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MODE SELECTION AND OTHER NONLINEAR PHENOMENA IN STELLAR OSCILLATIONS

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ABSTRACT Nonlinear effects determining the amplitudes of unstable modes of oscillations in stellar modes, are reviewed. The two basic processes – saturation of the driving mechanism and resonant mode coupling – are discussed within the framework of the amplitude equation formalism. There are difficult problems in the theory of multiple mode interaction that must be solved to make a prediction of amplitude spectra possible. The observed spectra for δ Scu stars and other multiperiodic variables, exhibit no simple pattern that would suggest a solution of the theoretical problems.

1.INTRODUCTION

Linear stability analyses of stellar models reveal that pulsational instability, if present, occurs most often in a large number of oscillation modes simultaneously. Consequences of the instability are therefore difficult to predict. Satisfactory numerical models are available only for the radial monomode pulsation in Cepheid-type stars. We understand selection between fundamental and first overtone pulsation, but we do not understand the role of nonradial modes of high degree, which are unstable under the same condition. A sustained double-mode pulsation has not yet been reproduced in realistic models, and the cause of such behavior in stars remains controversial.

In stars in the Main Sequence (MS) and early post-MS evolutionary phases, multiperiodic variability is a common phenomenon. Up to seven modes of low degree have been tentatively identified in individual objects. The identified modes are among those which are found to be unstable in corresponding stellar models. We do not understand, however, how the mode amplitudes are determined. We do not know which of the undiscovered modes have finite amplitudes, but just too low for the detection with the present-day techniques. Thus, for instance, we cannot yet assess the reward in terms of detected modes from the planned asteroseismological observation from the space.

In this review of the status of the nonlinear theory of stellar oscillations, I focus on the mode selection problem. This is certainly the most important aspect of the theory for the main stream problems of this meeting. It would be indeed very helpful to have an *a priori* assessment of the number and kind of modes that are likely to be discovered once the amplitude resolution is greatly improved. This is not yet possible. I will, thus, present here only elements of the theory that may lead in the future to a prediction of the amplitudes. I believe that this will be an interesting and useful exercise, even though the needed theory may not be developed before the launch of first asteroseismological satellite.

We will start, however, with a different aspect of the theory. In the next section I will briefly review nonlinear effects in amplitude spectra. We will see how, with the help of the nonlinear theory, one can recover useful asteroseismological observables from such spectra. Impact of the linear nonadiabatic theory on the modal selection problem will be discussed in Section 3. Subsequently, I will outline the amplitude equation formalism which is the only practical tool for handling the nonlinear theory of *nonradial* oscillation and discuss two distinct amplitude limiting effects. Specific roles of resonances in mode selection will be reviewed in two consecutive sections. At the end we will discuss existing data on multiperiodic objects.

2. NONLINEAR OBSERVABLES

Oscillation frequencies, ω, are by far the most important observables for asteroseismology. They are not only most accurately measured but, more importantly, the easiest to interpret in terms of parameters describing stellar interiors. However, one should try to try to extract all information contained in amplitude spectra of variable stars. This is particularly important in dealing with the sparse spectra such as all (with one exception) currently available for stars.

A complete model of a pulsating star allow us to represent any observable parameter, O, in the form of the following expansion

$$\begin{aligned} O &= O_0 + \sum_k A_k \Re(Y_{(k)} \exp i(\omega_k t + \psi_k)) + \\ &+ \sum_{k,j,\pm} \lambda_{kj\pm} A_k A_j \Re(Y_{(k)} Y_{(j)} \exp i[(\omega_k \pm \omega_j)t + \psi_{kj\pm}]) + ..., \end{aligned} \tag{1}$$

where amplitudes A and frequencies ω may vary with time, but on the time scale much longer than $1/\omega$, and

$$Y_{(k)} = Y_l^m(\theta, \phi) \quad \text{for} \quad l = l_k \quad \text{and} \quad m = m_k. \tag{2}$$

