Dynamical Evolution of Dense Stellar Systems
Proceedings IAU Symposium No. 246, 2007
E. Vesperini, M. Giersz & A. Sills, eds.

doi:10.1017/S1743921308015834

Black Hole Motion as Catalyst of Orbital Resonances

C. M. Boily[1], T. Padmanabhan[2] and A. Paiement[3]

[1]Observatoire astronomique & CNRS UMR 7550, Université de Strasbourg I, F-67000 Strasbourg
email: cmb@astro.u-strasbg.fr

[2]I.U.C.A.A., Ganeshkhind Post Bag 4, Pune, India
email: paddy@iucaa.ernet.in

[3]E.N.S.P. de Strasbourg, Parc d'innovation, Bd. Sébastien Brant, F-67412 Ilkirch

Abstract. The motion of a black hole about the centre of gravity of its host galaxy induces a strong response from the surrounding stellar population. We consider the case of a harmonic potential and show that half of the stars on circular orbits in that potential shift to an orbit of lower energy, while the other half receive a positive boost. The black hole itself remains on an orbit of fixed amplitude and merely acts as a catalyst for the evolution of the stellar energy distribution function $f(E)$. We then consider orbits in the logarithmic potential and identify the response of stars near resonant energies. The kinematic signature of black hole motion imprints the stellar line-of-sight mean velocity to a magnitude $\simeq 13\%$ the local root mean-square velocity dispersion σ. The high velocity dispersion at the 5:2 resonance hints to an observable effect at a distance $\simeq 3$ times the hole's influence radius.

Keywords. stellar dynamics, gravitation, Galaxy: centre, galaxies: nuclei

---

1. Introduction

Black hole (BH) dynamics in galactic nuclei has attracted much attention for many years (e.g., Merritt 2006 for a recent review). The influence of a BH on its surrounding stars is felt first through the large velocity dispersion and rapid orbital motion of the inner-most stars ($\sigma \sim v_{1d} \lesssim 10^3$ km/s). This sets a scale $\lesssim GM_{bh}/\sigma^2$ ($\simeq 0.015 - 0.019$ pc for the Milky Way, henceforth MW) within which high-angle scattering or stellar stripping and disruption takes place. For the MW, low-impact parameter star-BH encounters are likely given the high density of $\rho \sim 10^7 M_\odot/\mathrm{pc}^3$ within a radius of ≈ 10 pc (see e.g. Yu & Tremaine 2003; O'Leary & Loeb 2006; see also Freitag, Amaro-Seoane & Kalagora 2006 for a numerical approach to this phenomenon). Star-BH scattering occurring over a relaxation time (Preto, Merritt & Spurzem 2005; Binney & Tremaine 1987) leads to the formation of a Bahcall-Wolf stellar cusp of density $\rho_\star \sim r^{-\gamma}$ where γ falls in the range 3/2 to 7/4 (Bahcall & Wolf 1977). Genzel *et al.* (2003) modeled the kinematics of the inner few parsecs about Sgr A$\star$ with a mass profile $\rho_\star \sim r^{-1.4}$, which suggests of a strong interplay between the BH and the central stellar cusp. More recently, Schödel *et al.* (2007) presented a double power-law fit to the data, where the power index $\simeq 1.2$ inside a break radius r_{br}, and $\simeq 7/4$ outside, where $r_{br} \simeq 0.2$ pc. This is indicative of on-going evolution inside r_{br} not accounted for by the Bahcall-Wolf solution.

Most, if not all, studies of galactic nuclei dynamics assume a fixed BH (or BH binary) at the centre of coordinates. Genzel *et al.* (1997) had set a constraint of $\lesssim 10$ km/s for the speed of the BH relatively to the galactic plane, a constraint later refined to $\lesssim 2$ km/s (Backer & Sramek 1999; Reid & Brunthaler 2004). Stellar dynamics on scales of $\sim$ few pc surrounding Sgr A$\star$ is complex however, and the angular momentum distribution on that scale is a prime example of this complexity (Genzel *et al.* 2003). Reid *et al.* (2007) used maser emission maps to compute the mean velocity of 15 SiO emitters relatively to Sgr A⋆. They compute a mean (three-dimensional) velocity of up to 45 km/s from sampling a volume of ≃ 1 pc about the centre†. This raises the possibility that stars within the central stellar cusp experience significant streaming motion with respect to Sgr A⋆. The breaking radius $r_{br} \sim 0.2$ pc is suggestive of uncertain dynamics on that scale. Random, 'Brownian' BH motion may result from the expected high-deflection angle encounters (Merritt 2001, 2005; Merritt, Berczik & Laun 2007). Here we take another approach, and ask what net effect a BH set on a regular orbit will have on the stars. In doing so, we aim to fill an apparent gap in the modeling of BH dynamics in dense nuclei, by relaxing further the constraint that the hole be held fixed at the centre of coordinates. A full account will be found in Boily, Padmanabhan & Paiement (2007, MNRAS, in the press, hereafter BPP+07).

