

CORRIGENDUM

Axisymmetric global gravitational equilibrium for magnetized, rotating hot plasma - Corrigendum

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The authors wish to thank Antoine Cerfon and Dimitrios Andriopoulos of the Courant Institute at New York University for pointing out an error in our manuscript (Catto, Pusztai & Krasheninnikov 2015). This mistake only affects the results when a toroidal magnetic field is present. It arises because our Grad–Shafranov equation (3.11) should be corrected to read

$$\frac{d^2H}{d\mu^2} + \frac{\alpha(\alpha+1)}{1-\mu^2}H = \alpha \left\{ \beta \left[\frac{g}{2}H^{-1/\alpha} - \omega^2(1-\mu^2)H^{2/\alpha} - (\alpha+2) \right] - \frac{(\alpha+1)b^2e^{-\chi}}{(1-\mu^2)H^{2/\alpha}} \right\} H^{1+4/\alpha}e^\chi, \quad (3.11)$$

where the only change is to insert the missing $e^{-\chi}$ multiplying the toroidal magnetic field term proportional to b^2 since it cannot have any density dependence. There is also a typographical error in (3.6) since there, B_o should be replaced by B_{po} , with $B_{po} = -\alpha\psi_o/R_o^2$. The equations shown here and the material in quotes are the corrected content. The references remain the same as in the publication.

The preceding change in the Grad–Shafranov equation (3.11) changes (3.17) and the short sentence that follows to

$$\alpha + 2 = \frac{C^2b^2 - \beta\omega^2e^{-g-\omega^2(1-C^{-1})}}{C^3 + C^2b^2 + C\beta e^{-g-\omega^2(1-C^{-1})}}. \quad (3.17)$$

‘Consequently, we expect $\alpha + 2 < 0$ unless $C^2b^2 > \beta\omega^2e^{-g-\omega^2(1-C^{-1})}$, with $C = 1$ if $g = 0$ ’.

In §4 the only change occurs in the penultimate sentence that becomes the following ‘For example, we do not consider the limit $C^2b^2e^{g+\omega^2(1-C^{-1})} > \beta\omega^2 \sim \beta g/2 \gg \beta \gg 1$, which requires a toroidal magnetic field but allows the magnetic field to vanish at infinity’.

There are a number of changes in §5 where we keep the toroidal field. Equation (5.1) through to the end of the paragraph including (5.3) should be corrected to read:

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$$\begin{aligned}
 &(\alpha - 1)(\alpha + 2) \int_0^1 d\mu H + \alpha(\alpha + 1)b^2 \int_0^1 d\mu H^{1+2/\alpha} \\
 &= \alpha\beta \int_0^1 d\mu (1 - \mu^2)H^{1+4/\alpha} \left[\frac{g}{2}H^{-1/\alpha} - \omega^2(1 - \mu^2)H^{2/\alpha} - (\alpha + 2) \right] e^x. \tag{5.1}
 \end{aligned}$$

‘Using the vacuum magnetic field solution for the $\alpha = -2$ root of $H = 1 - \mu^2$, we find that the $b^2 \sim \beta g \sim \beta\omega^2 \ll 1 \sim \beta$ corrections to that result must satisfy’

$$\frac{2(\alpha - 1)(\alpha + 2)}{3} + \alpha(\alpha + 1)b^2 = \alpha\beta \int_0^1 d\mu \left[\frac{g}{2}\sqrt{1 - \mu^2} - \omega^2 - (\alpha + 2) \right] e^{-g[1 - \sqrt{1 - \mu^2}]}. \tag{5.2}$$

‘For $g \ll 1$ only a weak density departure from cylindrical symmetry is allowed, giving’

$$\alpha \simeq -2 + [b^2 + \beta(\pi g/8) - \beta\omega^2]/(1 + \beta). \tag{5.3}$$

‘Based on (3.17) we expect $b^2 > \beta\omega^2$ is required for a solution that keeps $0 > \alpha > -2$, thereby making the poloidal magnetic field fall off at large distances and pinch in slightly at the equatorial plane. Indeed, in this small g limit, finite $b^2 > \beta\omega^2$ seems to be required to find a numerical solution for $\alpha > -2$ ’.