The parameters occurring in this expansion may be directly compared with those occurring in the Fourier decomposition of the observed light or radial velocity curve. Linear adiabatic theory yields only ω. The amplitude ratios, A_k^a/A_k^b, and the phase differences, $\psi_k^a - \psi_k^b$, (superscripts a and b refer to various observables) may be evaluated within the framework of linear nonadiabatic theory. These quantities depend on stellar parameters, but they are also sensitive to the mode spherical harmonic degree so that they are useful for mode identification (Balona & Stobie, 1980)

Evaluation of the remaining parameters – A_k, $\lambda_{kj\pm}$, $\psi_{kj\pm}$, and corresponding higher order quantities – lies in the domain of nonlinear theory. In general, a connection between these parameters and those descrbing stellar internal structure is, rather complicated. However, in the case of the "bump" Cepheids, for instance, the phase difference, $\psi_{kj\pm} - (\psi_k \pm \psi_j)$, may be translated into information about the second overtone period (Simon & Lee, 1981; Buchler *et al.*,

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1990). Furthermore, secular changes in amplitudes and phases, other than those caused by stellar evolution, may be interpret only with use of nonlinear theory.

3. GROWTH RATES

The first step toward understanding modal selection in pulsating stars, which is the determination of mode stability, belongs to the linear theory domain. Qualitatively, solution of the linear nonadiabatic problem yields growth rate, $\gamma = -\Im\omega_c$, where ω_c is the complex eigenfrequency. Since the linear theory is relatively easy and credible, it is natural to ask whether γ-values could be used as a predictors of mode amplitudes.

To get an insight to the meaning of these values it is useful to consider the following well-known expression.

$$\gamma = \frac{W}{2\omega I}, \tag{3}$$

where W denotes the net rate of mode energy gain due to nonadiabatic effects and I mode inertia. Although W is a measure of driving efficiency, the order-of-magnitude differences in γ's between various modes are caused by the differences in I (the usual normalization of eigenfunctions at the surface is assumed).

In Figure 1 we show the behavior of the integrand of W and a chosen eigenfunction for unstable radial modes in a β Cep star model. The instability results from dominance of the driving occurring in the region of the metal opacity bump at $r \approx 0.97R$ over the damping occurring below (see e.g. Dziembowski & Pamyatnykh, in this volume). The values of γ for these as well as nonradial modes of $l = 1$ and 2 degrees are shown in Figure 2 are plotted against mode frequency. In the same figure also shown are normalized growth rates defined by Stellingwerf (1978), as follows

$$\eta = \frac{W}{\int_0^1 |\frac{dW}{dx}| dx}. \tag{4}$$

This parameter is a good measure of the instability robustness and it is independent of the mode inertia

Both γ and η depend primarily on frequency. Difference in the behavior of this two quantities follows mainly from the rapid decrease of I with increasing frequency. The pattern seen in this figure are not unlike ones found in less luminous Cepheid and RR Lyrae star models, in which also first three radial modes are found unstable with the p_2 having the largest γ and p_3 being only marginally unstable.

Figure 3 shows the behavior of the growth rates in two δ Scu star models. These two models are more evolved than the β Cep model used in the previous figures. In these more evolved models all unstable nonradial modes are of dual, gravity and acoustic, character which causes that their spectra are denser than those of radial modes. The complicated behavior of γ, which for the nonradial modes exhibit local minima and maxima reflects effect of a partial mode trapping. Modes trapped in the acoustic cavity are characterized by lower I and

