of the light synthesis program package by an execution file. An input file specifies the parameters to be optimized. DIFCORR solves for increments to the optimization parameters, predicts the next iteration individual residuals, calculates a covariance matrix and probable errors for the incremented parameters, calculates an array of simple correlation coefficients, an array of partial correlation coefficients, tests for incipient solution indeterminacy, sorts the residuals into a histogram, and applies the Kolmogorov-Smirnov goodness-of-fit test for a normal distribution of weighted residuals.

The program package produces a large number of files for useful off-line plots.

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II. Test on Simulated Algol Data

A more detailed discussion of the program and an application to synthetic data for a contact binary is in a separate publication (Linnell 1989). This paper concentrates on the analysis of synthetic data for a system like Algol, omitting the third component. The adopted true system parameters were from the papers by Hill, et. al. (1971) and Hill and Hutchings (1970). The true system parameters are q = 0.21739, Ω_s = 5.30, Ω_g = 2.28, i = 81.60°, T_s = 10800(K), T_g = 4600(K), A_s = 1.0, A_g = 0.5, b_s = 0.25, b_g = 0.08. For simplicity, this investigation used the black body law. Separate tests on a contact system simulation demonstrate proper operation with model atmospheres substituted. The optimization of physical parameters, based on analytic derivatives of the black body law, continues to produce convergence when model atmosphere values are substituted for normal intensities.

Modern automated observing techniques permit easy acquisition of substantial data bases. Thus high speed photometry of W UMa produced about 15,000 UBVRI observations in a single night (Linnell 1985). DeLandtsheer (1983a) obtained nearly 6000 observations of TV Cas in four spectral bands. Accordingly, simulated UBV light curves were produced with 2600 observations in each spectral band. Adopted observational dispersions were σ_V = 0.006, σ_B = 0.005, σ_u = 0.008, values consistent with actual observational experience.

There recently has been controversy concerning the convergence criterion (Wilson 1983, deLandtsheer 1983b) and the use of the Method of Multiple Subsets. Solution of simulated data, for which the model is precisely defined and the exact solution known in advance, can be of help in this controversy. Note that deLandtsheer performed more than 150 iterations on TV Cas.

The test solution had several objectives: (1) To test operation of the differentials correction program on an Algol-like system; (2) To determine the number of iterations necessary to recover known system parameters; (3) To measure the accuracy with which parameters can be recovered in the presence of observational errors similar to those actually found in practice; (4) To discover whether the optimization program can start from a relatively poor fit and achieve a high quality fit, without concern for possible secondary minima in the variance. (5) To test the commonly-adopted convergence criterion.

Popper (1980) has emphasized the observational difficulties in spectroscopic determinations of mass ratios for Algol systems. Photometric determinations of q are possible (see the remarks by Wilson 1980 p. 205), but typically use the boundary condition that the secondary fill its Roche lobe (Wilson 1980, p.263). It is desirable to relax this constraint. It is a more satisfying scientific result if the observations demand that the mass loser fill its Roche lobe than if that component satisfy the control condition because of a theoretical argument that it should do so. In the present simulation, the fact that the system is partially eclipsing places more extreme demands on the

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parameter recovery process than would be true for the case of complete eclipses.

The initially assumed parameters are on data line 2 of Table 2. Parameters subject to optimization are $\underline{q}$, Ω_1, Ω_2. $\underline{i}$, $\underline{T}_2$ and $\underline{L}_{ref}$. These initially assumed parameters give a poor fit to the observational data.

A comment is in order concerning limb darkening coefficients. For the mass loser, gravity varies over the photosphere by more than a factor 10, and temperature by more than 1000 K. Limb darkening coefficients, from model atmospheres, show an appreciable variation (Table 1). The photometric effect for Algol is not significant because the loser contributes so little light. The effect is more pronounced for contact binaries. Limb darkening coefficients do not occur in isolation in eclipsing binary model parameters. They also connect via the source function to model atmosphere fits to observed stellar spectra, which generally are quite good. For this reason it is preferable to use theoretical limb darkening coefficients for cases in which one or both stellar components are severely distorted from a sphere.

