PART II

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A. LUNAR MECHANICS

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DYNAMICS OF THE MOON

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Abstract. Koziel's results on the librations have been rediscussed. Some serious departures from independence of the errors were detected and have been allowed for. The results are

$\beta = 0.0006271 \pm 0.0000010$
$\gamma = 0.0002362 \pm 0.0000082$

These do not differ much from Koziel's values but the uncertainties are larger. Both are consistent with results previously derived by comparison of results of different authors, and γ is consistent with a reinterpretation of a result of Yakovkin.

The effect of elasticity is considered. It is shown that the elastic strain does not contribute to the librations but would affect the perturbations of a satellite. Allowance for this difference reduces Michael's estimate of 0.4015 for C/Ma^2 to 0.4001 ± 0.0030, which would be consistent with either uniform density or with the value I found in 1936 after allowance for compressibility and a possible thin surface layer, namely 0.3971 ± 0.0007.

Since last year's meeting on the Moon I have carried out some revision of the results described then. A serious mistake had been found by Habibullin and Schrutka-Rechtenstamm in Yakovkin's estimate of the term in the libration in longitude with a period close to 3 years. I was not altogether satisfied with Koziel's analysis of four series of observations, since possibility of correlation of the errors had not been checked. It was possible however to solve his separate sets of normal equations, and the results differed by more than random errors would explain. This was allowed for in a revised treatment. Available values of β and γ are now as follows.

$$\begin{aligned}
\beta &= 0.0006279 \pm 0.0000015 \text{ (Jeffreys, 1961, by comparison of 10 determinations)} \\
&= 0.0006294 \pm 0.0000006 \text{ (Koziel)} \\
&= 0.0006271 \pm 0.0000010 \text{ (Koziel revised)} \\
\gamma &= 0.0002398 \pm 0.0000092 \text{ (Yakovkin, corrected)} \\
&= 0.0002274 \pm 0.0000088 \text{ (Jeffreys, 1961, by comparison of 20 determinations)} \\
&= 0.0002310 \pm 0.0000032 \text{ (Koziel)} \\
&= 0.0002362 \pm 0.0000082 \text{ (Koziel revised).}
\end{aligned}$$

The revised values are within the standard errors of the others.

Elasticity has a well known effect on the free nutation of the Earth, and it seemed possible that it might have one on the Moon's librations. The treatment turned out to be easy. The elastic deformation is always in the direction of the disturbing body, and the couples are consequently unaffected by it. Hence the values of β and γ estimated from the librations do not include the parts contributed by elastic deformation. But these parts do affect the motion of a satellite travelling near the Moon. Now comparison of its perturbations, on the supposition that the Moon is rigid, with the librations leads to a determination of C/Ma^2 for the Moon. The best determination so

Urey and Runcorn (eds.), The Moon, 11–12.

far is by W. H. Michael, announced at the 1970 conference of the IAU, namely

$$C/Ma^2 = 0.4015 \pm 0.0030 .$$

Correction for elasticity reduces this to 0.4001 ± 0.0030. This would agree with either uniform density or with 0.3971 ± 0.0007, which I derived in 1936 by allowing for compression and the possibility of a granitic layer. At any rate Eckert's result from the motion of the Moon's node, leading to a structure like a tennis ball, must have some other explanation.

Reference

Jeffreys, H.: 1971, *Monthly Notices Roy. Astron. Soc.* **153**, 73.