SECTION VI

NONGRAVITATIONAL FORCES

NONGRAVITATIONAL FORCES ON COMETS: THE FIRST FIFTEEN YEARS

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ABSTRACT. The recent and current situation with regard to our analysis and understanding of the nongravitational effects in cometary motions is reviewed. Comets can be categorized according to the different physical situations that may exist in their nuclei. Further experimentation with theoretical models and empirical fits to observations is encouraged.

1. INTRODUCTION

Just 15 years ago this month, I published the first detailed paper that attempted to make a systematic study of the way in which nongravitational forces influence the motions of comets (Marsden 1969). The existence of such forces had been discussed on numerous occasions and in many contexts during the preceding 15 decades, but this was the first time that computations were presented in a uniform and mathematically rigorous manner for a number of different comets, using relatively general equations of motion that are capable of at least some physical interpretation. My motivation was twofold: (a) to improve the accuracy with which positions of comets could be predicted, and (b) to obtain insight into the physical nature of the forces.

This is not to say that earlier researchers did not have the same motivation. Encke (1820) was clearly concerned with the need for making accurate predictions when he introduced a nongravitational secular-acceleration term into his computations on the comet that bears his name, and he subsequently developed in considerable detail a theory (Encke 1831) relating this secular acceleration to the resistive coefficient of the medium in which the comet was supposed to move. Encke's theory was applied to other comets, and the resisting-medium hypothesis was finally abandoned only when it began to be suspected, a century later, that some comets experienced secular decelerations rather than accelerations. Although these secular variations in mean motion were available for only a handful of comets, Whipple (1950) put them to good use in his brilliant paper that introduced the concept of the icy-conglomerate model for a cometary nucleus. Since most of the predictions for the returns of comets were clearly affected by computational approx-

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imations and sometimes by downright errors, several astronomers (e.g., Roemer 1961) questioned whether physical nongravitational influences on cometary motions really existed. Clearly there was a need for a concerted attack on the problem, and the rapid advances in automatic computational capabilities during the 1960s provided the opportunity.

2. COMPUTATIONAL PROCEDURE

The n-body computer program by Schubart and Stumpff (1966) provided an excellent starting point, and with programing help from K. Aksnes it was combined with a differential-correction procedure in which the partial derivatives required are replaced by the differences in the comet's calculated coordinates that arise when small changes are made successively in each of the assumed orbital parameters. In a purely gravitational situation n would therefore be 17, the integration being done for the sun, the planets Mercury to Pluto, the nominal orbit of the comet and the six variations. This process is more accurate than the traditional one of calculating the partial derivatives analytically from the basic equations of the two-body problem, since the two-body equations are frequently a poor approximation to the true motion of a comet, particularly one that makes a close approach to Jupiter during the span of time covered by the observations under consideration. By iterating the solution, terms of higher order than the first are automatically eliminated, and the differences converge to true partial derivatives. The beauty of this process is that it can be readily adapted to include a nongravitational force. The nongravitational force can be any continuous function of the comet's position and velocity vectors and the time. Additional integrations must be done for each new nongravitational parameter introduced, and in a few of our computations n has been as high as 21. There is of course a danger that some of the additional parameters will be strongly correlated, and our experience has been that it is undesirable to introduce more than two additional parameters.

Accordingly, we considered the nongravitational force in the form of additional acceleration components F_1 directed along the comet's instantaneous radius vector outward from the sun, F_2 parallel to the line from the sun to the point in the instantaneous orbit 90° ahead of the comet, and F_3 perpendicular to the plane of the instantaneous orbit (to the "northern" side in the case of direct motion). Although the transverse component F_2, which can most closely be related to the secular variation of the earlier computations, obviously turned out to be the best determined of the components, it soon became clear that the radial component F_1 tended to be positive and an order of magnitude larger. This is of course precisely what is to be expected from the Whipple model, F_2 arising solely because there is a lag between the direction of maximum ejection of material from the comet nucleus and the subsolar point. The computations suggested that this lag is generally on the order of 5°-10°. When solutions were made for the normal component F_3, the results were comparable in magnitude to the transverse component, but since the determinacy of F_3 is poor it has generally been found advisable to ignore it. Since there seemed to be evidence that the secular acceleration of P/Encke was decreasing, perhaps exponentially to zero with time, exponential decay terms were introduced into our early nongravitational-force computations. The existence of P/Arend-Rigaux and P/Neujmin 1, each of which has both a generally inert appearance and no detectable nongravitational effect in its motion, added support to the hypothesis. However, with the recognition (Marsden 1970a) that the transverse component of P/Pons-Winnecke had changed sign the practice was abandoned.

