EIGHTH LECTURE

Some Aerodynamic Problems of the Helicopter

By H B SQUIRE, M A (Oxon)

A lecture presented to The Helicopter Association of Gt Britain on Saturday, 6th December, 1947, at Manson House, 26 Portland Place, London, W 1

H A MARSH, A F C , A F R Ae S , IN THE CHAIR

INTRODUCTION BY J A J BENNETT, D SC , F R Ae S

I have accepted with pleasure the Chairman's request to introduce our lecturer this afternoon It is twelve years ago since MR SQUIRE proved himself to be a worthy successor to the late MR H GLAUERT at the Royal Aircraft Establishment in analysing the basic aerodynamic problems of the helicopter The work of GLAUERT had been confined to vertical flight and to horizontal flight with the rotor axis vertical (like that of the Gyrodyne) MR SQUIRE extended the analysis to the flight of a helicopter in which the propulsive force for horizontal flight was obtained by a forward inclination of the rotor axis The classic report by MR SQUIRE on this subject was studied closely by the Sikorsky organisation and was put into practical effect a few years later in the well-known Sikorsky designs

MR SQUIRE is recognised as the leading aerodynamicist on helicopters in this country and is Chairman of the Helicopter Committee of the Aeronautical Research Council We are all looking forward to hearing from him this afternoon on certain aerodynamic problems of the helicopter which have remained a mystery to most of us until now

MR H B SQUIRE

This lecture is concerned largely with the velocities induced by a helicopter rotor due to the aerodynamic forces on the blades and the influence of these velocities on the stability of multi-rotor helicopters These questions are likely to become more important in the future than they are today and an understanding of them will help to elucidate some of the outstanding problems of the helicopter

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The Velocity Field of a Rotor

GENERAL

The helicopter rotor provides the lift which supports the weight of the aircraft, and to do this it must transmit momentum to the air in a downwards direction In general, the rotor axis is inclined to the vertical and the rotor thrust is inclined forwards Further, to obtain a horizontal component of the thrust air must be accelerated backwards Thus we see that the air passing through the rotor is directed backwards and downwards as a direct consequence of the existence of the rotor thrust (Fig 1) This extra speed which the air acquires is called the induced velocity

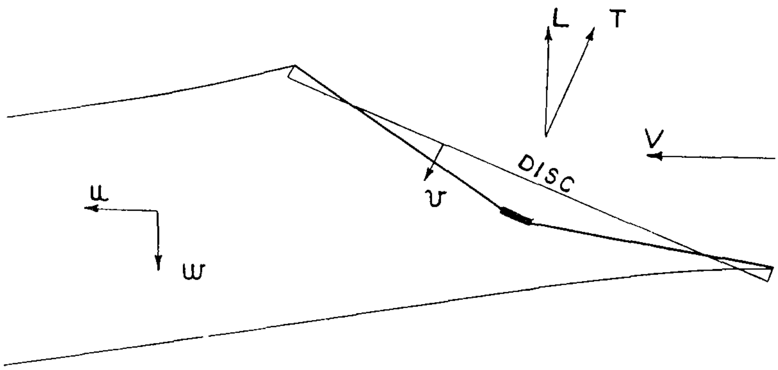


Fig 1

The order of magnitude of this induced velocity can be estimated purely from considerations of the conservation of momentum but the calculation of the exact distribution of induced velocity is a complicated problem I should therefore show that this is worth doing before I proceed to describe the results of our recent work The justifications for such an investigation are —

(1) Analogy with allied aerodynamic problems of wing theory and propeller theory, which have progressed very far, suggests that detailed theories would be useful also for rotors

(2) Elaborate theories of blade motion have been constructed based on crude assumptions as to the induced velocity distribution over the rotor disc a better method of calculation of the induced velocity distribution would enable the blade motion theory to be more soundly based

(3) The advent of the multi-rotor helicopter has shown that we need to know the induced velocity in order to calculate rotor interference effects

ORIGINAL INVESTIGATIONS OF GLAUERT AND LOCK

It is well worth while to study the original investigations of Glauert[1] and Lock[2] on the autogyro, both from their historical interest and practical applicability Glauert introduced the physical principles which are still used but a number of difficulties remained, some of which were cleared up by LOCK in his subsequent investigation Two major assumptions were introduced by Glauert to enable him to carry through this analysis These were —

