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ASPECTS OF GLOBULAR HIGHER CATEGORY THEORY

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The thesis is in the field of higher category theory, a new branch of mathematics which emerged over the last 20 years from the demands of several seemingly distinct areas such as mathematical physics, computer science, mathematical logic and algebraic topology.

The thesis consists of six chapters including an Introduction and a Conclusion. The main results are presented in Chapters 2–5. Chapters 2 and 3 are concerned with the definitions of ω-functors and higher transformations in the context of Batanin's theory of ω-categories (see [1]). In Chapter 4, a natural algebraic structure on the complex of these higher transformations is studied. In Chapter 5, an important definition of $(\infty; n)$-category, in the purely algebraic and globular context, is presented.

In summary, the achievements are as follows.

- A proposed definition of higher dimensional transformations for Batanin's ω-categories is presented (see [2]).
- A natural algebraic structure on the complex of these higher transformations is shown to exist. (The details are in a paper on the arXiv, 'ω-Operads of coendomorphisms for higher structures', which is to be submitted for publication.) It is conjectured that this structure is a weak ω-category structure and a reasonable strategy for proving this conjecture is outlined. It is important to understand that the existence of such a weak ω-category structure is one of the main unsolved hypotheses in higher category theory. My results show that the problem is reduced to a very concrete question in abstract homotopy theory. Abstract homotopy theory has developed powerful methods to approach such

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questions. Using these methods, we expect now that the original hypothesis will be solved in the not so distant future.

- The first purely algebraic definition of $(\infty; n)$-category is given. (The details are in a submitted paper, 'Algebraic definition of (∞, n)-categories'.) This is important because of the connections this definition has with another big and still unsolved hypothesis in higher category theory: the so-called 'Grothendieck homotopy hypothesis'. Again, my result opens the possibility that abstract homotopy theory methods will be applicable to finish the proof of the Grothendieck hypothesis.

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