A GENERALIZATION OF THE CORS METHOD TO DETERMINE CEPHEID RADII: THEORY AND APPLICATION

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Abstract The CORS method for the empirical determination of the radii of pulsating variables (Caccin et al., 1981; Sollazzo et al., 1981) is discussed in the framework of the quasistatic approximation to the variations of the atmospheric parameters (Unno, 1965) and reformulated in a way that does not make direct use of theoretical calibrations of the photometric system in terms of model atmospheres. The radii calculated with this approach are in good agreement with those previously obtained by means of Pel's calibrations of the VBLUW system (Sollazzo et al., 1981) which lead to a period-radius relation coinciding, within the errors, with the theoretical one (Cogan, 1978) and, consequently, mass determinations consistent with pulsational masses (Cox, 1979). The method can be immediately applied to any other multicolour system, and very promising preliminary results are obtained by using recent UBV data by Gieren (1982).

INTRODUCTION

The introduction of a two-colour dependence for the physically significant atmospheric parameters in multicolour photometry of pulsating variables, has its observational basis in the existence of closed loops in the colour-colour planes, the area of which shows a definite trend with the pulsation period. In two previous papers (Caccin et al., 1981; Sollazzo et al., 1981, hereafter paper I and II, respectively) we have shown that in the case of the surface brightness, this two-colour dependence allows a more precise determination of cepheid radii by means of the CORS method. We will show here that this assumption has its theoretical basis in the quasistatic approximation (QSA) by Unno (1965), and that colour-colour loops may be used to evaluate the area of the loop surface brightness vs. colour (necessary in the CORS method) without resorting to theoretical calibrations.

THE CORS METHOD

For the sake of clarity, we will recall in the following the

https://doi.org/10.1017/S0252921100109030 Published online by Cambridge University Press

CORS method described in paper I and II. The definition of the surface brightness, s_i, is:

$$m_i - s_i - 2\underline{a} \log_{10} R = \text{const.} \qquad (1)$$

where m_i is the apparent magnitude in the bank i, a is a constant (equal to -2.5 or 1.0 according to the scale of the photometric system used) and R is the radius. Manipulating this relation, we finally obtain the following equation, hereafter defined as the CORS equation:

$$B_{i,jk} - \Delta B_{i,jk} + [R]_{jk} = 0 \qquad (2)$$

were

$$B_{i,jk} = \oint m_i \, d\, c_{jk} \qquad (3)$$

$$\Delta B_{i,jk} = \oint s_i \, d\, c_{jk} \qquad (4)$$

$$[R]_{jk} = 2\, \underline{a} \oint \log(R_\circ - K\, P \int_{\phi_\circ}^{\phi} V_r(x)\, dx)\, d\, c_{jk} \qquad (5)$$

where o is the phase, $c_{jk} = (m_j - m_k)$ is the colour index, K is the ratio of pulsational to radial velocity, P is the period and $R_\circ$ is the radius at an arbitrarily chosen phase $\phi_\circ$. Eqn. (2) can be solved to obtain the radius, $R_\circ$, if one can evaluate the term B, the area of the loop surface brightness vs. colour. In paper II, this has been accomplished, for a set of data in the Walraven system, by using the theoretical calibrations by Pel (1978) of T_{eff} and g_{eff} in terms of $(V-B)_\circ$.

We show in the next section that, with the assumption of QSA, the term ΔB is proportional to the area of the colour-colour loop, a fact which, with suitable hypotheses, may allow the computation of R in any photometric system.

QUASI STATIC APPROXIMATION

The groundwork of the QSA for the atmospheric layers of pulsating variables has been set up by Ledoux and Whitney (1961) and by Unno (1965). Within this approximation, the photosphere of these stars is described at any time by a classical hydrostatic, plane-parallel model in radiative equilibrium and LTE. Each such model is identified by only two parameters (within the assumption of constant chemical composition): i) the effective temperature, T_e corresponding to the instantaneous values of luminosities and radius, and ii) the effective gravity g_e defined by

$$g_{eff} = \frac{G\, M}{R^2} + \frac{d^2\, R}{d\, t^2} \qquad (6)$$

As a consequence, the whole sequence of states can be suitably represented by a point describing a closed loop in a two dimensional space. This loop is described by two equations in functionaly dependant on the parameter ϕ:

https://doi.org/10.1017/S0252921100109030 Published online by Cambridge University Press

$$T_{eff} = T_{eff}\ (\phi) \tag{7}$$

$$g_{eff} = g_{eff}\ (\phi) \tag{8}$$

with the condition of periodicity, $T_{eff}\ (0) = T_{eff}(1)$, and $g_{eff}(0) = g_{eff}(1)$.