(1) *Induced velocity* —He assumed for the major part of his investigation that the induced velocity associated with the rotor lift force is uniform and directed along the axis of the rotor Some of the consequences of assuming that the induced velocity varied linearly from front to rear of the rotor were also considered , this latter kind of induced velocity field might be expected from wing theory which suggests that upwash would be present at the front of a rotor and downwash behind But there is no simple way of specifying the fore and aft variation of the induced velocity and this alternative proposal of Glauert has not been generally followed The assumption of a uniform induced velocity on the other hand led to definite conclusions when linked with Glauert's second assumption

(2) *The momentum equation* is assumed to be of the form

$$T = \rho\pi R^2 V' \quad 2v, \qquad (1)$$

where T = the thrust
R = the radius of the disc
V' = the resultant velocity at the disc
v = the induced velocity at the disc, so that V' is the velocity obtained by combining the air velocity V and the induced velocity v

This formula is a generalisation of the two extreme cases —

(*a*) The Froude momentum equation for axial flow In this case V and v are both directed along the axis of the rotor so that

$$V' = V + v$$

and

$$T = \rho\pi R^2(V + v)2v$$

(*b*) The relation for elliptic loading on a wing of span 2R at small incidence In this case the induced velocity v is normal to the stream velocity V and is small compared with it It is known that the lift of the wing L is then related to the induced velocity v by the formula

$$L = \rho\pi R^2 V\, 2v$$

It will be seen that equation (1) reduces to this form if we may replace the resultant velocity V' by the stream velocity V Thus we see that GLAUERT's momentum equation for the thrust is a generalisation of two limiting cases and this is its only real justification

LOCK started from the framework prepared by GLAUERT but he carried through the analysis of the aerodynamic characteristics and the blade motion much more thoroughly so that the nature of the approximations was clear at every stage The result is that his report is still a standard reference work However, he was forced to adopt the assumption of a uniform induced velocity for lack of any reliable theory of the induced velocity distribution and this is the main gap in his work

It is worth while to start briefly with the exact problem and consider stage by stage what approximations are introduced into our analysis The first approximation that we make is the restriction to a linearised theory, *i e*, we assume that the induced velocities are all small compared with the stream velocity This is permissible provided that the thrust coefficient C_T, defined by the equation

$$C_T = \frac{2T}{\rho V^2 \pi R^2},$$

is not too large The linearised theory is probably valid up to $C_T = 0\,5$ approximately

The next steps in the approximation are to ignore the rotation in the slipstream and to assume that there are sufficient blades to permit neglect of the periodicity in the flow The latter assumption is a drastic one but is essential since we already know that the solution of the axial flow propeller with a finite number of blades is very difficult the corresponding case of an oblique propeller with a finite number of blades is probably insoluble