The optimization run results are in Table 2. The last column gives the standard deviation of the residuals in $\underline{V}$. The usual criterion to stop iterations is to stop when the standard deviation of the formal parameter errors is as small as the calculated parameter increments. If Ω_s were chosen as the test parameter, this condition initially would be met in going from iteration 2 to iteration 3, but convergence is far from complete. The criterion is essentially satisfied for all geometric parameters in going from iteration 9 to iteration 10, yet some of the geometric parameters are capable of substantial improvement with further iterations. The value of σ does not decrease monotonically with successive iterations, a reminder that optimization is a highly nonlinear process and the least squares process uses linearized equations of condition. The change of the residuals σ with successive iterations is so slow that it appears nearly constant, and visual examination of plots of the residuals disclose no apparent trends that imply incomplete convergence. What criterion is appropriate to terminate iterations? Table 2 shows a monotonic average drift in some parameter values, eventually ceasing. A similar result has been found for a contact system. The working rule which this investigation suggests is: Iterations should be continued until average drift in each parameter ceases. It is important to emphasize the word average. As successive iterations occur, the change in a given parameter may not always be monotonic. The indication of this investigation and a corresponding study of a contact system is that perhaps 20 or more iterations may be necessary with the current program if the initial approximation is quite inaccurate. Note that the first two or three iterations may appear to diverge.

The proposed test to stop iterations is a stop-gap at best. This topic deserves more detailed study. Table 3 shows the final solution, in $\underline{V}$, together with the calculated parameter probable errors, the composite UBV solution, and the true values, repeated for easy

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comparison. The program produces a separate solution in each spectral band of observation and calculates a weighted combination for the next iteration parameter set. The calculated values of $\underline{q}$, Ω_s, Ω_g, and $\underline{i}$ are within 1 σ of their true values. $\underline{T}_g$ still differs from its true value by an amount five times the formally calculated probable error. This is perhaps not surprising since the component in question contributes only 5% of the system light in $\underline{V}$, and correspondingly less in $\underline{B}$ and $\underline{U}$. Also, the temperature distribution of the loser is strongly influenced by irradiation by the companion, and this temperature distribution in turn depends on the adopted bolometric albedo of the loser.

The remarkable accuracy of the composite $\underline{q}$ is fortuitous since a single iteration with assumed parameters exactly equal to true values produces a composite next iteration $\underline{q}$ of 0.21996. Nevertheless, the optimization program, together with a large data set, is able to recover $\underline{q}$ to within 1%, for this partially eclipsing system, while simultaneously determining the other geometric parameters to within a few tenths of a percent after 18 iterations.

Table 4 lists the simple correlation coefficients among optimized parameters. It is clear that an idealization such as this may prove very different, practically, from a corresponding data set on a real Algol system. There is the problem of night errors in photometry, familiar to all of us, which complicates the combination of data from several nights. During a single night there are azimuthal extinction problems requiring different extinction coefficients on different sides of the meridian. Observations from space may be necessary to eliminate these sources of error. The important point is that a sufficiently large data set and an efficient parameter optimization program recovered simulation model parameters with high accuracy in approximately 20 iterations, for the particular system investigated here. A separate simulation solution should accompany a solution of actual observational data in individual cases.

III. Conclusions

This study leads to the following conclusions:

1. The new optimization program, together with a synthesized data set 10 times larger than current standard observational practice, determined Roche model geometric parameters for a particular Algol system to 1% in $\underline{q}$ and a few tenths of a percent in the other parameters, without recourse to the Method of Multiple Subsets.
2. The calculated $\underline{T}_g$ may differ from its true value by a much larger amount than indicated by the formal probable error, if the corresponding component contributes a minor fraction of the system light.
3. The widely used criterion for stopping iterations, when the standard deviation of formal parameter errors equals or is greater than the calculated parameter increments, is inadequate in the case tested.
4. A working rule is to continue iterations as long as there is a secular average trend in any optimized parameter.

These conclusions, following from a single example, require much more extensive test before deserving general acceptance.