Next, equation (5.4) and the remainder of the paragraph that it appears in should be corrected to read as follows

$$\alpha \simeq -2 + b^2 + \beta(g - 2\omega^2)(\pi/8g)^{1/2}. \tag{5.4}$$

‘These results are the same as in Catto & Krasheninnikov (2015) except the toroidal field term has been retained and it further enhances the pinching in of the flux surfaces at the equatorial plane. The disk thickness from $e^{-g\mu^2/2}$ is as given by (4.10). Result (5.4) is verified by a numerical solution which is imperceptibly different from that shown in figure 6(a,b) for $\beta = 0.001$, $g = 100$, $\omega^2 = 40$ and $b^2 = 0.05$. Analytically we find $\alpha = -1.949$ and $\Delta/R = 0.14$ and a sensitive numerical solution is found for $\alpha = -1.951214$ and $\Delta/R = 0.12$. We only need $b^2 > \beta\omega^2$ in this limit to satisfy $\alpha + 2 > 0$ from (3.17) due to the exponential g factor and the use of the vacuum solution for H away from the equatorial plane. Equation (5.4) remains valid in the strict Keplerian case $g = 2\omega^2 \gg 1$, where we can evaluate the integrals in (5.2) a little more carefully to find $\alpha + 2 \simeq b^2/[1 + \beta(\pi/2g)^{1/2}]$, which is consistent with (5.4) when $\beta \ll g^{1/2}$. The numerical solution confirms that this strict Keplerian case is a valid limit’.

‘Catto & Krasheninnikov (2015) also find a disk solution localized to the equatorial plane by considering $g > 2\omega^2$ and then allowing $g - 2\omega^2 \gg 1 \gg \beta$, so that the exponential dependence e^x in the Grad–Shafranov equation provides the desired localization about $\mu = 0$ for the assumed small β terms. Therefore, we modify their treatment to find disk solutions with strong poloidal variation, but with the toroidal magnetic field retained to satisfy (3.17). This constraint was not considered in Catto & Krasheninnikov (2015). To begin, we need to find a solution in the disk different from the cylindrical solution $H = (1 - \mu^2)^{-\alpha/2}$ valid outside the disk. We find this inner disk solution by considering the approximate Grad–Shafranov equation’

$$\frac{d^2H}{d\mu^2} + \alpha(\alpha + 1)b^2e^{-(\alpha+2)(1-H)/\alpha} \simeq \alpha\beta \left[\frac{g}{2} - \omega^2 - (\alpha + 2) \right] e^{(g-2\omega^2)(1-H)/\alpha}, \tag{5.5}$$

‘where now both rotation and gravity enter the exponential density dependence, for which we use’

$$H^{1/\alpha} = e^{(1/\alpha)\ln H} \simeq e^{(1/\alpha)(H-1)} \simeq 1 + (H - 1)/\alpha + \dots \tag{5.6}$$

‘When $g - 2\omega^2 \gg 1 \sim \alpha + 2 > 0$ we obtain strong exponential decay away from the equatorial plane. Very near the equatorial plane $d^2H/d\mu^2 < 0$ in (5.5) if $g/2 > \omega^2 + \alpha + 2 > 0$ with $\alpha < 0$, but once the right-hand side of (5.5) decays away then the b^2 term can grow. For $\alpha + 2 \sim 1$ and b^2 not too large this growth occurs far enough away from the equatorial plane that the b^2 term may be ignored in the disk. These observations suggest, in agreement with Catto & Krasheninnikov (2015), that solutions that are strongly localized to the equatorial plane in the presence of gravity are not possible for $g < 2\omega^2$ and $0 > \alpha > -2$ since the rotation is too strong for the plasma to be gravitationally confined’.

‘Continuing as in Catto & Krasheninnikov (2015), we multiply (5.5) (with the b^2 term ignored) by $dH/d\mu$ and integrate from $H = 1$ (at $\mu = 0$) to $H < 1$ (for $\mu^2 > 0$) to find for $\alpha + 2 \sim 1$ ’

$$\frac{dH}{d\mu} \simeq \alpha \sqrt{\beta [1 - e^{(g-2\omega^2)(1-H)/\alpha}]}, \tag{5.7}$$

‘where we select the negative root to make $dH/d\mu < 0$. Using $\int dx/\sqrt{1 - e^{-x}} = 2 \tanh^{-1} \sqrt{1 - e^{-x}}$ we obtain’

$$\frac{g - 2\omega^2}{\alpha} (H - 1) = x = -\ln[1 - \tanh^2(\sigma\mu/2)] \rightarrow \begin{cases} (\sigma\mu/2)^2 + \dots & \sigma|\mu|/2 \ll 1 \\ \mp\sigma\mu - \ln 4 + \dots & \sigma|\mu|/2 \gtrsim 1, \end{cases} \tag{5.8}$$

‘where $\sigma \equiv (g - 2\omega^2)\sqrt{\beta}$ and the upper (lower) sign is for $\mu > 0$ ($\mu < 0$). A solution strongly localized at the equatorial plane is found for $-x = (g - 2\omega^2)(1 - H)/\alpha \gg 1$ that results in only a small departure from the gravity free solution $H = (1 - \mu^2)^{-\alpha/2}$ that remains an adequate approximation in the outer region. The behaviour $x \approx \mp\sigma\mu \approx \mp\sigma z/R$ implies a disk width $\Delta = R/\sigma$ so that $\sigma \gg 1$ is required’.

