The Universality of Laws in Space and Time[1]

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Part of our folklore is that genuine laws of nature must be universal in space and time. The purpose of this note is to explicate and compare various senses of this requirement. I am not concerned to argue here that the requirement, in any one of its explicated forms, should or should not be adopted.

1. Tooley's Garden

If it is hard to state straight out exactly what is demanded by universality in space and time, Michael Tooley has provided an example of a hypothetical law which fails the requirement in a significant sense:

> All fruit in Smith's garden at any time are apples. When one attempts to take an orange into the garden, it turns into an elephant. Bananas so treated become apples as they cross the boundary, while pears are resisted by a force that cannot be overcome. Cherry trees planted in the garden bear apples or they bear nothing at all. If all these things were true, there would be a very strong case for its being a law that all the fruit in Smith's garden are apples. And this case would be in no way undermined if it were found that no other gardens, however similar to Smith's in all other respects, exhibited behavior of the sort just described. (Tooley 1977, p. 686)

In this example, the lawfulness of 'All the fruit in Smith's garden are apples' is not meant to derive from some special property unique to Smith's garden. In fact, the term 'Smith's garden' need not be understood to pick out a physical entity at all, but can be taken simply to delimit a particular space-time region R. The gist of the example is that a statement of the form 'All F's are G's' holds in the space-time R and nowhere else, and, furthermore, holds in R not as an accidental generalization, but as a law of nature.[2]

What sense of "universality in space and time" does the law in Tooley's example violate?

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2. Earman's (U)

Earman (1978) considered a number of senses in which a law could be taken to be "universal in space and time". One of these, that laws have an unrestricted range in space and time, suggests itself as a candidate for the sense in which Tooley's example fails to be universal.[3]

Before giving Earman's explication of this condition, let me establish the notation to be used in the remainder. Upper case Latin letters A, B, etc. denote structures and models of theories and laws. M(A) denotes the space-time manifold of A. R, R', R'', etc. are space-time regions. A¶R denotes the restriction of A to region R. The symbol $\simeq$ is used to indicate isomorphism, in the appropriate sense for the types of entities involved. K ranges over classes of structures. L, L' etc. are statements of putative law. $A \models L$ means that A satisfies L. Mod L denotes the class of structures which satisfy L.

For the moment, let us restrict the class K of "permissible" structures to one for which there is a single fixed space-time, so that for all $A,B \in K$, M(A) = M(B). In this context, Earman formally interpreted "unrestricted range in space and time" to mean:

(U) There is no non-empty proper subregion R such that for any A there is a B such that $B \models L$ and B¶R $\simeq$ A¶R.

The idea, in English, is that there is no region R in which, as far as L is concerned, "anything goes". For this reason, I will officially call Earman's (U) the "No Chaos" condition.

As Earman pointed out, this condition is not easily generalized to contexts in which the space-time background varies from structure to structure. Nonetheless, Tooley's example can be considered in the context of a fixed space-time background. Suppose putative law L has the intended content of 'All fruit in Smith's garden are apples'. Since L itself in no way constrains what may happen outside Smith's garden, it violates the No Chaos condition. But suppose that putative law L' expresses the idea that all fruit in Smith's garden are apples and all fruit contain seeds. Since, according to L', no region of space-time may contain seedless fruit, L' meets the No Chaos condition. Nonetheless, L' fails to be fully universal in space and time in a univocal sense in which L fails.

If the No Chaos condition strikes you as relatively weak, it might surprise you that a "good guy" law like Newton's Second fails to meet it. Expressing this law in the form

$$(*) \qquad \sum_b \underline{f}_{ab} = m(a) \times (d^2\underline{r}(a)/dt^2),$$

where a and b range over point masses and $\underline{f}_{ab}$ denotes the force of b on a, consider any finite connected region R of Newtonian space-time. Impose on R whatever kinematically consistent particle trajectories, mass assignments, and interactions you will. R can always be extended to a model of (*) by situating outside of R additional particles with the right trajectories, masses, and interactions so as to balance the equation. If it is said that Newton's Second Law is really

$$(**) \qquad \underline{F}_a = m(a) \times (d^2\underline{r}(a)/dt^2),$$

where $\underline{F}_a$ is the total force acting on a, then there is another law in the edifice of Newtonian Mechanics relating component forces to total force, to wit

$$\underline{F}_a = \sum_b \underline{f}_{ab},$$

which suffers the same fate as (*).