A rough calculation helps to get some orientation into the problem. Consider a point mass falling from rest from a radius R_o in the background potential of the MW stellar cusp. Let the radial mass profile of the cusp $\rho_\star(r) \propto r^{-3/2}$, consistent with MW kinematic data. If we define the BH radius of influence ≃ 1 pc to be the radius where the integrated mass $M_\star(<r)$ = the BH mass ≃ 3 to $4 \times 10^6 M_\odot$ (Genzel *et al.* 2003; Ghez *et al.* 2005), then R_o may be expressed in terms of the maximum BH speed in the MW potential as $[\max\{v\}/100\,\mathrm{km/s}]^{4/5} = R_o/1\,\mathrm{pc}$. For a maximum velocity in the range 10 to 40 km/s, this yields $R_o \simeq 0.3 - 0.5$ pc, or the same fraction of its radius of influence‡. We ask what impact this motion might have on the surrounding stars. To proceed further, let us focus on a circular stellar orbit outside R_o in the combined potential of the BH and an axisymmetric galaxy. When the BH is at rest at the centre of coordinates, the star draws a closed circular orbit of radius r and constant velocity v. We now set the BH on a radial path of amplitude R_o down the horizontal x-axis. Without loss of generality, let the angular frequency of the stellar orbit be $\omega_\star$, and that of the BH $\omega \geqslant \omega_\star$. The ratio $\omega/\omega_\star \geqslant 1$ is otherwise unbounded. The net force $\boldsymbol{F}$ acting on the star can always be expressed as the sum of a radial component $\boldsymbol{F}_r$ and a force parallel to the x-axis which we take to be of the form $F_x \cos(\omega t + \varphi)$; clearly the constant $F_x = 0$ when $R_o = 0$. The net mechanical work done on the star by the BH as the star completes one orbit is

$$\delta W = \int \boldsymbol{F} \cdot \boldsymbol{v} \mathrm{d}t = \int F_x v \sin(\omega_\star t) \cos(\omega t + \varphi)\, \mathrm{d}t \tag{1.1}$$

where φ is the relative phase between the stellar and BH orbits. The result of integrating (1.1) is set in terms of the variable $\nu \equiv \omega/\omega_\star$ as

$$\frac{2\delta W}{vF_x} = \frac{1}{\nu+1}\left[\cos(2\pi\nu + \varphi) - \cos(\varphi)\right] + \frac{1}{\nu-1}\left[\cos(2\pi\nu - \varphi) - \cos(\varphi)\right] \tag{1.2}$$

when $\nu > 1$, and $2\delta W/vF_x = 2\pi \sin(\varphi)$ when $\nu = 1$. Equation (1.2) encapsulates the essential physics, which is that δW changes sign when the phase φ shifts to $\varphi + \pi$. Thus whenever the stellar phase-space density is well sampled and all values of $\varphi : [0, 2\pi]$ are

† Statistical root-n noise ∼ 25% remains large owing to the small number of sources but is inconsequential to the argument being developed here.

‡ These figures are robust to details of the stellar cusp mass profile, so for instance a flat density profile ($\gamma = 0$) would yield R_o in the range 0.3 to 0.6 pc.

https://doi.org/10.1017/S1743921308015834 Published online by Cambridge University Press

realised with equal probability, half the stars receive mechanical energy ($\delta W > 0$) and half give off energy ($\delta W < 0$). In other words, stars in the first quadrant exchange energy with those in the third quadrant of a Cartesian coordinate system. (Similarly for those in the second and fourth quadrants.) By construction, the BH neither receives nor loses energy but merely acts as a *catalyst* for the redistribution of mechanical energy between the stars. Our goal, then, is to explore the consequences of this mechanism quantitatively for realistic stellar distribution functions.

We present a subset of results lifted from BPP+07 for the case of an BH orbiting in a logarithmic background potential. We focus on the effect of resonances and their likely detection at a distance equal to several times the BH radius of influence and show that streams that develop at the 5:2 resonance have larger Toomre parameters (hot streams).