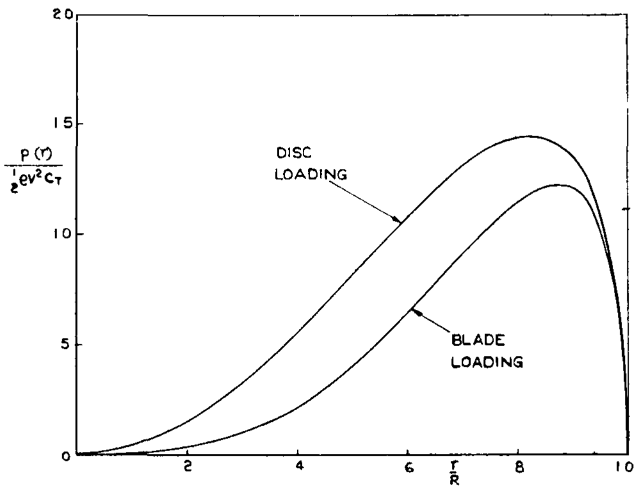


Fig 2
Assumed Loading Distribution

We have now reduced our problem to a disc loading problem we have a thrust T which is distributed over a disc of radius R The disc we define as the plane from which the path of the blade tips deviates as little as possible In addition to the thrust which is assumed to act normal to this reference plane there may be small forces in the plane of the disc, but these are in practice less than 5% of the thrust and may be ignored in considering the induced velocity field We may further assume that for a rotor with hinged blades and small hinge offset the thrust T acts through the centre of the disc since otherwise there would be a rolling or pitching moment present at the hub We are, therefore, led to the assumption that the loading is symmetrical round the disc, this is certainly not more than a rough approximation but it may be possible to improve it in the future We further know that the lift vanishes at the root and at the tips of the blades, *i e*, the load vanishes at the centre and at the rim of the disc There is one particular load distribution which satisfies these conditions for which it is not too difficult to make calculations This is for the loading over the disc given by

$$p(r) = \frac{15}{8}\rho V^2 \left(\frac{r}{R}\right)^2 \sqrt{1-\left(\frac{r}{R}\right)^2}\, C_T \qquad (2)$$

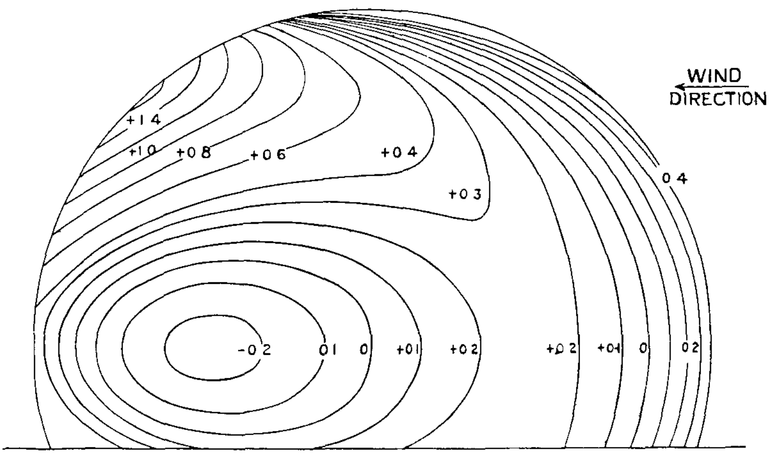
This is shown in Fig 2 and is the loading assumed for all the induced velocity calculations The corresponding load distribution along a blade is

$$l(r) \propto \left(\frac{r}{R}\right)^3 \sqrt{1-\left(\frac{r}{R}\right)^2}$$

and is also shown in Fig 2 The mathematical analysis of the flow through a rotor with this load distribution over the disc has been carried through by W Mangler and the computations by P Sibbald, and a detailed report on this work will be issued in due course We proceed to consider some of the results, beginning with the induced velocity distribution over the disc itself

The contours of induced velocity normal to the disc, denoted by v, for the above loading and taking $C_T = 1\ 0$ are given in Figs 3 and 4 for disc incidences (i) of 0 and 15° Calculations have also been made for 30, 45 and 90 degrees

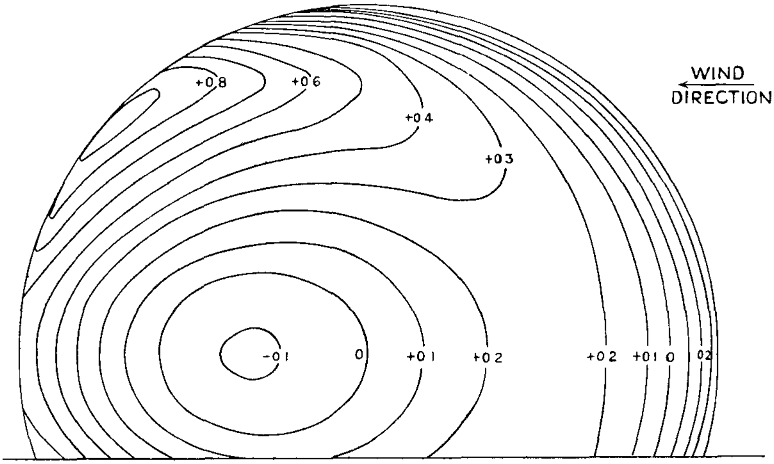
If we consider in particular Fig 4 ($i = 15°$) we notice first that the induced velocity is anything but uniform over the disc The average value of the downwash parameter $\frac{v}{VC_T}$ is about 0 25 but it varies from an upwash