The difficulty here is not that (*) fails to apply to some regions of space-time, but that (*) subjugates the various regions in harmony. Its reign is wholistic, not piecemeal. This suggests an important division of laws, to which I shall return below.

3. Uniformity

The putative law L' above, although it pre-empts completely "lawless" regions, singles out some regions for harsher treatment than others. It acts like a statute that prohibits everywhere the sale of alcohol to minors but also restricts a particular township to the sale of only beer and wine. We need a condition which asserts that, whatever the law forbids, its prohibitions apply uniformly to all regions of space and time.

(Uniformity) Let L be a putative law, R and R' non-empty and isomorphic space-time regions, and A an arbitrary model of L. Then if $R \subseteq M(A)$, there is some $B \in \text{Mod } L$ such that $R' \subseteq M(B)$ and $A¶R \cong B¶R'$.

The idea of Uniformity is, in English, that if R and R' are isomorphic space-time regions, then, according to L, whatever can happen in R can happen in R'.

Two comments are in order. First, the Uniformity condition does not suppose a fixed space-time background, and thus is already completely general. Second, and more importantly, the notion of "region isomorphism" must be delicately construed. Clearly, if $R \cong R'$, then R and R' must be at least homeomorphic. But the isomorphism must further respect any additional absolute space-time structure present on the regions. I realize that the distinction between absolute and dynamical elements of a space-time may be problematic,[4] but the distinction is workable in many cases. For example, in Newtonian space-time, if R is a region bounded by simultaneity sheets and a set of inertial trajectories, then $R \cong R'$ only if R' is similarly bounded. Similarly, in Aristotelian space-time, if each spatial hypersurface of R is a sphere concentric about the center of the universe, then (since absolute place is a distinguished absolute feature of Aristotelian space-time) $R \cong R'$ only if each spatial hypersurface of R' is similarly centered. In General Relativity, region isomorphism reduces simply to region homeomorphism, since the metric is a dynamical element of the theory.

This said, I think it is reasonably clear that each law of the usually studied theories in space-time formulations--Newtonian mechanics, relativistic mechanics, electrodynamics, General Relativity, and even Aristotelian physics--satisfies the Uniformity condition. Furthermore, the sorts of laws represented by L and L' above do not. Let me briefly remark on the relation of Uniformity to other conditions.

First, Uniformity does not entail No Chaos. This should be evident from Newton's Second Law construed as (*) above. For reinforcement, consider the following quaint example. Suppose in Aristotelian space-time (see Fig. 1) there is a region R, roughly halfway between the earth and the moon, in which "anything goes". Suppose the remainder of the space-time is governed by the usual Aristotelian natural philosophy. Because R is uniquely distinguished with respect to the center of the universe, any subregion of R is at best isomorphic to only other subregions of R (e.g., R' and R'' in Fig. 2). Hence, Uniformity prevails. But any subregion of R, including R itself, suffices as a counterinstance to the No Chaos condition.

Nor does No Chaos entail Uniformity. Consider Maxwell's equations in a context where there is a single, fixed Minkowski space-time. Because the field equations satisfy the No Chaos condition, so does any stronger theory. But by strengthening the theory so as to eliminate any model in which, for some fixed region R, the charge density is non-zero, we violate Uniformity.

Second, Uniformity does not coincide with the so-called "No Reference" requirement, viz., that laws do not make reference to specific spatio-temporal locations. Uniformity is at least a weaker condition, since specific regions can be singled out by their relation to absolute features of a space-time and laws formulated with reference to them without violating Uniformity (witness the Aristotelian example above). Whether or not Uniformity is entailed by No Reference is undecidable pending an exact formulation of No Reference, a project which may not even be possible. It would seem that a putative law, to fail Uniformity, must somehow make reference to one or more special regions. But what mechanisms of reference are illegitimate is not clear.[5] There are cases in which reference can proceed without names or definite descriptions. Consider a context having a fixed space-time background and suppose that that space-time, although globally E^4, is rather bumpy in curvature. There is a "big bump" and in addition to that numerous lesser bumps, though the latter are distributed in such a way that the manifold has no nontrivial automorphisms. The metric is an absolute geometric feature, but the "big bump" per se is not. Yet it can be picked out by its unique metrical features. And hence, so can any other region by its metrical relation to the "big bump". Despite all this, we may still have non-trivial isomorphism classes of regions. All the apparatus is in place to write down a law violating Uniformity but apparently satisfying the strictures of No Reference.