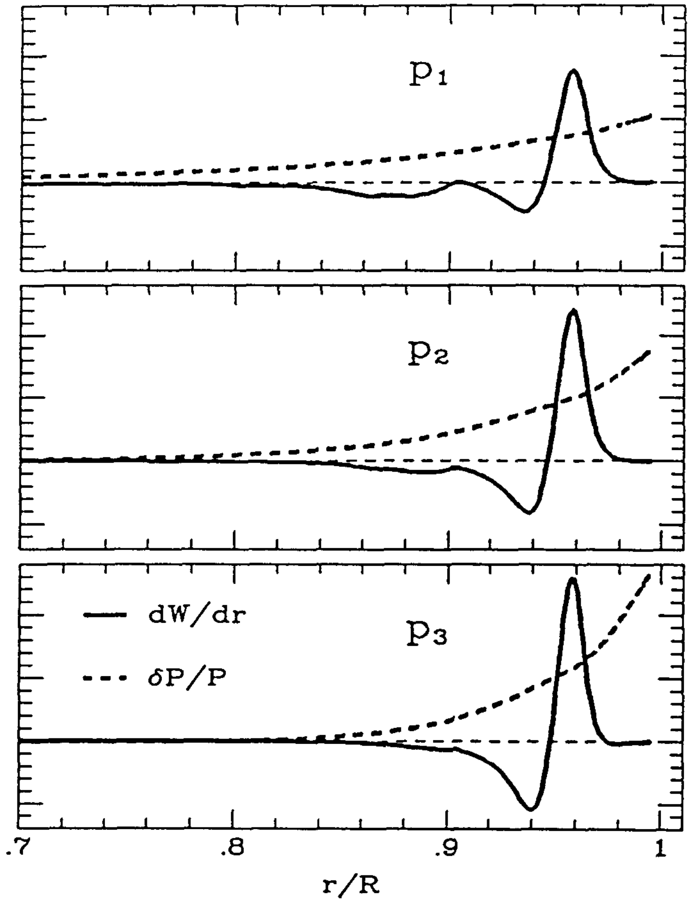

FIGURE I The differential work integral dW/dr (in arbitrary units) and the eigenfunction, $\delta P/P$, for the three first radial modes in a β Cephei star model, characterized by the following parameters $M/M_\odot = 12$, $\log T_{\rm eff} = 4.376$, $\log L/L_\odot = 4.208$, $Z = 0.03$, $X_0 = 0.7$, $X_c = 0.222$.

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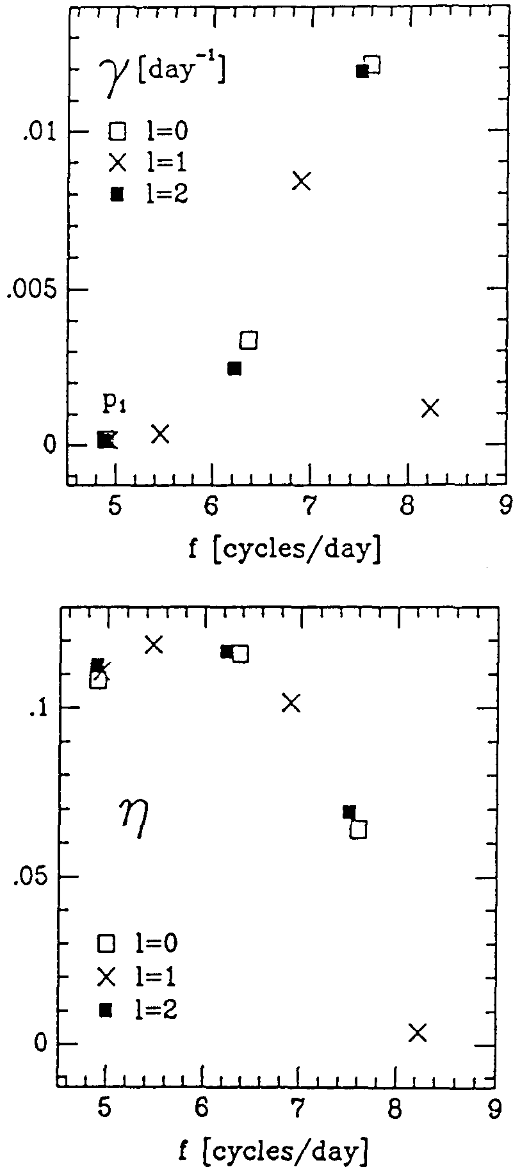

FIGURE II The growth rate, γ, and normalized growth rate, η, plotted against cyclic frequency, $f = \omega/2\pi$, for unstable modes of low spherical harmonic degree, l, in the same model as used in Figure I