INDUCED VELOCITY OVER DISC
CONTOURS OF $\frac{v}{VC_T}$ FOR $\alpha = 0°$

Fig 3

of 0 3 at the front of the disc to a downwash of 0 9 on the rear edge of the disc halfway out along the span It is desirable here to mention another feature which is characteristic of this kind of investigation and which is already well known in aerofoil theory this feature is that the induced velocity varies rapidly with any change in the load distribution over the disc This



INDUCED VELOCITY OVER DISC
CONTOURS OF $\frac{v}{VC_T}$ FOR α - 15°

Fig 4

is because the induced velocity depends on the gradient of the loading and it follows that large local changes in induced velocity can be obtained by quite small changes in the load distribution on the disc Consequently local peaks in upwash or downwash may not be present in practice because they can be removed by quite small changes in loading , we must therefore only take the general features as significant These general features are best considered in relation to the vortex system shown in Fig 5 which represents the system roughly

In Fig 5 the number of arrows shown is a measure of the vortex strength It must be remembered of course that we are actually dealing with a continuous variation of vorticity, and the diagram only illustrates the main features We note the following

(1) There is an upwash near A at the forward part of the disc
(2) There is a downwash along BO which is followed by upwash over OC and downwash over CD These are all in accordance with the detailed calculations
(3) There is zero downwash at the centre of the disc
(4) There is a large upwash at the lateral tips of the disc EE and a large downwash near the points FF , as explained above these may be modified by minor changes in load distribution

On the right of Fig 5 is shown the span loading distribution which is obtained for the disc loading given by equation (2), treating the rotor as an aerofoil It will be seen that this loading distribution differs considerably from the ideal elliptic loading which gives minimum induced drag for aerofoils

We now turn from the case of the induced velocity in the rotor disc to the case of the induced velocity away from the disc Since we are here mainly interested in the mean value with respect to time of the induced

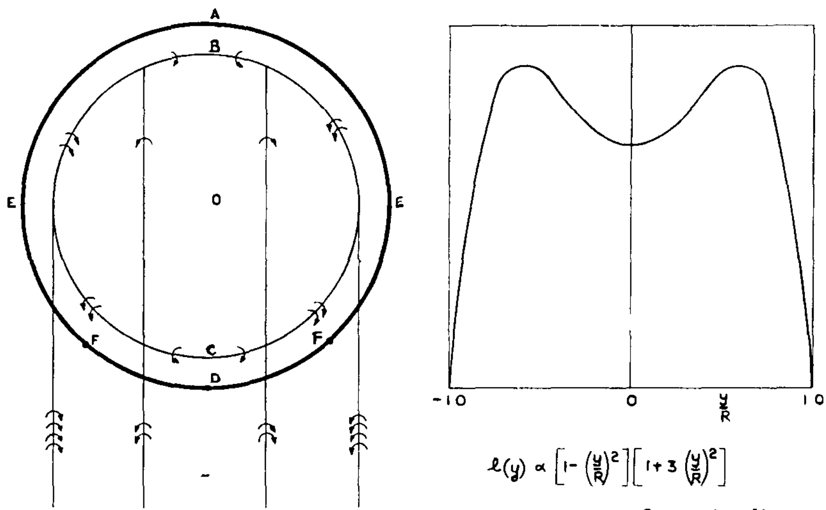


Fig 5 Vortex Distribution and Span Loading

velocity at a point we can here satisfactorily represent the rotor as a disc with a steady load distribution over it The same loading as in the former case has been adopted, given by (2), and the calculation of the induced velocity distribution has been made by the same method as for points on the disc Here we are of course interested in induced velocities normal to the wind direction since the results are to be applied to calculate interference effects Figs 6 and 7 show the downwash $\frac{w}{VC_T}$ in the plane far downstream for a rotor at zero incidence and at an incidence of 15° The plane of the figures is normal to the stream direction so that in Fig 6 the projection of the rotor disc is shown as a line and in Fig 7 as an ellipse, and w is the induced velocity directed vertically downwards in this plane