Finally, there appears to be a close connection between Uniformity and symmetry principles, though perhaps not in the way anticipated. Uniformity by itself is completely impotent to enforce any requirement of the form 'Laws must be invariant under space-time transformations of type x'. But in a context with a fixed space-time background, the symmetries of the space-time take regions onto isomorphic images of themselves in precisely the sense of "isomorphism" need to implement the Uniformity condition. Consequently, Uniformity requires that for a theory formulated in the context of a fixed space-time background, the

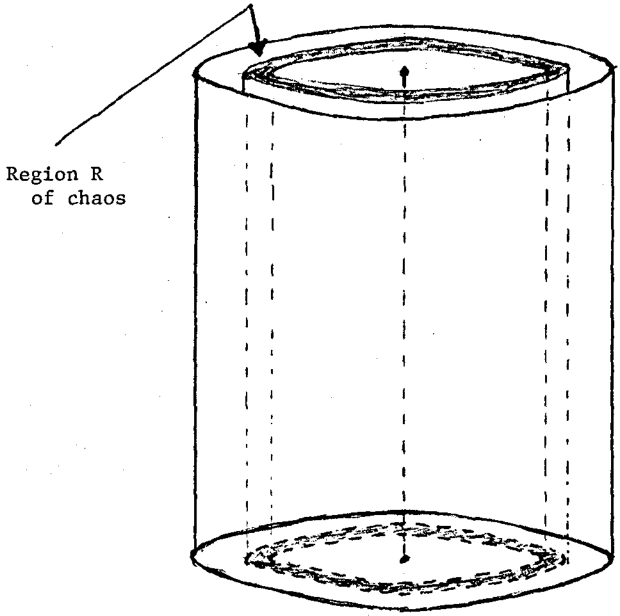
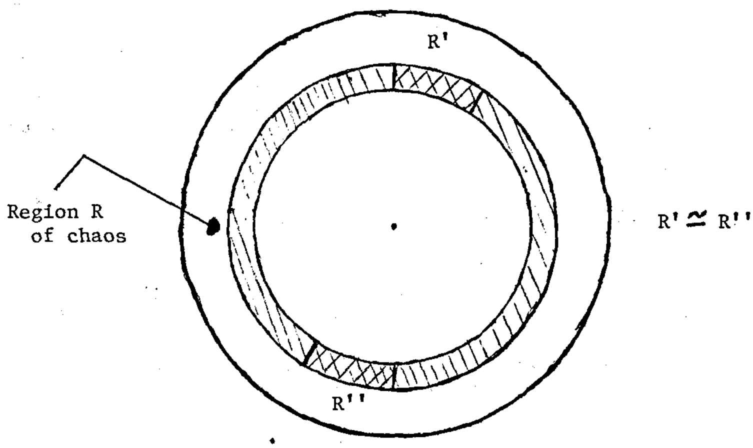


Fig. 1: Aristotelian Space-time

Fig. 2: Spatial Slice of Fig. 1



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local symmetries of the theory[6] must include the symmetries of the space-time. Uniformity, however, does not prevent the symmetries of the theory from outstripping the space-time symmetries, as is the case in Newtonian mechanics formulated in Newtonian space-time with absolute space. It should also be noted that even when non-trivial space-time symmetries are lacking, Uniformity can still have "punch". The reader is invited to consider the "bumpy" space-time in the preceding paragraph.

4. Armstrong and Aristotle

Does Uniformity appropriately capture the sense in which Tooley's garden variety law fails to be universal in space and time? Uniformity rules out such cases, but permits such laws as those of Aristotelian physics. Armstrong (1983), however, thinks the two cases are on a par anent the issue of universality. Armstrong provides no reasons for this judgment beyond the fact that the Aristotelian system endows the center of the universe with a "special nomic role" (p. 26). Indeed it does, but in the same sense that Newton gave the velocity of the center of mass of the system of the world a special nomic role, viz. as an absolute feature of the fixed space-time background. I do not see any difference in principle between the imposition of absolute position on a space-time and the imposition of absolute velocity or absolute acceleration. According to Armstrong's lights, we would have to consider the laws of classical mechanics as mere local uniformities, since they serve to distinguish regions "comoving" with inertial frames from regions "comoving" with accelerated frames.