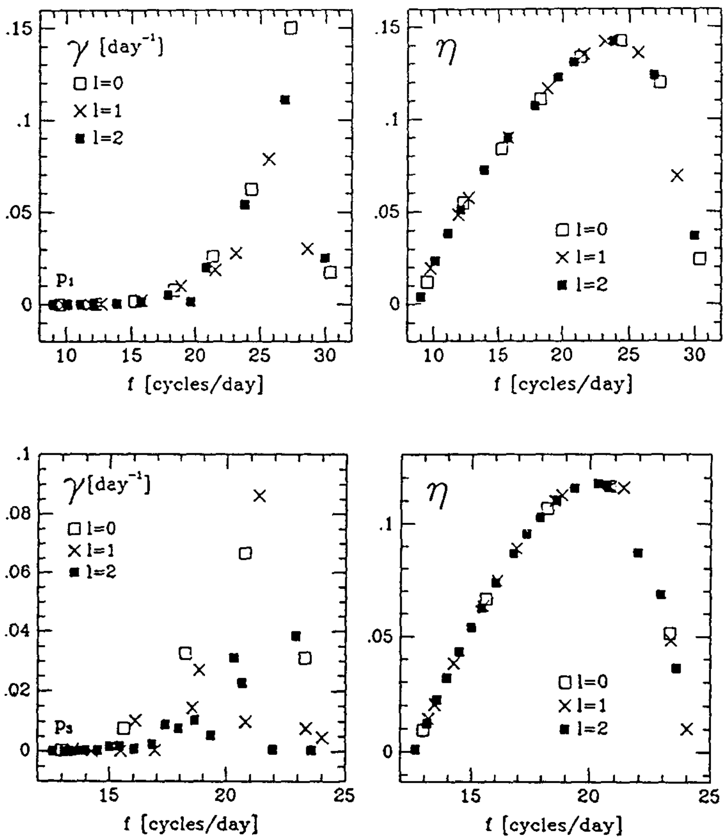

FIGURE III The growth rate, γ, and normalized growth rate, η, plotted against cyclic frequency, $f = \omega/2\pi$, for unstable modes of low spherical harmonic degree, l, in two models of δ Scu stars. The model used in the upper plots has the following parameters: $M/M_\odot = 2$, $\log T_{\rm eff} = 3.878$, $\log L/L_\odot = 1.343$, $Z = 0.02$, $X_0 = 0.7$, $X_c = 0.060$ The model used in the lower plots has the following parameters: $M/M_\odot = 2$, $\log T_{\rm eff} = 3.890$, $\log L/L_\odot = 1.490$, $Z = 0.02$, $X_0 = 0.7$, $X_c = 0$

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consequently higher γ. Possibility that the trapping in the outer cavity is a crucial factor in mode selection, was first suggested by Winget *et al.*(1981) in the context of ZZ Ceti stars, a similar effect in δ Scu was discussed by Dziembowski & Królikowska (1990). The dual character modes appear also more evolved β Cep star. The frequencies of such modes in both types of variable stars depends on the details of the convective core evolution.

The instability in δ Scu and β Cep star models continues to higher l-values. There is indeed a very large number of modes which are simultaneously unstable. This gives a ground to the hope that, some day, many of them will be detected. Measurement of their frequencies would then enable us to make a detailed seismic
There is indeed a very large number of modes which are simultaneously unstable. This gives a ground to the hope that, some day, many of them will be detected. Measurement of their frequencies would then enable us to make a detailed seismic sounding of the evolving stellar interiors. This is not certain, however. Complex nonlinear processes determine whether an unstable mode will be present in the terminal development of the instabilities, what will be its amplitude and whether it will be constant or subject, rather, to modulation. Only after considering specific nonlinear effects it may be possible to decide which, if any, of the two growth rates is a useful proxy for the amplitude.

4. AMPLITUDE EQUATION FORMALISM

Nonlinear equations for stellar oscillations are very difficult to solve. Fortunately, in many important applications the amplitude equation formalism applies, which reduces the problem from a partial differential equation system in time and three spatial variables to a system of ordinary differential equations in time. Basic assumptions underlying this simplification are, validity of the power expansion in the oscillation amplitudes, and $|\gamma| \ll \omega$ for all involved modes. It is remarkable that even for Cepheid models, the solutions of the amplitude equations agree quite well with exact nonlinear models (Buchler and Kovács 1986), in spite of relatively large amplitudes and γ's. The details of the derivation may be found for instance in papers by Vandakurov (1981) and Buchler and Goupil(1984).