These curves can be applied to a number of important problems in helicopters Consider first the interference effect on induced drag for a pair of rotors If the rotors are side by side each will produce an upwash on the other and a reduction of the induced drag will result If on the other hand, there is one rotor behind the other the front rotor will produce a downwash on the rear one with a consequent increase in induced drag These effects are of course well known in wing theory We obtain the results given in Table I for the side-by-side configuration

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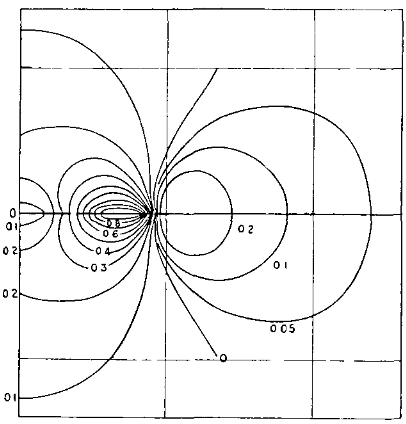
TABLE I INDUCED DRAG OF SIDE-BY-SIDE ROTOR COMBINATION DISC INCIDENCE ZERO	Horizontal Gap / Diameter	Induced drag / Induced drag without interference
	0	0 875
	0 05	0 906
	0 1	0 924

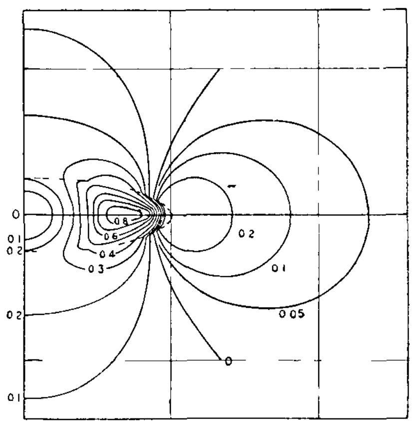
The induced drag of an isolated rotor at zero incidence with the disc loading distribution shown in Fig 2, where the span loading distribution is as shown in Fig 6, is calculated to be 1 176 times the induced drag for the ideal case of elliptic loading across the span , this result has been used in the calculation of induced drag for a combination

For gap-diameter ratios of less than 0 1 the favourable effect may be rather greater than that shown because the effective disc loading may possibly change in such a way that the two discs act more and more like a single disc



INDUCED VELOCITY FAR DOWNSTREAM CONTOURS OF $\frac{w}{VC_T}$ FOR α 0°

Fig 6



INDUCED VELOCITY FAR DOWNSTREAM CONTOURS OF $\frac{w}{VC_T}$ FOR α 15°

Fig 7

It is clearly desirable to place the discs as close together as possible without increasing the induced drag, since this reduces the length of the outriggers I don't know the answer to this problem but my guess is that the best arrangement would be to use a small overlap on the discs , this would not normally give any risk of the blades striking one another since the rotors are usually geared together

The next important case of induced drag is for the helicopter with tandem rotors Here in the extreme case of no gap between the rotors the

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Vertical Gap / Diameter	Induced drag / Induced drag without interference
0	2
0 1	1 573
0 2	1 394
0 35	1 259
0 5	1 182

TABLE II

INDUCED DRAG OF TANDEM ROTORS

DISC INCIDENCE ZERO

induced drag may be twice as great as the drag of the two independent rotors The results for different vertical gaps between the rotors are given in Table II and it will be seen that the interference effect diminishes rapidly with increase in gap This interference effect is independent of the fore and aft distance between the rotors but the gap should really be estimated as the distance of the wake of the front rotor from the rear one , due to the deflection of the wake this may be somewhat greater than the geometrical gap

ROTOR INTERFERENCE FOR MULTI-ROTOR HELICOPTERS

In order to analyse or predict the stability characteristics of multi-rotor helicopters it is necessary to estimate the interference between the rotors This requires the calculation of the downwash induced by the front rotor(s) on the rear rotor(s) The other interference effects are not likely to be important with the possible exception of the effect of the rotor slipstream on the fuselage at low forward speeds

