SIXTH LECTURE

Helicopter Rotors

By A McCLEMENTS, A M I MECH E
A R T S (GLASGOW)

A lecture presented to the Helicopter Association of Great Britain, on Saturday, 20th September, 1947, at Manson House, 26 Portland Place, London, W 1

H A MARSH, A F C, A F R Ae S, IN THE CHAIR

Introduction by the Chairman

Ladies and Gentlemen,

It is with pleasure that I introduce to you today Mr McClements, who is going to talk to us on the subject of "Helicopter Rotors"

Mr McClements is a relatively newcomer to the rotary wing industry and only claims a modest two years or so of direct connection, first with the Ministry of Supply and now as experimental engineer to the Helicopter Development Unit of British European Airways Corporation Apart from this, it is quite obvious that he has given a great deal of thought and work to the subject of his lecture

Speaking from personal experience, I can assure you that he was invaluable to all those in the industry during his time at the Ministry of Supply and will, I am sure, be equally so in his present appointment

On behalf of the Association, may I welcome our guests and trust you will be well rewarded for coming along

Mr A McClements

INTRODUCTION

In this paper certain mathematical relationships are derived which it is thought will be helpful in the detailed consideration of rotor designs and during study of the influence of the rotor on the aircraft as a whole

In general, the contents of the paper are straightforward insofar as they are statements of fact However, this is not so right throughout the work because some of the assumptions made are based on incomplete data and are therefore likely to lead to controversy In making such assumptions this possibility is appreciated, but, rather than omit them, they are included in the belief that the resulting discussion will be of general interest

https://doi.org/10.1017/S2753447200000123 Published online by Cambridge University Press

In considering the factors which influence the effect of the blade on the aircraft as a whole it soon becomes apparent that their number of possible combinations can lead to complexity unless simplifications are resorted to In order, then, to derive expressions which are both manageable and useful, the treatment here adopted is limited where necessary to extreme cases within which operational conditions are thought likely to prevail

A complete study of this subject would take account of the power parameter throughout This is not done in the present work and the omission should be noted at the outset for the purpose of appreciating the limitations of some of the expressions derived

In order to explore general trends it has been necessary to assume values for certain constants and for such parameters as disc loading and tip speed The values so used for the constants are based on current design experience and the tip speed and disc loading adopted are chosen as representative of a conventional medium speed machine

2 SUMMARY

(The paper is in 2 parts)

In PART 1 basic relationships of a general nature are derived between such variables as blade weight, centrifugal force, moment of inertia, aerodynamic lift, coning angle, and such dimensions as the radial positions of the blade centre of gravity, centre of percussion, radius of gyration and centre of resultant lift These basic relationships are used to derive the general equation of blade equilibrium The general equation of blade equilibrium, used in conjunction with the basic relationships, enables expressions to be determined which define the following within the limits of the assumptions stated in para 4

(*a*) Coning angle for maximum axial rotor lift on the assumption that the blade has no acceleration in the flapping plane about the flapping hinge
Note —Consideration of power requirements would show that this angle would never be used as a steady design condition , it is, however, of academic interest

(*b*) Blade weight/Aircraft weight ratio This ratio is determined in terms of rotor angular velocity, rotor radius and coning angle
Note —While it is shown that increase in coning angle results in a depreciation of this ratio, it does not follow that it is a good thing from the weight viewpoint to adopt lirge hovering coning angles Large coning angles necessitate an increase in rotor power for the same all-up-weight and the power parameter must be introduced to get an overall appreciation of the effect of coning angle on payload This is done in (*c*) under

(*c*) Best hovering coning angle from viewpoint of weight economy for any given engine power

(*d*) Loss in useful load when a coning angle other than the best from the weight viewpoint is adopted

(*e*) Change in coning angle with change in blade lift

https://doi.org/10.1017/S2753447200000123 Published online by Cambridge University Press