Furthermore, since the emergent flux is uniquely determined by T_{eff} and g_{eff}, then any photometric quantity derived from the spectra can be expressed in terms of these two same parameters, in particular:

$$s_i = s_i(T_{eff}, g_{eff}) \tag{9}$$

$$c_{jk} = c_{jk}(T_{eff}, g_{eff}) \tag{10}$$

$$c_{hl} = c_{hl}(T_{eff}, g_{eff}) \tag{11}$$

Supposing the invertibility of these equations, we may write:

$$S_i = s_i\ (c_{jk}, c_{hl}) \tag{12}$$

and therefore the area of the loop surface brightness vs. colour (see Onnembo et al., 1984) is:

$$\Delta\ B_{i,jk} = \alpha\ .\ \oint c_{hl}\quad d\ c_{jk} \tag{13}$$

being $\quad \alpha = (\ s_i /\ \ c_{hl})_o$.

The hypothesis of invertibility of Eqns. (9), (10), (11) is satisfied if the two colour indices are independent quantities. This is true, e.g., for the pairs (V-B,B-U) in the Walraven system,(B-V,U-B) in Johnson photometry and (b-y,c_1) in Strömgren's system.

APPLICABILITY AND RESULTS

The important of this method arises from the fact that it is essentially the first successful attempt to keep full account of the well established theoretical and observational result that the surface brightness if a function of two colours. The effect of not considering this point is an underestimation of the radius, which is most sensitive at longer periods. Most of the Baade-Wesselink type methods suffer from this shortcoming. The Δ B term in Eqn. (2) expresses this improvement. It can be evaluated, at the moment, in either of the following two ways:

i) using theoretical calibrations to compute s_i from (9) and ΔB from (4)- as we did in paper II. This is the most direct and safest method. Such calibrations, which express T_{eff} and g_{eff} as a function of two suitable colours, are now available for most photometric systems. Dereddened colours are needed.

ii) computing the area of the colour-colour loop, and then estimating the proportionality constant as described in

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Onnembo et al., (1984) (paper III). Dereddened colours are not crucial to this procedure.

It is worth noting that, as a first approximation, even if ΔB is set to be zero, the radius is intrinsically better determined because of the global treatment of the data.

In the following table we present some preliminary results using procedure ii).

Star		Period	R_{BW}	R_{CORS}	Phot. System	Reference
V419	Cen	$5^{d}.51$	39.2	45.3	Walraven	Pel, 1976
β	Dor	9.84	64.6	76.4	Walraven	Pel, 1976
AD	Pup	13.59	84.3	91.6	Walraven	Pel, 1976
SV	Mon	15.23	84.1	98.6	Walraven	Pel, 1976
SZ	Aql	17.14	98.8	117.0	Walraven	Pel, 1976
RU	Sct	19.70	94.7	109.7	Walraven	Pel, 1976
U	Car	38.77	137.1	163.7	Walraven	Pel, 1976
S	TrA	6.32	44.5	55.5	Johnson	Gieren, 1982
U	Sgr	6.74	60.1	59.2	Johnson	Gieren, 1982
V496	Aql	6.81	48.0	65.2	Johnson	Gieren, 1982

APPENDIX: PRACTICAL SUGGESTIONS

Feeling the need for a clearer exposition of the practical application of the method presented here, we will outline below a step-by-step procedure for the actual computation of radii.

For a given star, a radial velocity curve, V_r, (baricentric velocity subtracted) and photometry in three passbands, i, j, k, (e.g. V, B, U) are needed. The first step is to perform (possibly interactively) some kind of fitting (Fourier, spline fit etc.) of the observed data, to obtain $V(\phi)$, $(B-V)(\phi)$, $(U-B)(\phi)$, $V_r(\phi)$, as smooth equispaced curves. These smoothed quantities are used to generate, with some reliable numerical algorithm, the integral function of the radial velocity and the derivative functions of magnitude and colours with respect to the phase. Then, the term B defined by (3) can be calculated in a trivial way; for (5), the reference phase can be profitably chosen to be the minimum of V_r. As far as ΔB (in Eqn. (4)) is concerned, it can be computed in either of the two ways discussed in the previous section. The left-hand side of Eqn. (2) is now defined as a function of R_o and can be solved to obtain the radius of the Cepheid at ϕ_o. Finally, the function $R(\phi)$ can be obtained from the integral of the radial velocity curve, and the mean radius can be unambiguously calculated.

The FORTRAN code for the complete application of the CORS method is available upon request from G.R. or C.S. (tape in VAX format).

https://doi.org/10.1017/S0252921100109030 Published online by Cambridge University Press

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