A description of calculations of the induced velocity field of a rotor has been given above and it is only necessary to indicate briefly the application to the calculation of stability here For the tandem rotor helicopter the rear rotor is in a stream which is inclined downwards due to the downwash from the front rotor and this would cause a reduction in thrust if it were not counterbalanced by a change in pitch of the blades or a change in inclination of the rotor axis This downwash varies with forward speed and with the height of the rear rotor above the front one, so that it is not possible to draw many general conclusions as to the effect on stability of the rotor interference Since the effect increases with reduction of forward speed but vanishes altogether in the hovering condition, to which the theory does not apply, there will be some speed for which the effect is a maximum

As the speed decreases the slipstream of the front rotor is deflected more and more from the wind direction and consequently further and further away from the rear rotor Since our theory is a first order theory which assumes that the loading of the rotor and the deflection of the slipstream are both small the theory becomes less reliable as the speed decreases It may, however, be possible to make some allowance for second order effects by estimating the height of the rear rotor above the deflected slipstream of the front rotor

Stability

SINGLE ROTOR AIRCRAFT

(*a*) *Hovering*

The helicopter is least like an aeroplane when it is hovering and it is in this condition that we should expect that the stability characteristics would be most unusual This is in accordance with experience which shows that the hovering helicopter in its simple forms is unstable this instability takes the form of an increasing oscillation It is desirable to be clear about this and not to be confused by meaningless statements about " pendular stability "

It has been often forgotten that VON KARMAN gave an adequate account[3] of the stability of the hovering helicopter in 1921 His work had been done in connection with the Karman-Petroczy helicopter developed in Austria-Hungary during the 1914-1918 war I repeated this investigation in 1938 in an unpublished paper[4] in ignorance of VON KARMAN's work, and in the same year HOHENEMSER[5] published a much more detailed investigation of the subject * Fortunately this instability of the hovering helicopter turns out to be of long period and fairly slow rate of growth For example, for the Sikorsky R 4 the period of oscillation is 14 sec and the increase in amplitude in one period is about 5

(*b*) *Forward flight*

In forward flight we should logically begin by considering static longitudinal stability which is measured by the variation of the position of the fore-and-aft control with forward speed It appears, however, that this movement is small and independent of C G position so that we should conclude that the static longitudinal stability of the aircraft is neutral or that the concept is inapplicable

I shall not discuss the dynamic stability here as work on this will be described by Mr Stewart in a lecture to the R Ae Soc in January, 1948

Helicopter Stability—General

The single rotor helicopter of the Sikorsky type is an aircraft whose stability characteristics are very different from the stability characteristics of aeroplanes the small effect of C G position on the stability of the Hoverfly I helicopters is sufficient evidence of this Helicopters with two side-by-side rotors behave like single rotor helicopters for motion in the plane of symmetry

But when the lifting surfaces are spread out along the line of flight, as in tandem rotor helicopters, then the resemblance, from the stability point of view, to aeroplanes becomes closer In the following paragraphs I shall for definiteness refer to the tandem rotor helicopter but it must be understood that the remarks apply in general to other configurations, such as the Cierva Air Horse, which has three rotors at the corner of an equilateral triangle

In considering the general problems of the stability of the multi-rotor helicopter we must take advantage of the many years of study devoted to aircraft stability We follow here the theory of Gates and Lyon[8] These

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* The corresponding theory of rotor stability in forward flight has been worked out by HOHENEMSER[6] and SISSINGH[7]

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writers have shown that the principal parameters determining aircraft stability are the static margin K_n and the manoeuvre margin H_m and that for satisfactory behaviour it is necessary that both these quantities should be positive

The definitions of K_n and H_m are* —

(1) $$K_n = -\frac{\delta C_m}{\delta_\alpha}\frac{d\alpha}{dC_L} - \frac{\delta C_m}{\delta V}\frac{dV}{dC_L},$$

with $$\frac{dV}{dC_L} = -\frac{V}{2C_L},$$

(2) $$H_m = -\frac{\delta C_m}{\delta_\alpha}\frac{d\alpha}{dC_L} - \frac{\delta C_m}{\delta_q}\frac{dq}{dC_L},$$

with $$\frac{dq}{dC_L} = \frac{V}{l}\frac{1}{2\mu_1},$$

where α = incidence
C_L = lift coefficient $2L/\rho V^2 S$
C_m = pitching moment coefficient $2M/\rho V^2 Sl$
l = arm for pitching moments
q = angular velocity in pitch in a circle at constant speed
μ_1 = $W/g\rho Sl$