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Table 1
Theoretical limb darkening coefficients
Coefficients are in the sequence first order, second order

	Gainer	Loser
Pole	0.759,-0.265	0.772,0.035
Point	0.762,-0.265	0.807,-0.102
Back	0.762,-0.265	0.767,0.046
Side	0.763,-0.265	0.748,0.072

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Table 2
Log of Optimization Run

IT	$\underline{q}$	Ω_s	Ω_g	$\underline{i}$(deg)	$\underline{T}_g$(K)	σ
true	0.21739	5.30000	2.28000	81.6000	4600.0	
0	0.22500	5.35000	2.30000	84.0000	4800.0	
1	0.23560	5.48924	2.31724	81.2405	4817.1	0.02502
2	0.24467	5.41813	2.33662	81.3421	4702.3	0.00661
3	0.23422	5.40818	2.34618	81.4013	4656.8	0.00839
4	0.23064	5.38334	2.33622	81.2561	4583.1	0.00686
5	0.22438	5.30090	2.32820	81.6240	4549.0	0.00811
6	0.21944	5.26987	2.31333	81.5653	4511.6	0.00796
7	0.21501	5.19514	2.29995	81.8872	4483.7	0.00901
8	0.21165	5.21246	2.28078	81.7778	4466.3	0.00738
9	0.21009	5.22726	2.26081	81.7456	4471.4	0.00692
10	0.21173	5.24460	2.26039	81.6805	4495.5	0.00584
11	0.21394	5.26448	2.27322	81.6379	4529.3	0.00586
12	0.21573	5.26847	2.27034	81.5892	4527.7	0.00588
13	0.21776	5.28339	2.28392	81.6128	4549.5	0.00587
14	0.21688	5.29048	2.27977	81.5956	4543.7	0.00581
15	0.21699	5.28935	2.27816	81.5853	4542.6	0.00580
16	0.21738	5.29065	2.27998	81.5923	4545.6	0.00579
17	0.21736	5.29221	2.28024	81.5923	4546.1	0.00579
18	0.21734	5.29221	2.27998	81.5891	4545.7	0.00579

Table 3
$\underline{V}$ Solution, Composite, True Values

$\underline{q}$	Ω_s	Ω_g	$\underline{i}$(deg)	$\underline{T}_g$(K)
0.21595 ± 0.00098	5.3043 ± 0.0088	2.2772 ± 0.0020	81.589 ± 0.029	4561.2 ± 8.3
0.21734	5.2922	2.2800	81.589	4545.6
0.21739	5.3000	2.2800	81.600	4600.0

Table 4
Simple Correlation Coefficients

Param.	$\underline{q}$	Ω_s	Ω_g	$\underline{i}$(deg)	$\underline{T}_g$(K)	$\underline{L}_{ref}$
$\underline{q}$	1.0000	-.1826	0.8574	-.5074	-.2700	-.4721
Ω_s	-.1826	1.0000	-.4725	-.5701	0.2618	0.3729
Ω_g	0.8574	-.4725	1.0000	-.0121	-.1753	-.4080
$\underline{i}$	-.5074	-.5701	-.0121	1.0000	0.2651	0.2562
$\underline{T}_g$	-.2700	0.2618	-.1753	0.2651	1.0000	0.9589
$\underline{L}_{ref}$	-.4721	0.3729	-.4080	0.2562	0.9589	1.0000

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DISCUSSION

Wilson asked if Linnell never needed to use the method of multiple subsets or only had not needed it in this example. Linnell replied that he did not need it in this case; he did not yet know about other cases. Wilson also asked if Linnell could solve simultaneously light-curves in different colours. Linnell replied that he solved each colour (e.g. UBV) separately and then, to obtain the geometric parameters appropriate for all wavelengths, he made a weighted mean final solution. Leung commented that a correlation coefficient of 0.85 was not very high and that it should be possible to obtain a solution without using the subsets approach. Linnell replied that he had had no difficulty but could not tell what would happen with correlation coefficients of 0.9 or more.

Budding thought the study would be very useful in helping us to understand the information content of light-curves. If Linnell would vary the number and distribution of his data points and the size of the simulated observational errors, it would be possible to find out how many parameters could be specified by light-curves of various qualities. Linnell agreed that his program lent itself to such numerical experiments.

Hill thought the model good but suggested that computing time could be saved, in accordance with an idea put forward by Lucy some years ago, by calculating the differentials at a reduced quadrature. The differential-correction technique could be grafted onto the end of the Simplex algorithm to provide errors. Responding to Budding, Hill also commented that accurate errors could be computed by the method of "bootstrap statistics", but only at the cost of computing about a thousand light-curves - so Hill does not recommend the method. Wilson finally commented that although the application of Leibnitz' theorem was elegant, it did not save much computing time since the part of the computations for which it was used were not very time-consuming anyway. Linnell replied that his procedure also had the advantage of avoiding the differencing of nearly equal numbers.