In computations of this type there is in any case usually a large correlation between a parameter and its rate of change, and more insight, as well as greater determinacy, can be obtained by comparing values of the parameter from discrete solutions over spans of time long enough that the parameter can be reasonably determined, but short enough that there is no injustice to the residuals. Although the gradual deactivation of a comet with time presumably plays a role, at least for some comets, it is apparent that the nongravitational parameters as we have defined them can also be influenced to some extent by large changes in a comet's orbit following a close approach to Jupiter, and presumably even more so by variations in the orientation of the comet's axis of rotation.

Although the idea that some of the Apollo objects and other unusual minor planets are defunct cometary nuclei has been somewhat out of vogue, it has recently received new fuel with the discovery of the Geminid parent 1983 TB, as well as the discoveries of 1982 YA, 1983 SA, 1983 XF and 1984 BC and their anticipated close approaches to Jupiter. The possibility that some objects that appear to be asteroidal should show nongravitational effects (Ziołkowski 1983) is of course interesting, but the necessary computations are extremely intricate, and there is always the danger that the small residuals from the gravitational solutions have some other cause. A "nongravitational effect" suspected in the motion of (944) Hidalgo could be much more logically explained by an adjustment to the adopted mass of Saturn (Marsden 1970b). In any case, not all cometary nuclei may deactivate. If a comet nucleus should instead completely disperse into meteoroids, the nongravitational force, representing relative mass loss, should show a large increase. Erratic behavior of the rotation axis in a comet's dying stages would be expected to produce even wilder changes in the nongravitational parameters. Such changes were apparent in the motion of P/Brorsen, and they appear to have become extreme between the comet's 1873 return and its last observed appearance in 1879.

From a physical point of view it is highly desirable to consider the variation of the nongravitational force with the comet's heliocentric distance r. After utilizing for some time an entirely arbitrary dependence on a combination of an inverse power and a diminishing exponential, we adopted for the F_i what has been variously termed "Style II" or the "standard model", namely (Marsden et al. 1973):

$$F_i = A_i\, g(r) \qquad (i = 1, 2, 3), \tag{1}$$

where the A_i are constants and

$$g(r) = C\,(r/r_o)^{-m}\,[1 + (r/r_o)^n]^{-k}, \tag{2}$$

with $m = 2.15$, $n = 5.093$, $k = 4.6142$, and the normalizing constant $C = 0.1113$. The dependence is essentially that of an inverse-square law out to the vicinity of $r_o = 2.808$ AU, and beyond that distance there is a much more rapid decrease. The form of $g(r)$, suggested by Z. Sekanina, was fitted by A. Delsemme to a curve (Delsemme and Miller 1971) showing the vaporization flux of water snow with heliocentric distance. Since other likely constituents of cometary ices are much more volatile than water, their corresponding transition distances r_o are much larger--almost 8 AU even in the case of ammonia. Application to actual nongravitational orbit solutions for several comets suggested that r_o was not larger than 4 AU, thereby apparently confirming the general belief that water is the principal icy constituent of a comet nucleus.

3. RESULTS

Although the majority of the available solutions for nongravitational parameters are still those published by myself, frequently in collaboration with Sekanina, generally in a series of papers in the Astronomical Journal up to 1974, D. K. Yeomans has also been a very important contributor to this work, beginning with his study of P/Giacobini-Zinner (Yeomans 1971). The day has not yet arrived when predictions for the returns of periodic comets routinely allow for nongravitational effects, but since about 1978 a number of nongravitational orbit solutions, all using the "standard model", have also been made by S. Nakano, by W. Landgraf, and most recently by G. Forti. Because F_2 (or A_2) is a measure of the rate of change of revolution period, nongravitational orbit solutions generally become appropriate when a comet has been observed at a third perihelion passage.

The first five sections of Table I list the 63 comets that fall into this category and summarize the situation with regard to the computation of nongravitational forces. In each section the comets are arranged in order of increasing perihelion distance. Section 2, which is the largest, contains comets that have non-zero transverse parameters A_2 that are either steady or decreasing in magnitude with time. For the most part these are well-behaved, predictable comets, generally consistent with the deactivation/rotation-variation scenario. Some of the large values of A_2 seem to be associated with objects that had recently been thrown in by Jupiter from orbits of larger perihelion distance, but the extreme value for P/Gunn might be influenced by the fact that in this case the solution actually included a pre-approach, prediscovery observation. Section 1 shows comets for which no definite nongravitational parameters have been detected. In addition to the afore-mentioned "asteroidal" comets, this section contains the three three-apparition comets with perihelion distances above the transition distance for water--another indication that water is the principal cometary constituent; and in spite of the outbursts in brightness of P/Schwassmann-Wachmann 1 that might at first sight be expected to influence the comet's motion. The apparent absence (or near absence) of nongravitational forces acting on P/Crommelin over an interval of more than a century is surprising in view of this comet's rather small perihelion distance and known large brightness surge near perihelion. Computations by Kamieński (1959) indicated that P/Wolf did experience a secular de-