2. Results

We set our problem in the framework of the logarithmic potential, which we write as

$$\Phi_g(\boldsymbol{r}) = -\tfrac{1}{2} v_o^2 \ln \left| \frac{R^2}{R_c^2} + 1 \right| \tag{2.1}$$

with v_o the constant circular velocity at large distances, and the radius R_c defines a volume inside of which the density is nearly constant. Thus when $r \ll R_c$ we have again harmonic motion of angular frequency $\omega = v_o/R_c$. If we let $q = 1$ and define $u \equiv r/R_c$, the integrated mass $M_g(u)$ reads

$$M_g(< u) = \frac{v_o^2 R_c}{G} \frac{u^3}{u^2 + 1}\,. \tag{2.2}$$

The mass $M_g(u \gg 1) \propto u$ diverges at large distances, however this is not a serious flaw since we consider only the region where $u \sim 1$. The mass $M_g(u = 1) = v_o^2 R_c/2G$ fixes a scale against which to compare the BH mass M_{bh}. Since the BH orbits within the harmonic core, we set

$$M_{bh} \equiv \tilde{m}_{bh} \frac{v_o^2 R_c}{2G} \tag{2.3}$$

with $0 < \tilde{m}_{bh} \leqslant 1$, and

$$\mathcal{M}(u) = 1 + \tilde{m}_{bh} \frac{1 + u^2}{u^3} \equiv \left(\frac{\omega_\star}{\omega} \right)^2$$

defines the position of orbital resonances when the frequencies are commensurate. The core radius offers a reference length to the problem. The position and velocity of the BH at any time follow from the equation of a harmonic oscillator, $\ddot{\boldsymbol{R}}(t) = R_o \sin(\omega t + \phi_o)\hat{\boldsymbol{x}}$, where the amplitude R_o defines the dimensionless number $u_o = R_o/R_c$ and $\hat{\boldsymbol{x}}$ is a directional unit vector. Our goal is to quantify the time-evolution of a large number of orbits in the combined logarithmic and BH potentials. If we pick parameters such that

$$m_\star \ll M_{bh} < M_g(\max\{u\})$$

then we may neglect the collective feedback of the stars on the BH and galactic potential and study only the response of individual orbits evolving in the time-dependent total potential. This approach will remain valid so long as the response of the stars are relatively modest. The time-evolution of orbits was done numerically using a standard integration scheme, see BPP+07 for details. As a specific case we chose dimension-less parameters $\tilde{m}_{bh} = u_o = 0.3$ (case 'C3' of BPP+07) with physical parameters $v_o = R_c = G = 1$. The radius of influence of the BH is then ≈ 0.58. We neglect the stars self-gravity. The stellar

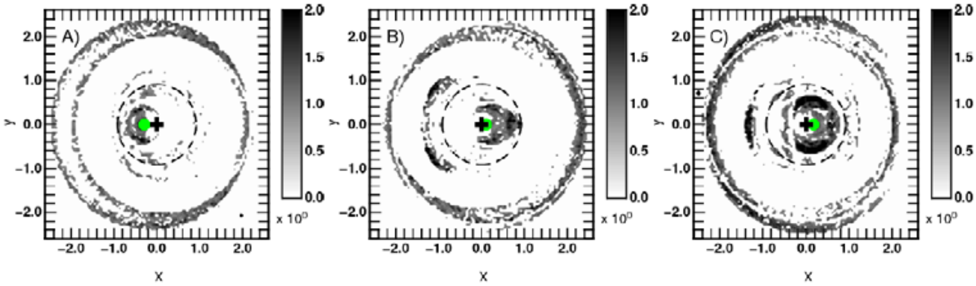

Figure 1. Maps of the Toomre number Q_J for a calculation with $\tilde{m}_{bh} = u_o = 0.3$ shown at times A) t = 15; B) t = 16 and C) t = 17. The dash circle marks the core length of the logarithmic potential, the cross is the origin of coordinates and the light dot marks the position of the BH. The shaded ring of radius $\simeq 2.2$ indicates large dispersion at the 5:2 resonance.

orbits were all co-planar with that of the BH but an extension to 3D only comforted our findings (see BPP+07).

2.1. *Resonances and hot streams*

Given the potential (2.1) it is straightforward to isolate for the radii where the circular orbital frequencies are in a commensurate ratio $m : n$ (see Table 2 of BPP+07). Inspection of the energy distribution function of the stars shows that BH motion induces highs and troughs when compared to the d.f. where the BH is fixed at the coordinate centre. The peaks seen in the d.f. match (roughly) the position of resonances, but, significantly, the d.f. is never steady because the potential varies continuously in time. A time-average of several snapshots, when all orbits are projected in space, shows that the largest resonances are still easily identifiable in the d.f., a result that would favour the detection of long-lasting hot streams (high velocity dispersion) at places where none is expected.