(*f*) Aircraft acceleration in the direction of the rotor axis of rotation and maximum angle of blade flap during acceleration
Note —It should be understood that accelerations arising from increases in blade lift coefficient of appreciable magnitude are likely to be of short duration and made possible by the necessary power increase being supplied from the rotor kinetic energy Such accelerations, while unlikely to be maintained, are important from the stressing viewpoint

(*g*) Time taken for the blade to move from one coning angle to another when the blade lift coefficient is suddenly changed

In Part 2 the aircraft is broken down into various components and the ratio of the weight of each of these components to the all-up-weight is studied in relation to the rotor radius On the basis of the assumed manner in which the aircraft component parts vary with the rotor size, and, in particular, that —

(i) the ratio of part of the transmission weight to the all-up-weight varies as the square of the rotor radius , and
(ii) the disc loading and tip speed do not vary as the rotor radius is changed ,

expressions are derived from which the following quantities can be studied —

(*a*) rotor size for maximum useful load lifted , and
(*b*) rotor size for maximum useful load/all-up-weight ratio

PART 2 of this paper is meant to apply only to machines having conventional engines, since the expressions derived are not necessarily applicable to jet driven rotors with simplified transmission systems

3 LIST OF SYMBOLS

Blade radial length	R Ft
Blade weight per unit length at radius R	w Lbs/Ft
Total blade weight	Wb Lbs
Moment of blade weight about flapping hinge	MWb Lbs Ft
Resultant blade centrifugal force	C_F Lbs
Moment of resultant blade centrifugal force about flapping hinge	Mc_F Lbs Ft
Aerodynamic lift per unit length of blade at radius R	l Lbs/Ft
Resultant aerodynamic lift on blade	L Lbs
Resultant aerodynamic lift during hovering	L_H Lbs
Maximum resultant aerodynamic lift on blade	L_M Lbs
Moment of aerodynamic lift about flapping hinge	M_L Lbs Ft
Axial component of resultant blade lift	L cos β_0Lbs
Coning angle of blade	β_0 RADIANS
Coning angle of blade for maximum axial lift component, when the blade has no flapping acceleration	β_{M_1} RADIANS

Coning angle fixed by maximum lift coefficient	β_M RADIANS
Coning angle of blade during hovering	β_H RADIANS
Hovering coning angle for maximum useful load	β_{H_1} RADIANS
Maximum angle of blade flap resulting from blade acceleration about flapping hinge	β_F RADIANS
Blade flapping velocity	β_0 RADS/SEC
Blade flapping acceleration	β_0 RADS/SEC^2
Blade lift coefficient	C_L
Maximum blade lift coefficient	C_{L_M}
Blade lift coefficient during hovering	C_{L_H}
Angular velocity of rotor about axis of rotation	ω RADS/SEC
Blade tip speed	$\cos\beta_0 \times \omega R$ FT/SEC
Vertical acceleration of aircraft when β_0 is zero	$N\beta_0$ g FT/SEC^2
Vertical acceleration of aircraft (general case)	N g FT/SEC^2
Maximum vertical acceleration which aircraft can experience when β_0 is zero	N_{β_0} (max) FT/SEC^2
Radial position of blade centre of gravity	k_1 R FT
Radial position of resultant centrifugal force on blade (i e, centre of percussion)	k_2 R FT
Position of resultant aerodynamic lift on blade	$k_3 \tilde{R}$ FT
Moment of inertia of blade about flapping hinge	I Lb FT SEC^2
Radius of gyration of blade about flapping hinge	k R FT
Ratio $\frac{\text{Axial lift component}}{\text{Blade weight}}$, when $\beta_0 = 0$	$\mathcal{E}\beta_0$
Ratio $\frac{\text{Axial lift component}}{\text{Blade weight}}$ (general case)	$\mathcal{E}$
Rotational energy of blade about flapping hinge when vel is β_0	= KE FT Lbs
Time taken for blade to cone from one angle to another	t SECS
Revolutions made by rotor in time t	n REVS
Aircraft all-up-weight	W Lbs
Weight of transmission parts dependent on rotor torque	WT_1 Lbs
Weight of transmission parts independent of rotor torque	WT_2 Lbs