‘Using (5.7) and (5.8) on the right-hand side of the integral constraint (5.1), with the cylindrical solution $H = (1 - \mu^2)^{-\alpha/2}$ inserted on the left-hand side, yields the approximate result’

$$\alpha + 2 \simeq \frac{b^2}{1 + b^2} + \sqrt{\beta} - O(\beta\omega^2) \simeq \frac{b^2}{1 + b^2}. \tag{5.9}$$

‘Gravity is assumed negligible outside the disk in this $\beta \ll 1$ limit, where the solution becomes cylindrical (with $C \simeq 1$). Then (5.9) is in agreement with (3.14) provided we assume $1 \sim b^2 \gg \beta\omega^2 \sim \sqrt{\beta}$ so the outer solution is well approximated by $H = (1 - \mu^2)^{-\alpha/2}$. The plasma disk width is given by’

$$\Delta/R = 1/[\beta^{1/2}(g - 2\omega^2)] \ll 1, \tag{5.10}$$

‘requiring $1/(g - 2\omega^2)^2 \ll \beta \ll 1$. Strict Keplerian motion is not allowed in this low β thin disk limit’.

‘The new figure 7(a,b) shows the flux surfaces, density contours and H for $\beta = 0.01$, $g = 120$, $\omega^2 = 10$ and $b^2 = 1$, for which the analytic results give $\Delta/R = 0.1$ and $\alpha = -1.5$. The numerical solution gives $\Delta/R = 0.0938$ and $\alpha = -1.4715$. In (b) we also plot $H = (1 - \mu^2)^{-\alpha/2}$ for reference. In this case the density decreases with radius (since $\alpha > -1.5$). For the same parameters but with $b^2 = 1$ we find $\alpha = -1.33$ and $\Delta/R = 0.1$ versus the numerical values of $\alpha = -1.314$ and $\Delta/R = 0.094$. We have not found solutions for $\alpha \rightarrow 0$, as claimed in Catto & Krasheninnikov (2015), even for finite b^2 ’.

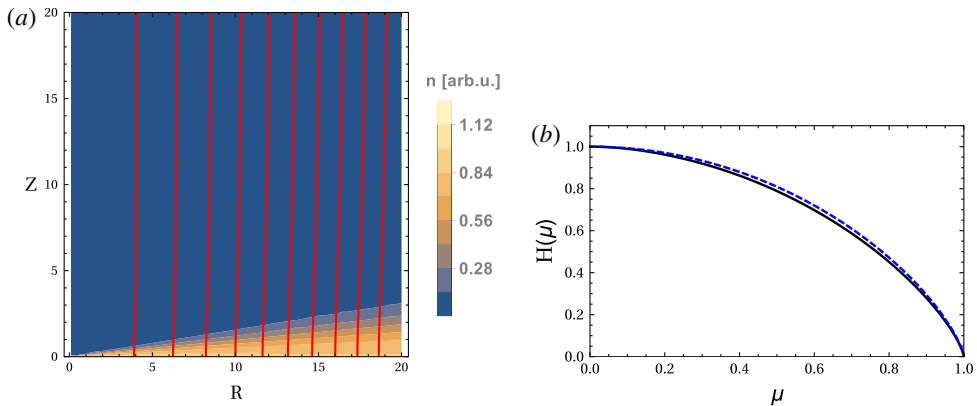


FIGURE 7. Parameters $g = 120$, $\omega^2 = 10$, $\beta = 0.01$ and $b^2 = 1$, giving $\alpha = -1.4715$. (a) Magnetic surfaces (red curves) and density (colour shading) as functions of R and Z . The density is normalized to unity at $R=1$ and $Z=0$. (b) The solution $H(\mu)$ is plotted versus μ . The numerical result is shown as the solid curve and $(1 - \mu^2)^{-\alpha/2}$ is shown dashed for reference.

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