TABLE I. SUMMARY OF NONGRAVITATIONAL INVESTIGATIONS

Comet		q	P	N	Interval	A_1	A_2	Comp.
1. No-nongravitational effects								
Crommelin		0.7	28	5	1873-1984	0.0	0.000	L,M,Y
Arend-Rigaux	a	1.4	7	5	1951-1978	0.0	0.000	M
Tempel 1		1.5	6	7	1967-1983	0.0	0.000	M
Tsuchinshan 1		1.5	7	3	1965-1978	0.0	0.000	L,S
Neujmin 1	a	1.5	18	5	1913-1966	0.0	0.000	M
Reinmuth 2		1.9	7	6	1947-1967	0.0	0.000	M
Neujmin 3		2.0	9	3	1929-1972	0.0	0.000	M
Holmes	o	2.2	7	6	1964-1980	0.0	0.000	M
Wolf		2.5	8	13	1925-1967	0.0	0.000	P,Y
Oterma		3.4	8	3	1942-1962	0.0	0.000	M
Smirnova-Cher.		3.6	9	3	1967-1983	0.0	0.000	N
Schwass.-W. 1	o	5.4	15	5	1902-1983	0.0	0.000	M
2. Stable or decreasing nongravitational effects								
Encke		0.3	3	53	1967-1980	-0.1	-0.004	M
Halley		0.6	76	30	1835-1982	+0.1	+0.015	L,Y
Tempel-Tuttle		1.0	33	4	1865-1965	0.0	+0.009	Y
Grigg-Skjellerup		1.0	5	14	1952-1972	0.0	-0.001	M,S
Tuttle		1.0	14	10	1939-1980	+0.1	+0.013	Y
Finlay		1.1	7	10	1960-1974	+0.3	+0.020	Y
Olbers		1.2	70	3	1815-1956	+0.2	+0.065	Y
Wirtanen		1.3	6	5	1948-1975	+0.5	-0.087	M
d'Arrest		1.3	6	14	1963-1977	+0.6	+0.120	M,Y
Churyumov-G.		1.3	7	3	1969-1982	0.0	+0.012	M
Borrelly		1.3	7	10	1932-1975	+0.1	-0.038	Y
Wolf-Harrington		1.6	7	6	1951-1978	+0.2	-0.049	N,S
Stephan-Oterma		1.6	38	3	1867-1981	+0.2	-0.003	Y
Daniel		1.7	7	6	1937-1964	+1.1	+0.078	M
Tsuchinshan 2		1.8	7	3	1965-1978	-1.2	-0.004	L
Arend		1.8	8	5	1951-1976	+0.1	-0.029	M
Brooks 2	s	1.8	7	12	1946-1961	+1.1	-0.191	M
Reinmuth 1		2.0	8	7	1928-1973	+0.2	-0.028	M
Schwass.-W. 2		2.1	6	9	1956-1981	+2.0	-0.174	F,M,N
Johnson		2.2	7	6	1956-1977	+0.8	-0.027	N
Kearns-Kwee		2.2	9	3	1963-1982	0.0	-0.404	F
Ashbrook-Jackson		2.3	7	5	1948-1979	0.0	-0.012	F
Gunn		2.5	7	4	1954-1982	+2.4	+0.613	M
Whipple		2.5	7	7	1947-1978	+0.6	-0.044	M

3. Slightly increasing nongravitational effects

Honda-Mrkos-P.		0.6	5	6	1969-1980	+0.1	-0.046	M
Giacobini-Zinner		1.0	7	11	1965-1978	-0.2	-0.046	Y
Schaumasse		1.2	8	6	1944-1960	+0.4	-0.041	M
Pons-Winnecke		1.3	6	19	1951-1976	0.0	+0.002	M
Tempel 2		1.4	5	17	1956-1978	+0.1	+0.002	M
Kopff		1.6	6	12	1958-1977	+0.3	-0.084	Y
Faye		1.6	7	17	1954-1977	+0.1	-0.003	L,M,N
Comas Solá		1.9	9	7	1960-1979	+0.8	-0.093	F,M