The neutral point is the C G position for which K_n vanishes and in the simplest cases K_n is proportional to the distance of the C G ahead of the neutral point Further the static margin K_n is proportional to the rate of movement of the longitudinal control with speed, and the latter vanishes if the C G is at the neutral point This relation between K_n and the longitudinal control provides a good way of understanding the significance of the static margin We consider the succession of equilibrium conditions of the aircraft which corresponds to increases in speed at constant power Then there will be a corresponding movement of the longitudinal control lever which may be assumed to tilt the rear rotor forward and back just as if we had a gigantic all-moving tail plane , if this movement is such that the backward tilt of the rear rotor increases relative to that of the front rotor with increase of speed (corresponding to downward movement of the elevator with increase of speed for a stable aeroplane) then the helicopter is statically stable

The neutral point and the static margin are linked with slowly developing changes in motion and do not give the whole picture An aircraft which is statically unstable may be quite controllable , nevertheless it is desirable that helicopters which are to be used for military or civil purposes should be

* Ignoring the distinction between the lift coefficient C_L and the resultant force coefficient C_R

statically stable for speeds above some lower limit, this lower limit might be the best climbing speed

On the other hand the manoeuvre margin H_m is a measure of the effects of rapid changes and it is essential that H_m should be positive If H_m is negative large accelerations can be developed as a result of small movement of the longitudinal control lever and this is unacceptable

For the tandem rotor helicopter we can carry our analysis a little further If the aircraft is given a rate of pitch q then the front rotor hub will acquire an upwards velocity and the rear rotor hub a downwards velocity (Fig 8)

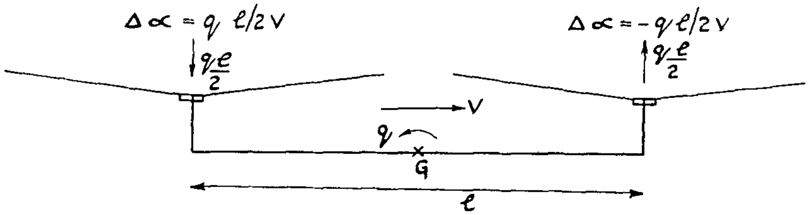

Fig 8 *Crown copyright reserved*

This will produce a decrease in thrust of the front rotor and an increase of thrust of the rear rotor and hence a negative pitching moment For the twin rotor helicopter rough estimation gives

$$\triangle M = \frac{1}{2}\frac{dL}{d\alpha}\triangle\alpha = -\frac{l^2 q}{4V}\frac{dL}{d\alpha}$$

and hence
$$\frac{\delta C_m}{\delta q} = -\frac{1}{4V}\frac{dC_L}{d\alpha},$$

where $\frac{dC_L}{d\alpha}$ is the overall variation for both rotors Substitution in the equation defining H_m then gives

$$H_m = -\frac{\delta C_m}{\delta\alpha}\frac{d\alpha}{dC_L} + \frac{1}{8\mu_1}\frac{dC_L}{d\alpha}$$

Now μ_1 is of order unity and $\frac{dC_L}{d\alpha}$ is of order unity for a tip speed ratio $\mu = 0\ 1$ and of order 0 4 for a tip speed ratio $\mu = 0\ 3$ Hence the second term in the right hand side of the expression for H_m is of order 0 1 for $\mu = 0\ 1$ and falls with increase of tip speed ratio This rough calculation is sufficient to show that the second term in the expression for H_m is important, probably more important than for aeroplanes We must therefore be on our guard against the simple criterion that satisfactory stability characteristics are assured if $\frac{\delta C_m}{\delta\alpha}$ is negative, as this is not the whole story

As an example, calculations have been made of the stability characteristics

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at sea level of a hypothetical tandem rotor helicopter of the following dimensions —

Weight	9,500 lb	Solidity	0 05
Rotor power	2 × 350 h p	Tip speed	550 ft /sec
Rotor diameter	45 ft	Distance between rotor hubs l=45 ft	