4.1 Modal selection in the absence of resonances

The coupling between modes in the absence of resonances occurs only through a collective saturation of the driving mechanism. The amplitude equations have an exceptionally simple form in this case .

$$\frac{dA_k}{dt} = \tilde{\gamma}_k A_k, \tag{5}$$

where

$$\tilde{\gamma}_k = \gamma_k (1 + \sum_j \alpha_{kj} A_j^2) \tag{6}$$

We assume $\gamma > 0$ and $\alpha < 0$ for each j and k. Monomode solutions

$$A_n = \frac{1}{\sqrt{-\alpha_{nn}}} \tag{7}$$

always exists in such conditions, but they are stable only if

$$\chi_{kn} = \frac{\alpha_{kn}}{\alpha_{nn}} > 1 \tag{8}$$

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for each k. There may be more than one stable monomode solution, like in the EO domain of the Cepheid instability strip where either the fundamental mode or the first overtone pulsation is possible (Stellingwerf, 1975). Choice of the mode depends in such a situation on the direction of stellar evolution.

Double mode pulsation occur represent a stable solution of Eq.(4) if

$$\chi_{1,2} < 1, \qquad \chi_{2,1} < 1 \tag{9}$$

and

$$\chi_{k,1}(1-\chi_{1,2}) + \chi_{k,2}(1-\chi_{2,1}) + \chi_{1,2}\chi_{2,1} > 1 \tag{10}$$

Ishida (1990) considered the case of three-mode coupling, which is the largest number of modes considered so far. He found a domain in the parameter space, where no stable fixed point solution exists and shown that chaotic solutions are possible there.

Investigation of the collective saturation involving much larger number of modes is a needed research project. It should be stressed that even in the case of classical pulsating stars there is a large number of high-l modes that are unstable (Dziembowski, 1977; Osaki, 1977). Presence of such modes cannot be directly detected, but they may play a role in the final amplitude development of instability. One difficulty in treating the multiple mode saturation is the fact that coefficients α involve up to third derivatives of opacity, $\kappa(\rho, T)$. Standard opacity tables are not suitable for evaluation derivatives of this high order.

4.2 Resonant mode coupling

Coupling to a stable mode is an alternative way of amplitude limitation. Here, we shall limit ourselves to considering the simplest and most important case of two-mode resonance case, assuming that for the two modes, we have

$$\omega_2 = 2\omega_1 + \Delta\omega, \tag{11}$$

where $\Delta\omega$ is a small quantity. The amplitude equations for this case are

$$\frac{dA_1}{dt} = \gamma_1 A_1 + C_1 A_2 A_1 \cos\Phi, \tag{12}$$

$$\frac{dA_2}{dt} = \gamma_2 A_2 - C_2 A_1^2 \cos\Phi, \tag{13}$$

$$\frac{d\Phi}{dt} = \Delta\omega + (\frac{C_2 A_1^2}{A_2} - 2C_1 A_2)\sin\Phi, \tag{14}$$

where $\Phi = \Delta\omega t + \Phi_2 - 2\Phi_1$. The coupling coefficients, C are integrals involving triple products of the eigenfunctions, two for mode 1 and one for mode 2. In particular, both coefficients

$$C \propto \int Y_{(1)}^2 Y_{(2)}^* \sin\theta d\phi d\theta. \tag{15}$$

There are two clearly distinct situations. One describing the case of unstable lower frequency mode, $\gamma_1 > 0, \gamma_2 < 0$, is refered to as the 2:1 resonance. The opposite one, $\gamma_2 > 0, \gamma_1 < 0$, is called the 1:$\frac{1}{2}$ or parametric resonance. The latter is much more likely to occur. Note, for instance, that in this case an unstable may be coupled to modes of various l, while in the 2:1 resonance case only to another $l = 0$ mode.