The charged will no doubt be leveled that Tooley's law can be converted into a law satisfying Uniformity simply by introducing into the space-time backgound a distinguished object associated with the region of Smith's garden. Doing this will indeed restore Uniformity, but then we shall have a different example. Let Φ be the absolute object introduced, say a fixed scalar field vanishing everywhere except in the garden. The law of the garden would then take the form 'If $\Phi \neq 0$, then all fruit are apples'. The difference between the two cases is not a small one. In the former case, fruit in the garden has its distinctive behavior there for no particular reason. In the latter case, the field Φ can be held responsible. If this is judged unsatisfactory, I remind you that in Newtonian mechanics, given frames F and F' relatively accelerating, there is no particular reason in virtue of which F could be said to be inertial and F' non-inertial.[7]

I submit that whatever qualms one might feel about Tooley's garden with scalar field Φ derive, not from a violation of "universality in space and time", but from the introduction of what Einstein called "factitious causes". In his paper on the foundations of General Relativity (1916), Einstein poses the case of two fluid bodies, S and S', in constant relative rotation about the line joining them and sufficiently separated from one another and all other masses so that the gravitation interaction becomes negligible. Suppose further S measures to be a sphere while S' measures to be a flattened ellipsoid. According to Einstein, to explain the flattening of S' with reference to the class of inertial frames of Newtonian mechanics (or of special relativity) is to invoke a factitious cause.

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The prohibition against factitious causes can be interpreted in at least three different senses, none of which can plausibly be taken as an expression of the requirement of "unversality in space and time". (1) Einstein's immediate reading of it was as an epistemological principle that only an "observable fact of experience" is an admissible cause. (2) Einstein's derivative reading of it two paragraphs later was as a form of Mach's Principle, to wit that all motion is relative motion. (3) Finally, the prohibition might be taken as a prohibition against absolute objects in Anderson's sense of an object which can influence other objects without itself being influenced (Anderson 1964, 1967). Unless the notion of "observable" is extremely liberalized, reading (1) is simply an expression of a principle of strict empiricism. The principle expressed by reading (2) is not satisfied in any space-time theory with historical currency, including General Relativity.[8] The prohibition of reading (3) is satisfied in General Relativity, but fails in Newtonian Mechanics, electrodynamics, Special Relativity, and the like--theories which we want to say satisfy the requirement of "universality in space and time".

It might be suggested that (3) be strengthened in such a way to exclude only those absolute objects of a "local" character. One attempt at this might be to say that an absolute object G is non-local if for any regions R and R' and any homeomorphism ϕ from R to R', $\phi^*(G|R) = G|R'$, where ϕ^* is the mapping of G induced by ϕ. But this is too strong, since, according to it, the inertial structure of, e.g., Newtonian space-time is not non-local. Another attempt might be to insist that a space-time contains absolute objects of a "local" character just in case the space-time has no non-trivial automorphisms. This, however, will not serve Armstrong's purposes, since Aristotelian space-time is preserved under spatial rotations and time translations. Consider also a space-time which is topologically $E^3 \times E^1$ where the E^3 component is an infinite three dimensional "checkerboard", each cube of which contains as a subregion an isomophic copy of the Tooley garden with absolute scalar field Φ.

The appropriate conclusion, I believe, is that "universality in space and time" is not a feature of laws given by the "permitted phenomena" alone, but by the phenomena plus the details of space-time. If this seems unacceptable, then I leave as a challenge the formulation of comparable criterion of universality independent of the details of the structure of space-time.

5. Other Senses of Local, Global, and Universal

In the short space remaining, let me mention some other senses in which laws can be said to be local laws, global laws, or universal laws. Full treatment of the details will have to be reserved for another occasion.