4. Significantly increasing or wild nongravitational effects

Brorsen	d	0.6	5	5	1868-1879	+1.3	+0.134	L,M
Pons-Brooks	op	0.8	71	3	1812-1954	-0.1	-0.027	Y
Biela	ds	0.9	7	6	1832-1852	+1.2	-0.094	L,M
Tuttle-G.-K.	o	1.1	6	6	1951-1973	+0.7	+0.022	M
Tempel-Swift	d	1.2	6	4	1869-1908	+0.1	-0.113	M
Perrine-Mrkos	d	1.3	7	5	1896-1955	-0.1	-0.060	M
Forbes		1.5	6	6	1961-1980	+0.5	-0.078	M

5. Generally unstudied comets of more than two apparitions

Swift-Gehrels		1.4	9	3	(1889-1982)	+
Jackson-Neujmin		1.4	8	3	(1936-1978)	-
Clark		1.6	6	3	(1973-1984)	0
Harrington		1.6	7	3	(1953-1980)	+
de Vico-Swift		1.6	6	3	(1844-1965)	+
du Toit-N.-D.		1.7	6	3	(1941-1983)	-
Harrington-Abell		1.8	8	5	(1955-1984)	0
Väisälä 1		1.8	11	5	(1939-1982)	0
Taylor	s	2.0	7	3	(1915-1984)	-
Shajn-Schaldach		2.2	7	3	(1949-1978)	0
Van Biesbroeck		2.4	12	3	(1954-1979)	0
Slaughter-B.		2.5	12	3	(1958-1981)	+

6. Two-apparition comets, possible nongravitational effects

Brorsen-M.	p	0.5	72	2	(1847-1919)
Denning-F.		0.8	9	2	(1889-1978)
Schwass.-W. 3		0.9	5	2	(1930-1979)
Gale	dp	1.2	11	2	(1927-1938)
du Toit-H.	s	1.2	5	2	(1945-1982)
Westphal	dp	1.3	62	2	(1852-1913)
Neujmin 2	d	1.3	5	2	(1916-1927)
Peters-Hartley		1.6	8	2	(1846-1982)

The columns q, P and N give the perihelion distance (in AU), period (in yr) and number of apparitions. A_1 and A_2 are representative values, valid for the interval shown. The column Comp. shows the orbit computers: F = G. Forti, L = W. Landgraf, M = B. G. Marsden, N = S. Nakano, P = E. I. Kazimirchak-Polonskaya, S = G. Sitarski, Y = D. K. Yeomans. The notes preceding the column q are: a = asteroidal, d = disappeared, o = outbursts, p = poor fit, s = split.

celeration, particularly before the comet's close approach to Jupiter in 1922. Nongravitational effects whould also presumably be detectable for objects like P/Tempel 1 and P/Holmes if the older observations of these comets (each of which was lost for a long time) were tied in with the recent ones.

Section 3 of Table I shows comets for which A_2 is currently slightly increasing, and Section 4 shows those where there has perhaps been a larger rate of increase, or possibly some particularly erratic behavior. Although ascription of a comet to one or the other of these sections could be debated, it is thought that the comets in Section 3 are well-behaved objects whose A_2 values have increased due to straightforward variations in the axis of rotation, whereas Section 4 contains comets that have split, experienced irregular outbursts, disappeared--or are perhaps about to do so.

Section 5 lists comets for which satisfactory nongravitational-force studies have not yet been made, although in some cases the probable sign of A_2 can be indicated. Section 6 lists eight comets that have been observed at only two perihelion passages but that might merit nongravitational investigation. Among these are potential Section 4 comets gravitational orbit solutions for which are known to be unsatisfactory (P/Gale, P/Westphal, P/Neujmin 2), as well as cases where the apparitions are widely separated in time.

Nongravitational effects are sometimes evident in the motions of comets that have been observed at only a single perihelion passage. There are in fact at least eight known long-period comets for which one cannot make satisfactory orbit determinations if nongravitational forces are ignored, and when nongravitational solutions have been attempted the results are not out of line with those for the short-period comets. Since a positive radial nongravitational component has the effect of making a comet's inverse semimajor axis larger than it would otherwise be, allowance for nongravitational effects could clearly eliminate those few cases of comets that otherwise seem to have "original" orbits that are hyperbolic. This has implications for the size of the Oort cloud (Marsden et al. 1978).

