

SIMPLICITY OF TWISTED C^* -ALGEBRAS OF TOPOLOGICAL HIGHER-RANK GRAPHS

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In a recent series of papers, Kumjian, Pask and Sims [2–5] have investigated the effect of ‘twisting’ C^* -algebras associated to higher-rank graphs using a categorical 2-cocycle on the graph. This work has included a characterisation of simplicity for these twisted C^* -algebras in terms of the underlying graphical and cohomological data. In this thesis, we initiate the study of twisted C^* -algebras associated to *topological* higher-rank graphs using groupoid techniques and we characterise simplicity of these C^* -algebras.

For each cofinal, proper, source-free topological higher-rank graph, and each continuous 2-cocycle on the associated boundary-path groupoid, we consider the twisted groupoid C^* -algebra in the sense of Renault. We show that the quotient of the boundary-path groupoid by the interior of its isotropy subgroupoid acts on the Cartesian product of the infinite-path space of the graph and the dual group of a particular subgroup of the periodicity group of the graph that is dependent on the cohomological data. We refer to this action as the *spectral action*.

To prove our simplicity characterisation, we first extend results of Brown, Nagy, Reznikoff, Sims and Williams [1] to characterise injectivity of homomorphisms of the reduced twisted C^* -algebra associated to any Hausdorff étale groupoid \mathcal{G} and continuous 2-cocycle on \mathcal{G} in terms of injectivity of homomorphisms of the reduced twisted C^* -algebra associated to the interior of the isotropy of \mathcal{G} . We apply this result to prove that a twisted C^* -algebra of a topological higher-rank graph is simple if the associated spectral action is minimal. We complete the proof of our characterisation by assuming that the spectral action is not minimal and constructing a nonzero representation of the twisted C^* -algebra with nontrivial kernel. Our characterisation of simplicity generalises the analogous result of Kumjian, Pask and Sims pertaining to twisted C^* -algebras of (discrete) higher-rank graphs.

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