Note on a Theorem connected with the area of a $2n$-sided polygon.

By THOMAS MUIR, M.A., F.R.S.E.

The theorem is:—*If* $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_{2n}$ *be the middle points of the sides of any convex polygon* $A_1A_2A_3\ldots A_{2n}$ *then as regards areas*

$$\alpha_1\alpha_2\ldots\alpha_{2n} = \tfrac{1}{2}A_1A_2\ldots A_{2n} + \tfrac{1}{4}A_1A_3\ldots A_{2n-1} + \tfrac{1}{4}A_2A_4\ldots A_{2n}.$$

The following proof depends only on the theorem that the line joining the points of bisection of two sides of a triangle cuts off a triangle equal in area to a quarter of the original triangle. For convenience in writing, let us take the case where $n = 4$. Then

$$\left.\begin{aligned} &\tfrac{1}{4}A_1A_3A_5A_7 = \\ &\qquad \tfrac{1}{4}(A_1A_2A_3\ldots A_8 - A_1A_2A_3 - A_3A_4A_5 - A_5A_6A_7 - A_7A_8A_1) \\ \text{and }\ &\tfrac{1}{4}A_2A_4A_6A_8 = \\ &\qquad \tfrac{1}{4}(A_1A_2A_3\ldots A_8 - A_2A_3A_4 - A_4A_5A_6 - A_6A_7A_8 - A_8A_1A_2) \end{aligned}\right\}$$

$$\left.\begin{aligned} \therefore \tfrac{1}{4}A_1A_3A_5A_7 \\ + \tfrac{1}{4}A_2A_4A_6A_8 \end{aligned}\right\} = \tfrac{1}{2}A_1A_2A_3\ldots A_8 + \alpha_1\alpha_2\alpha_3\ldots\alpha_8 - A_1A_2A_3\ldots A_8$$

and $\therefore \alpha_1\alpha_2\alpha_3\ldots\alpha_8 = \tfrac{1}{2}A_1A_2A_3\ldots A_8 + \tfrac{1}{4}A_1A_3A_5A_7 + \tfrac{1}{4}A_2A_4A_6A_8$
as was to be proved.

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Spherical Geometry.

By R. E. ALLARDICE, M.A.

The object of this paper is to bring together the principal properties of figures described on the surface of the sphere that can be established without the use of Solid Geometry or of Trigonometry.

The following properties of the spherical surface, which correspond to the definitions and axioms in Plane Geometry, are assumed. They may be considered as derived from one's general notion of the sphere.

a. On the surface of the sphere certain circles (great circles) can be drawn, which are closely analogous to straight lines in a plane. These great circles are all equal in circumference.

b. Through any two points one great circle can be drawn, and in general only one; if the distance between the two points be half a great circle, any number may be drawn.

c. Any two great circles intersect in two points (called antipodal points), the distance between which is half a great circle.