The results are given in Fig 9 which shows static margin, manoeuvre margin and longitudinal control setting plotted against the tip speed ratio μ for two c g positions, one midway between the rotors and another forward of it by 5 per cent of the length between the hubs It will be seen that the manoeuvre margin is positive in all cases and this suggests that there is no risk of rapid divergence On the other hand with the rear c g position the static margin is negative throughout the speed range With forward c g we have static stability at high speeds and static instability at low speeds It seems likely that static instability may occur for all tandem rotor helicopters below a certain speed and it may be only practicable to require that this speed shall be below the best climbing speed

The longitudinal control setting is defined as the angle of tilt of the rear rotor axis relative to the front rotor axis, a backward tilt being positive If we wish to travel faster the immediate movement is to increase this angle

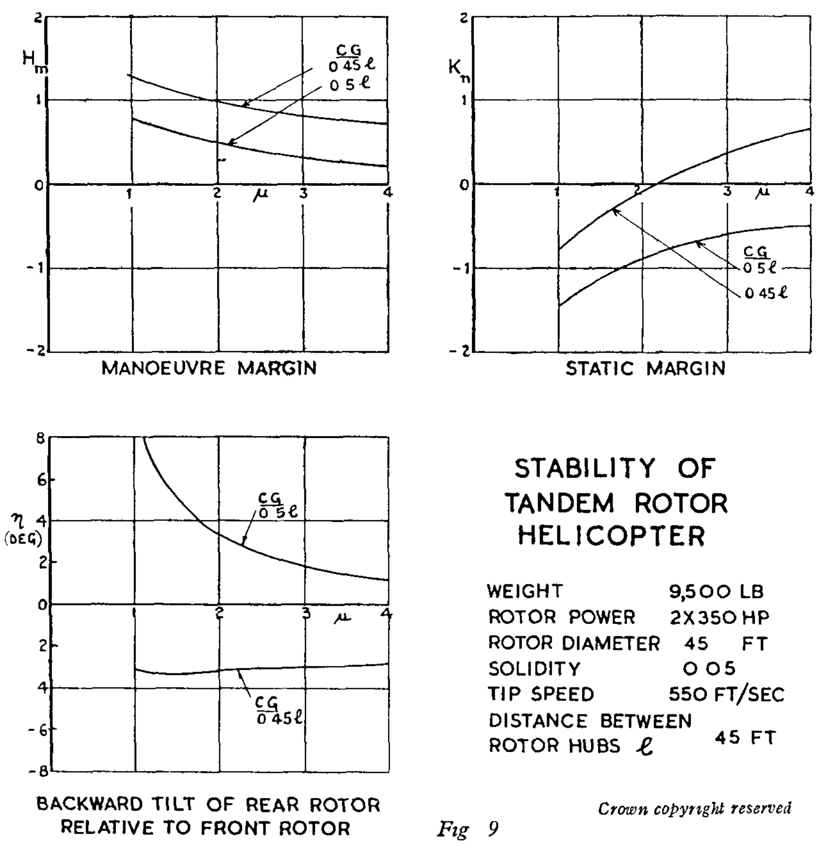

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Fig 9

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as it will give greater lift on the rear rotor and this will tilt the nose down It is very desirable that such a movement should not have to be followed by too much reversed movement when we reach the new condition This happens for a statically unstable aircraft as is shown for the helicopter with rear c g position in Fig 9 With the forward c g position on the other hand the longitudinal trimmer setting hardly varies at all with speed which is much more satisfactory

I am not sure how far it is safe to generalise from this one example which was calculated in rather a hurry In particular, the effect of drag and of vertical height of the c g have been neglected But it is probably correct to conclude that the maintenance of static stability over a sufficient part of the speed range is the matter to which most attention should be given

Concluding Remarks

I have left a number of matters not properly illustrated by means of examples There has been no opportunity so far to do much of this but we shall try to do some more in the future I will conclude my lecture by expressing the hope that perhaps some members of the audience may be willing and able to take some part in this work

The Chairman's Vote of Thanks to Mr Squire

The Chairman said he had very much pleasure in proposing a hearty vote of thanks to Mr Squire for his most interesting lecture, and felt all present were well rewarded for coming there that afternoon

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