Equations (11)–(13) admit the following constant-amplitude solution

$$A_1 = \sqrt{-\frac{\gamma_1\gamma_2}{C_1C_2}(1+q^2)}, \tag{16}$$

$$A_2 = \sqrt{\frac{\gamma_1^2}{C_1^2}(1+q^2)}, \tag{17}$$

$$\tan\Phi = -q, \tag{18}$$

where $q = \Delta\omega/(\gamma_2 + 2\gamma_1)$.

Nonlinear correction to frequencies are given by

$$\frac{d\Phi_1}{dt} = C_1 A_2 \sin\Phi \quad \text{and} \quad \frac{d\Phi_2}{dt} = C_2\frac{A_1^2}{A_2}\sin\Phi. \tag{19}$$

They imply that frequency synchronization occurs, so that even if $\Delta\omega \neq 0$ the 2:1 resonance manifest itself only through an enhancement of λ_{11+}. Information about $\Delta\omega$ is contained in the phase difference $\phi_{11+} - 2\phi_1$ (*conf.* Equation (1)).

These fixed point solutions are not always stable. The necessary stability condition is $2\gamma_1 + \gamma_2 < 0$. In the case of the 2:1 resonance this also the sufficient condition. In the case of the $1{:}\frac{1}{2}$ resonance there is an additional condition that sets a lower bound on $|\Delta\omega|$. In the unstable cases, amplitude limitation may still occur, but in a time-dependent form. The amplitudes and frequencies are then subject to periodic or chaotic modulation occurring on much shorter time scale than the evolutionary one. Regardless its form, the resonant amplitude limitation, by preventing saturation of the driving mechanism, resonances promotes multimode pulsation.

5. THE ROLE OF RESONANCE IN CLASSICAL PULSATING VARIABLES

In most cases only one mode is observed in these stars. Rapid changes in pulsation characteristics are only seldom observed. Numerical models of radially pulsating stars reproduce light and radial velocity curves for Cepheids and RR Lyr stars, in a satisfactory way. This agreement points out to the saturation of driving mechanism as the most important nonlinear effect. Effect of resonances has been invoked only to account for anomalous behaviors.

The 2:1 resonance between the fundamental mode and a higher mode has been proposed by Woltjer (1935) as the cause for the occurrence of a secondary bump in light curves of certain Cepheid. This idea was fully confirmed in modern times (Simon & Schmidt, 1976; Klapp *et al.*, 1985).

It has been suggested (Dziembowski & Kovács, 1984; Buchler & Kovács, 1988) that the same resonance may be responsible for the occurrence of double mode pulsation in certain Cepheids and RR Lyr. This could happen if the mode, that would have been the only one present in the final amplitude state, is in the 2:1 resonance with a stable mode and therefore cannot attain its full amplitude needed to saturate the driving mechanism. In such a situation, another unstable mode may grow. Moskalik (1986) proposed that a periodic modulation of light curves, observed in some RR Lyrae and known as the Blazhko effect may be caused by a similar resonance if the constant amplitude solution is unstable.

Chaotic behavior in models of luminous pulsating stars associated with occurrence of higher-order resonance was discovered by Kovács and Buchler (1988). The relevant resonances are of *half-integer* type viz., $3\omega_1 \approx 2\omega_2$. These authors propose that the deterministic chaos they found in the models is the cause of the irregular variability of the W Vir and RV Tau stars.

6. EFFECTS OF THE PARAMETRIC RESONANCE

It has been shown (Dziembowski & Królikowska, 1985) that, in Main Sequence stars, a parametric g-mode excitation occurs at very low (mmag) amplitudes of the unstable acoustic modes. The general case of three mode resonance of the type

$$\omega_a = \omega_{g1} + \omega_{g2} + \Delta\omega, \tag{20}$$

was considered in that paper. However, the two-mode formalism developed in Section 4 may be used here, because in most important cases we have

$$l_{g1} \approx l_{g2} \gg l_a, \tag{21}$$

and

$$\omega_{g1} \approx \omega_{g2} \approx \frac{\omega_a}{2}. \tag{22}$$

Excitation occurs at amplitude, A_a, which is nearly equal to the equilibrium amplitude. Thus, we get from Equation (18)

$$A_a = \sqrt{\frac{\gamma_g^2}{C_g^2}[1 + (\frac{\Delta\omega}{(\gamma_a + 2\gamma_g})^2]}. \tag{23}$$

This amplitude may be very small indeed, because the large number of potential resonant pairs allows a fine tuning of the resonance, $|\Delta\omega| \ll \omega$, and because the g-mode are very nearly adiabatic, $|\gamma_g| \ll \omega_g$. In such a situation, only evaluation of a probability distribution for A_a is feasible.