Weight of engine and power plant	W_E Lbs
Weight of airframe, undercarriage, furnishings, etc	W_F Lbs
Weight of tail rotor blades	Wb_T Lbs
Weight of crew	Wc Lbs
Useful load	Wu Lbs
Maximum useful load	W_u (max) Lbs
Aircraft all-up-weight minus blade weight	W_A Lbs
Maximum value of W_A	W_A (max) Lbs
Ratio $\frac{\text{Wu (max)}}{\text{WA (max)}}$	×
Disc loading	DL Lbs /FT^2
No of blades per rotor	Z

PART 1

4 ASSUMPTIONS

(*a*) The blade flapping hinge is on the axis of rotation While rotors frequently have off-set flapping hinges the amount of off-set is usually small and unlikely to have any significance in the formulae derived

(*b*) The blade lift acts normal to the blade surface , thus the effect of radial air flow is ignored Since the effect of any radial flow on the direction of the resultant lift vector can only be of secondary importance it is felt that this assumption is justified

(*c*) If the power input to the rotor is constant and the rotor angular speed is constant, the length of the resultant blade lift vector is constant for all coning angles from zero up to those in which we are likely to be interested This assumption is unlikely to hold over a large range of coning angles, but it is probably accurate to a close order of approximation for coning angles from zero up to at least 15°

(*d*) The aircraft has air speed only in the vertical direction

(*e*) Bending in the blade is ignored Blade deflection will reflect on the values of the aircraft momentary accelerations derived but are unlikely to seriously influence the limiting values

(*f*) Air damping on the blade in the flapping plane is ignored The effect of this assumption will be to under-estimate the time taken for the blade to move from one coning angle to another, but the assumption is unlikely to influence the order of the result which is of interest as distinct from its absolute value

(*g*) In investigating general trends the values of the blade constants are assumed to be —

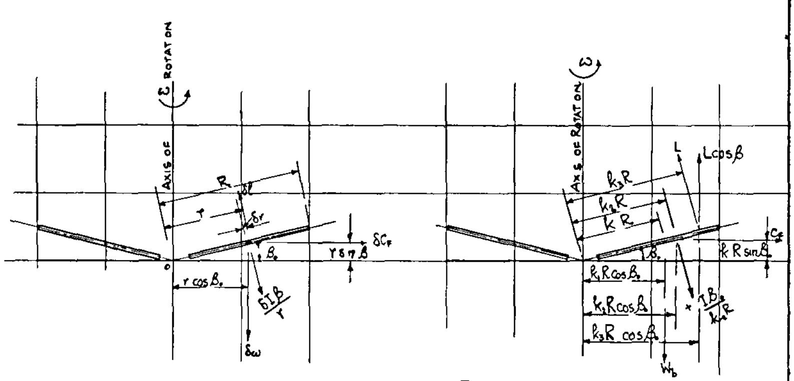
k_1 = 0 42 where k_1R is the radial position of the blade C G

k_2 = 0 56 where k_2R is the radial position of the blade centre of percussion

k_3 = 0 72 where k_3R is the radial position of the resultant blade lift

The value of these constants will vary from one design of blade to another but probably not greatly since blades are similar insofar as they are long and narrow and of like mass distribution The chosen values are based on current design experience

(*h*) Momentary loads on the blades resulting in movements in excess of the hovering coning angle are assumed to cause no loss in blade rotational speed This is an extreme case which is probably approached in practice because of the high rotational inertia of the rotor maintaining angular speed constant during short periods of excess blade flapping displacement



FORCES ACTING ON BLADE ELEMENT OF LENGTH δl

RESULTANT FORCES ACTING ON BLADE

FIG 1

FIG 2

5 BASIC RELATIONSHIPS

The following basic relationships used throughout the paper are derived from figure 1 and are applicable to any blade

