BULL. AUSTRAL. MATH. SOC.
VOL. 23 (1981), 157-158.

76B15

WATER WAVES ABOVE A SILL

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The theory of trapping of water waves in three-dimensional flows, where the bottom topographies model the natural variations of the ocean floors, is difficult (Meyer [6], [7]). Hence, the idealised problem of a submerged circular sill has received the most analysis (Longuet-Higgins [2]; Summerfield [9]; Pite [8]). All these works are based on linear shallow-water equations. In order to verify the large near-resonances predicted by these theories, W.G. Pritchard (Private Communication) performed laboratory experiments and produced results which were inconsistent with the theoretical results. This thesis develops a full linear theory for both inviscid and viscous cases (Havelock [1]; Mahony and Pritchard [3]), and thus serves to define the limitations in the results obtained by using shallow-water theory. The general trend of these results is that wave-trapping is a phenomenon which exhibits less peaky resonance than has been generally assumed. However, Pritchard's experiments do show marked peaks at frequencies not associated with resonances predicted by shall-water theory.

Since subharmonic resonance can lead to large responses, a weakly nonlinear theory (McEwan [5]; Mahony and Smith [4]), in which the calculations must be organised in a non-standard way, is described. The velocity potential is conveniently written as that of the linear solution plus the components excited by the nonlinear interactions. The calculation of the modal decomposition of the latter is complex and its difficulties are resolved in Chapter 4. A possible subharmonic resonance is presented, together with a particular laboratory condition, under which it can occur.

Received 20 November 1980. Thesis submitted to the University of Western Australia, July 1980. Degree approved, October 1980. Supervisor: Professor J.J. Mahoney.

The response curves exhibit the unusual property that the forced linear mode is amplified as it feeds energy into the half-frequency mode. It is concluded that usch interactions are unlikely to have occurred in Pritchard's experiments, and that the inconsistencies in his results may be due to resonances associated with modes of the wave tank.

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