Star rotation further reduces the acoustic mode amplitude at the onset of parametric instability. Lifting the m-degeneracy of frequencies results in increasing chances of close resonances and consequently in reducing expected A_a. The effect becomes significant already at equatorial velocities of some 40 km/s (Dziembowski *et al.*, 1988).

In stars near ZAMS, for p_1, p_2 and p_3 radial modes and corresponding low-l nonradial modes, the constant amplitude solutions are most probable. In this situation, $-\gamma_g \gg \gamma_a$ and A_a is virtually independent of γ_a. There is a fair probability of periodic limit cycle and as well as chaotic solutions. The time scale of the amplitude and period variations is of the order of γ_g^{-1}.

For higher order modes and for all acoustic modes in more evolved objects, there is an increasing probability of coupling to g-modes trapped in the inner cavity. In such situations we have $-2\gamma_g < \gamma_a$, which implies not only that the constant amplitude solutions are unstable but that the interaction does not halt the amplitude growth. It may be expected that the amplitude is ultimately limited as a result of interaction with *next generation* of parametrically excited g-modes, but a theory of such multi-mode interactions remains to be developed. If this effect indeed governs nonlinear development of acoustic mode instability then, at this stage, we can only predict that modes which have relatively low amplitudes in the inner g-mode propagation zone have the best chances to attain large surface amplitudes. These are radial modes and nonradial modes that are partially trapped in the acoustic cavity.

7. NONLINEAR PHENOMENA IN MULTIPERIODIC VARIABLES

7.1 δ Scu stars

Most of these stars are low amplitude variables. Furthermore, there are many objects in the δ Scu domain of the H-R diagram which variability was searched for and was not detected. There is an evidence for period and amplitude changes occurring on the time scale of years (Breger, 1992) in several objects of this type.

Part of δ Scu stars are well studied multimode pulsators. In Table I I compiled information about amplitude spectra for few such objects. Mode identification given in this table, in most cases must, be regarded as tentative. The exception is GX Peg, in which case the identification is based on comprehensive model calculations (Goupil *et al.*,1992). Results of that work, as pointed out by its authors, give certain support for the idea of preferential excitation of modes trapped in the envelope.

Spectra of the remaining objects are much harder to understand. In particular, in 4 CnV, the only pair of modes that on the basis of the period ratio could be identified with radial modes are the two of the highest frequencies. Presence of *only* two, at most, radial modes in this broad-frequency spectrum cannot easily be explain in terms of nonlinear effects discussed in previous sections. It is also difficult to understand absence of $l = 1$ modes in the 44 Tau oscillation spectrum. I suppose, however, that an alternative interpretation of this spectrum might be possible.

Discovery of three small amplitude modes, in addition to the fundamental and first overtone pulsations, in the spectrum of AI Vel (Walraven & Balona, 1992) is a surprise. I used to believe, that the two radial modes, owing their large amplitudes, fully saturated the opacity mechanism. This discovery is a good news for asteroseismology, though, interpretation of the present spectrum is not easy.

β Cephei stars

There are many multimode objects of this type. With possible exception is 12 Lac, in all β Cep frequencies of the detected modes in individual stars occur in ranges that are significantly narrower than the distance between consecutive radial modes. Observational identification of the spherical harmonic degrees, do not reveal any obvious preference to $l = 0$ values. These properties seem to favor the saturation as the dominant nonlinear effect.