(i) Blade weight $W_b = \int_0^R w\,dr$

(ii) Moment of blade weight about flapping hinge $M_{W_b} = \cos\beta_0 \int_0^R wr\,dr$

(iii) Blade centrifugal force $C_F = \dfrac{\omega^2 \cos\beta_0}{g} \int_0^R wr\,dr$

$= \dfrac{\omega^2 M_{W_b}}{g}$

(iv) Moment of blade centrifugal force about flapping hinge $M_{C_F} = \dfrac{\omega^2 \cos\beta_0 \sin\beta_0}{g} \int_0^R wr^2\,dr$

(v) Moment of inertia of blade about flapping hinge

$$I = \frac{1}{g}\int_0^R wr^2\,dr = \frac{M_{C_F}}{\omega^2 \cos\beta_0 \sin\beta_0}$$

(vi) Lift on blade

$$L \propto C_L\ \omega^2 \cos^2\beta_0 \int_0^R r^2\,dr$$

(vii) Moment of blade lift about flapping hinge

$$M_L \propto C_L\ \omega^2 \cos^2\beta_0 \int_0^R r^3\,dr$$

The resultant forces mentioned above are shown acting on the blade in Fig 2 From Fig 2 it follows that—

(viii) $M_{W_b} = Wb\ k_1\ R \cos\beta_0$

(ix) $M_{C_F} = C_F\ k_2\ R \sin\beta_0$

(x) $M_L = L\ k_3\ R$

(xi) $I_{\beta_0} = \frac{Wb}{g}\ k^2R^2\ \beta_0$

Now $\frac{Wb}{g}\ k^2R^2 = \frac{1}{g}\int_0^R wdr\ k^2R^2$ from (i)

$= \frac{1}{g}\int_0^R wr^2\,dr$ from (v)

i e $k^2 = \dfrac{\int_0^R wr^2\,dr}{R^2\int_0^R wdr}$ (a)

Also, $M_{W_b} = \cos\beta_0 \int_0^R wr\,dr$ from (ii)

$= \cos\beta_0 \int_0^R wdr\ k_1\ R$ from (i) & (viii)

i e $k_1 = \dfrac{\int_0^R wr\,dr}{R\int_0^R wdr}$

https://doi.org/10.1017/S2753447200000123 Published online by Cambridge University Press

$$\text{Again, } M_{CF} = \frac{\omega^2 \cos \beta_0 \sin \beta_0}{g} \int_0^R wr^2\, dr \qquad \text{from (iv)}$$

$$= \frac{\omega^2 \cos \beta_0 \sin \beta_0}{g} \int_0^R wr\, dr\, k_2 R \qquad \text{from (iii) \& (ix)}$$

$$k_2 = \frac{\int_0^R wr^2\, dr}{R \int_0^R wr\, dr} \qquad \text{(b)}$$

$$\text{Hence, } k_1 \times k_2 = \frac{\int_0^R wr^2\, dr}{R^2 \int_0^R w\, dr} = k^2 \qquad \text{from (a)}$$

$$\text{Hence, } I\beta_0 = \frac{Wb}{g} k_1\, k_2\, R^2\, \beta_0$$

$$= \frac{MW_b\, k_2\, R\, \beta_0}{g \cos \beta_0} \qquad \text{(c)}$$

(xii) If position of resultant inertia force on blade $= k_4$ R,

$$\text{then } \frac{k_4 R}{g} \int_0^R wr\, dr\, \beta_0 = \frac{1}{g} \int_0^R wr^2\, dr\, \beta_0 \qquad \text{from (ii) \& (v)}$$

$$\text{i e } k_4 = \frac{\int_0^R wr^2\, dr}{R \int_0^R wr\, dr} = k_2 \qquad \text{from (b)}$$

Hence, the resultant centrifugal and inertia forces act through the same point, *i e*, at a radius equal to k_2 R which is the blade centre of percussion