Two other expressions have been discussed for handling the cometary nongravitational effects in terms of equations of motion in rectangular coordinates. The first of these (Brady and Carpenter 1971) consisted simply of a radial component proportional to the inverse-square of heliocentric distance and changing linearly with time. Although this expression amply described the motion of P/Halley since 1682 and produced a perfectly satisfactory prediction for 1986, it is difficult to ascribe any physical meaning to it, and the explanation (Brady 1972) in terms of perturbations by a Jupiter-sized planet in a highly-inclined orbit 65 AU from the sun is obviously unacceptable. Although he continues to postulate that the nongravitational effect consists solely of the traditional secular variation in the mean motion, Sitarski (1981) has expressed this directly in terms of the equations of motion in rectangular coordinates. He has also developed an accurate and highly ingeneous procedure for solving simultaneously for the Keplerian elements and secular variation and has successfully applied it to a few comets. A rigorous comparison between Sitarski's model and ours is not possible, but to assume a constant secular variation in mean motion roughly supposes that F_2 is proportional to $1/r$.

4. NEW DEVELOPMENTS

In recent years several groups (e.g., Weissman and Kieffer 1981) have attempted to model the distribution of temperature over a cometary nucleus, and Rickman and Froeschlé (1982) have used such a thermal model to examine the variation of the nongravitational force with heliocentric distance in the case of P/Halley. They have found the "standard model" to be lacking in the sense that the reaction of the comet to variations in surface temperature can cause the parameters A_1 and A_2, which we have taken to be constant (or at least not to have any short-term variations), to vary by perhaps as much as a <u>factor</u> of 100 as r ranges from 0.6 to 4.0 AU! The actual variation--in fact, whether A_1 and A_2 are increasing or decreasing over this range--is very dependent on the comet's thermal inertia, but it would seem that, near the sun, A_1 is essentially constant and A_2 varies as r^2, with the result that F_2 is then constant. Beyond some transition distance the nongravitational force would diminish much more rapidly with distance than given in the Delsemme formula. There is also some asymmetry with respect to perihelion. Landgraf (1984) has very recently applied the Rickman-Froeschlé model in an exhaustive examination of the orbit of P/Halley over 1607-1984, solutions being made over a range of a factor of eight in thermal inertia and a factor of five in the comet's rotation period. In addition to considering the thermally induced effect, he assumed the presence of a factor $1 - Bt$ in both F_1 and F_2. The mean residuals of the observations from his various computations are identical, and he in fact obtained a result with precisely the same mean residual when he applied the "standard model" (modified again with the factor $1 - Bt$) to the same data. The spread of the values he obtained for A_1 and B according to the Rickman-Froeschlé model is not large and essentially encompasses the values given by the standard model. On the other hand, his various Rickman-Froeschlé values for A_2 differ by up to a <u>factor</u> of six, and the smallest value is 70 times greater than the A_2 given by the standard model!

Since A_2 (or the corresponding secular variation of the mean motion) is the basic quantity appearing in all studies of cometary nongravitational motion, it might appear that some drastic rethinking is necessary. More experimentation with the nongravitational forces associated with various thermal models is clearly very desirable, but in spite of the current interest in P/Halley and in the need for accurate predictions to ensure the success of the space missions in 1986, we should not delude ourselves into thinking that we shall be able to come up with <u>the</u> model that will give an extremely accurate prediction. The success of the probe will still be governed by the accuracy of the astrometric observations made just before encounter. If the function g(r) is systematically deficient in the way a comet nucleus actually reacts to the ejection of material, it would be appropriate to abandon it in favor of something else. That something else would presumably be different for F_1 and F_2, and there might be dependence on $\dot{r}$ as well as on r. If a satisfactory, not overly complicated, continuous function, applicable to all comets, cannot be produced, however, the "standard model" will continue to be useful in predicting the orbits of comets and in allowing some kind of comparison to be made between the characteristics of one comet and another.

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DISCUSSION

Weissman: We have just modeled P/Halley using Sekanina's new rotation rotation pole and 41.5-hr period, and we find that the temperature distribution is so symmetrical that it is difficult to believe there could be any transverse nongravitational force. Thus, if we assume any physically reasonable surface material, we have trouble believing the long rotation period. We would prefer the 10.3-hr or 14-hr periods that have been suggested.

Marsden: In view of the discouragingly large range of parameters suggested by Rickman and Froeschlé and considered by Landgraf, it is nice to know that there are perhaps a few constraints on the situation. Unfortunately, the recent photometric data do not seem to give an unambiguous value for the rotation period, and I am inclined to agree with R. M. West that the light curve is being significantly affected by the comet's intrinsic activity, even at its present great distance.