Our approach does not integrate the full response of the stars to their own density enhancements. These could become bound structures which would alter the dynamics globally. To inspect whether this could have an influence over the evolution of the velocity field, we computed the Toomre number

$$Q_J \equiv \frac{\sigma\Omega}{G\Sigma} = \frac{\sigma^2}{G\Sigma \mathrm{d}l}$$

on a mesh of 30×30 points in real space. We computed the dispersion σ with respect to the initial equilibrium flow; hence $\sigma = 0$ when the BH is at rest. The surface density Σ is calculated with an CIC algorithm. Stars are stable against self-gravitating local modes of fragmentation when $Q_J \gtrsim 1.7$ (Binney & Tremaine 1987). We applied a modified criterion for stability, because the disc is presumed initially stable against such modes, that is, when the BH is at the centre of coordinates. Because we subtracted from the local mean square velocity dispersion the value computed for the initial configuration, we set a conservative threshold for stability such that $Q_J > 1$. When that condition is satisfied, the BH contributes through its orbital motion more than 58% of the square velocity dispersion required to prevent local self-gravitating fragmentation modes from growing. Since the black hole already accounts for more than 50% to the gravity everywhere inside its radius of influence, it also provides the extra dispersion required to kill off all self-gravitating modes.

Fig. 1 maps out Q_J at three different times for the reference calculation; the dark shaded area have $Q_J > 1$ with an upper cutoff at 2, so white means instability on that figure. The outer dark ring at $u \simeq 2.2$ on the figure matches the position of the 5:2

https://doi.org/10.1017/S1743921308015834 Published online by Cambridge University Press

resonance. Thus it is very likely that structures that would cross this area would be heated up and disrupted as a result of BH motion.

2.2. *Comparison with MW data*

The ring seen on Fig. 1 may have consequences for the streams of stars observed at the centre of the MW (Genzel *et al.* 2003). The dimensions of this ring, about three times the BH radius of influence, would correspond to a radius of 3 pc in the MW. This should be an element to incorporate into future modelling of the MW centre since actual resolution power already resolves sub-parsec scales.

The line-of-sight velocity is also of interest. This is derived from individual snapshots by projecting the orbits on a 1D mesh and averaging by number. The largest values of v_{1d} were obtained from a viewing angle parallel to the motion of the BH. Contrasting these values to the local root mean square velocity dispersion σ, we find a ratio of $<|v_{1d}|>/\sigma \approx 25\%$ at maximum value, which occurs inside the hole's radius of influence. Applying this to MW data, where the mean velocity dispersion rises to $\sim$ 180 km/s inside 1 pc of Sgr A$\star$ (Genzel *et al.* 1996), we obtain streaming velocities in the range $\sim$ 40 km/s, a rough match to the values reported recently by Reid *et al.* (2007). The MW surface density profile shows a break at radius $r_{br} \sim 0.2$ pc (Schödel *et al.* 2007). Inside r_{br}, the volume density is fitted with a power-law index $\gamma \simeq 1.2$ which falls outside the range 3/2 to 7/4 of the Bahcall-Wolf solution. BH motion of an amplitude $R_o \sim r_{br}$ might cause such a break. The ratio $r_{br}/r_{bh} \sim 0.2$ is however lower than the value ≈ 0.5 adopted for the calculation discussed here. More detailed modelling in a galactic cusps is underway for a closer match to MW data.

Acknowledgments

CMB received a travel grant from IFAN which made possible a visit to IUCAA in 2007 where much of this work was carried out. We thank V. Debattista, T. Lauer and D. Merritt for comments.

References

Backer, D. C. & Sramek, R. A. 1999, *ApJ*, 524, 805
Bahcall, J. N. & Wolf, R. A 1977, *ApJ*, 216, 883
Binney, J. J. & Tremaine, S. D. 1987, *Galactic Dynamics*, Princeton: University Press
Boily, C. M., Padmanabhan, T., & Paiement A. 2007, *MNRAS*, astro-ph:0705.2756v2
Freitag, M., Amaro-Seoane, P., & Kalogera, V. 2006, *ApJ*, 649, 91
Genzel, R., Thatte, N., Krabbe, A., *et al.* 1996, *ApJ*, 472, 153
Genzel, R., Eckart, A., Ott, T., *et al.* 1997, *MNRAS*, 291, 219
Genzel, R., Schödel, R., Ott, T., *et al.* 2003, *ApJ*, 594, 812
Ghez, A. M., Salim, S., Hornstein, S. D., *et al.* 2005, *ApJ*, 620, 744
Merritt, D. 2001, *ApJ*, 556, 245
Merritt, D. 2005, *ApJ* 628, 673
Merritt, D. 2006, *RPPh*, 69, 2513
Merritt, D., Berczik, P., & Laun, F. 2007, *AJ*, 133, 553
O'Leary, R. M. & Loeb, A. 2006, submitted to *MNRAS*, astro-ph/0609046
Preto, M., Merritt, D., & Spurzem, R. 2004, *ApJ*, 613, L109
Reid, M. J. & Brunthaler, A. 2004, *ApJ* 616, 872
Reid, M. J., Menten, K. M., Trippe, S., *et al.* 2007, *ApJ*, 659, 378
Schödel, R., Eckart, A., Alexander, T., *et al.* 2007, astro-ph/0703178
Yu, Q. & Tremaine, S. D. 2003, *ApJ*, 599, 1129