Amplitude are most often small and their rapid changes have been reported. The best known example is *Spica* (α Vir), observed as a pulsating star in 1960's

TABLE I Amplitude spectra for some δ Scu stars

star	f[c/day]	P[hours]	A[mmag]	identification
4 CnV	5.0475	4.755	7.7	$l \neq 0$
$\log T_{\rm eff} = 3.839$	5.8508	4.102	9.1	$l \neq 0$
reference:	6.9763	3.440	5.1	$l \neq 0$
Breger *et al.*	7.3778	3.253	5.2	$l = 0$, p_4
(1990)	8.5950	2.792	12.8	$l = 0$, p_5
	6.898	3.479	27.8	$l = 0$, p_1
44 Tau	7.006	3.426	21.2	$l = 2$, p_1
$\log T_{\rm eff} = 3.833$	7.304	3.286	5.9	$l = 3$, p_2
reference:	8.960	3.678	8.6	$l = 0$, p_3
Mantegazza *et al.*	9.116	2.633	14.7	$l = 2$, p_2
(1992)	9.562	2.510	6.1	$l = 3$, p_1
	11.520	2.083	6.8	$l = 0$, p_3
AI Vel	8.9627	2.678	195.0	$l = 0$, p_1
$\log T_{\rm eff} = 3.875$	9.1376	2.626	6.5	$l \neq 0$
reference:	11.5998	2.069	144.0	$l = 0$, p_2
Walraven & Balona	15.9725	1.503	8.9	$l \neq 0$
(1992)	22.5218	1.066	10.8	$l = 0$, p_6
θ^2 Tau	13.2300	1.8141	6.6	$l = 0$, p_2 or p_3
$\log T_{\rm eff} = 3.903$	13.4807	1.7803	2.6	$l = 2$
reference:	13.6956	1.7524	4.5	$l = 1$
Breger *et al.*	14.4176	1.6646	2.7	$l = 1$
(1989)	14.6165	1.6420	1.2	?
GX Peg	16.17	1.484	1.62	$l = 0$, p_2
$\log T_{\rm eff} = 3.892$	19.66	1.221	1.78	$l = 1$, $m = 1$, p_3
reference:	19.91	1.205	2.08	$l = 0$, p_3
Michel *et al.*	20.52	1.170	1.80	$l = 1$, $m = 0$, p_3
(1992)	20.78	1.155	2.18	$l = 1$, $m = -1$, p_3

and showing no detectable pulsation in 70's and early 80's (Sterken *et al.*, 1986. The occurrence of an amplitude modulation on the time scale of years cannot be used as an evidence for the role of resonances. As discussed in Section 4.1, the amplitude changes may occur if more than two modes is involved in the saturation of the driving mechanism.

Rapidly oscillating Ap stars

Kurtz (1982) demonstrated that variability in these stars is due to excitation of high-order p-modes of $l = 1$ which are symmetric about magnetic field axis. Recently, Kurtz *et al.* (1992) showed that in the case of Ap star HR3831, there is a significant contamination with harmonic of other degrees.

The driving mechanism is not yet understood. A peculiarity of the nonlinear behavior of Ap star oscillations is the occurrence of harmonics in their power spectra already at millimagnitude amplitudes. This is in stark contrast to δ Scu stars where such nonlinearities are seen only at amplitudes of tenths of a magnitude. We do not know why it is so, but it seems likely that this property of Ap star oscillations might be the clue to finding the excitation mechanism.

Variable white dwarfs

These are best studied multiperiodic variable stars. An unprecedented amplitude resolution was reached in observations made with the *Whole Earth Telescope*.

In the power spectrum of PG 1159–035 Winget *et al.* (1991) identified 101 peaks as consecutive modes in $l = 1$ and 2 multiplets. The period- spacing clearly reveals the existence of at least two trapping cavities (a consequence of the element stratification). The lack of correlation between the spacing and the amplitudes seems to disprove, in this case, the hypothesis of the preferential excitation of the modes trapped in the outer cavity. There is now a a firm evidence for amplitude changes in this star, occurring on the time scale of years.

Power spectra for some oscillating white dwarfs such as G117-B15A (Kepler *et al.*, 1992) or BPM31594 (O'Donoghue *et al.*, 1992) are dominated by nonlinear effects - harmonics, amplitude modulation. Such objects are less rewarding from the point of view of asteroseismology but they are source of valuable information about nonlinear effects in stellar oscillations that we still have to learn to use it.

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