There is a well entrenched sense of "local", current with mathematicians and physicists, in which Tooley's law is not a local law at all. This sense is that in which a space-time is locally E^4 but globally may be in a different homeomorphism class. Such distinctions are habitual in topology and have the standard form "X is locally Q iff every $p \in X$ has a neighborhood U such that $\varphi(U)$", where φ is characteristic of the property Q. Adopting the topologist's sense of

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"local", a law L is said to hold locally in A if every $p \in M(A)$ has a neighborhood U such that L holds in U. Of course, this requires that, for the context in which the law is formulated, the class K of "permissible" structures be closed under restrictions to all open subregions of the respective manifolds, something we do not normally require in talking about the class of all "possible" worlds. But once done, it is easy to see that there are laws which hold locally but not globally, or globally but not locally. In fact, so-called cosmological principles appear to be laws of the latter sort.

Once we open the semantic doors to "fragments" of possible worlds, we can formulate another sense of "universality" which connects directly with an often proposed syntactic constraint on the form of law statements, namely that laws, or law statements, must be universal sentences. (I hasten to add that this sense of "universality" is a fruit different from the condition of Uniformity.) Familiar to model theorists is the result that any class of structures (for a first-order language) closed under the formation of substructures is axiomatizable by a set of universal sentences. Thus, if our structures are such that the space-time manifold constitutes the domain of each structure, we can say that a law L is universal in this sense just in case for any model A of L and any non-empty region $R \subseteq M(A)$, A¶R is also a model of L. Field theories will typically be universal in this sense; theories involving sums over discrete masses, such as classical mechanics, will not. This is the sense mentioned above in which the laws of mechanics rule wholistically.

There is a twist to this. Consider the question whether or not in elementary logic the theory of Boolean algebras is universal. It all depends. If a Boolean algebra is regarded as a distributive complemented lattice, the answer is no. If thought of as an algebraic structure with the operations meet, join, and complement and the distinguished elements top and bottom, the answer is yes. The difference is this. In the former case a Boolean algebra is a relational structure, and any restriction of the relation to a subset of the domain generates a substructure. But not every subset of a lattice is necessarily a sublattice. In the latter case, since the semantics of elementary logic interprets function symbols as total functions, only restrictions to subsets which contain the top and bottom and are closed under the algebraic operations count as substructures. The point here is that "universality" in this sense is not a property preserved under interpretations between theories. Thus, it may be that field theories are "universal" in their space-time formulations. But interpreted in, say, set theory, they are not.

6. Conclusion

I have considered here a number of senses in which laws might be taken to be "universal in space and time". The condition of Uniformity strikes me as the appropriate explication of an important, and often intended, sense. In addition to this concept, I have sketched a few others--local, global, and univeral in the sense of preservation under substructures--which should be kept distinct from it.

Must laws satisfy Uniformity, or any of the other senses of "universal"? I don't think one can say without an understanding of what makes for lawfulness. Sure, there are a number of accounts available. But the fact that they fail to resolve such issues suggests that our grip on nomology is still quite feeble.

Notes

[1] I would like to thank John Earman for several helpful comments on an earlier draft of his paper.

[2] One can also pose a case in which 'All F's are G's' is true in every space-time region, but holds as a law of nature exclusively in R.

[3] The other senses Earman considered are:
-laws do not make reference to specific spatio-temporal locations
-laws do not explicitly contain space or time coordinates
-laws are invariant under spatio-temporal translations.

[4] See Anderson (1964, 1967), Earman (1974), and Friedman (1983).

[5] See Earman (1978) for a similar remark.

[6] The notion of "the symmetries of a theory" is given implicitly in Earman (1974) and explicitly in Earman (forthcoming). The sense of "local" here is that discussed in section 5. If the class of models of the theory is closed under model isomorphism, then each of these local symmetries will be global as well. But closure under model isomorphism alone suffices to guarantee that each space-time symmetry is also a global symmetry of the theory. An imaginative reader can verify that Uniformity and closure under model isomorphism are independent conditions.

[7] To say "In F there are no centrifugal and coriolis effects but in F' there are", no more answers the question than does the reply, in the Tooley case, that Φ has the character it does in R because "in R, all fruit are apples".

[8] For a discussion of this see, e.g., Earman (1974) pp. 269-271 and Sklar (1974) pp. 216-221.

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