6 FUNDAMENTAL EQUATION DEFINING EQUILIBRIUM OF BLADE ABOUT FLAPPING HINGE

This relationship is obtained by considering the equilibrium of the blade about its flapping hinge The couples acting on the blade in the flapping plane are added algebraically and equated to zero The directions of all the couples are constant except the inertia couple which acts in opposition to the lift couple while the blade is being accelerated upwards and with the lift couple when the upward motion of the blade is being retarded

The inertia couple is taken as —ve and +ve respectively when it acts with and against the lift couple Taking moments about the flapping hinge, it follows that —

$$M_L - M_{W_b} - M_{C_F} - I\beta_0 = 0$$

From basic relations (iii) and C

$$M_L - M_{W_b} - \frac{\omega^2}{g} M_{W_b}\, k_2\, R \sin\beta_0 - \frac{M_{W_b}\, k_2\, R\, \beta_0}{g\cos\beta_0} = 0$$

i e, from basic relations (viii) and (x)

$$\frac{L}{W_b} = \frac{k_1 \cos\beta_0 \left[1 + \frac{k_2 (\omega R)^2 \sin\beta_0}{gR} + \frac{k_2 R \beta_0}{g\cos\beta_0} \right]}{k_3} \qquad 1\text{A}$$

The axial component of lift = L cos β_0

Let $\mathcal{E} = \frac{\text{Axial Component of lift,}}{\text{Blade Weight}}$

so it follows that

$$\mathcal{E} = \frac{L\cos\beta_0}{W_b} = \frac{k_1 \cos^2\beta_0 \left[1 + \frac{k_2 \sin\beta_0 (\omega R)^2}{gR} + \frac{k_2 R \beta_0}{g\cos\beta_0} \right]}{k_3} \qquad 1$$

It will be observed from equation 1 that the axial component of lift from a particular rotor running at constant ω R can be defined alone by constants, coning angle and blade acceleration in the flapping plane Since blade acceleration is a transient condition the ratio $\mathcal{E}$ may be considered within the limits of two conditions, viz —

(*a*) with respect to β_0 alone

and (*b*) with respect to β_0 and β_0

Consideration of (*a*) will lead, among other things, to the choice of coning angle which will result in maximum useful load , also, from condition (*a*) the acceleration imposed on the aircraft when the lift coefficient is changed slowly may be calculated Consideration of (*b*) will show the effect of sudden change in lift coefficient on the normal acceleration experienced by the aircraft

7 CONING ANGLE FOR MAXIMUM AXIAL LIFT WHEN $\beta_0 = 0$

This is the condition when the blade has been accelerated to a coning angle (β_{M_1}) at which the flapping acceleration is zero and the resultant axial force acting on the aircraft is a maximum This condition might

https://doi.org/10.1017/S2753447200000123 Published online by Cambridge University Press

arise, say, if C_L momentarily increased and the blade coned and stabilised itself at a new angle without appreciable loss in rotational speed

When $\beta_0 = O$ equation 1 becomes

$$\mathcal{E}\beta_0 = \frac{L \cos \beta_0}{W_b} = \frac{k_1 \cos^2\beta_0}{k3} \left[1 + \frac{k_2 \sin\beta_0 (\omega R)^2}{gR} \right] \qquad 2$$

The value of $\mathcal{E}\beta_0$ thus defined is a maximum when

$$\sin \beta_{M_1} = \frac{-2 + \sqrt{4 + 12 \left[\frac{k_2 (\omega R)^2}{gR}\right]^2}}{\frac{6 k_2 (\omega R)^2}{gR}} \qquad 2A$$

Consideration of equation 2A shows that to a close order of approximation β_{M_1} is constant for all rotor sizes and tip speeds in which we are likely to be interested, *i e*,

$$\sin \beta_{M_1} = \frac{\sqrt{12}}{6} \text{ or } \beta_{M_1} = 35^\circ$$

This means that any helicopter rotor is giving its maximum axial lift when coned at an angle of about 35° This point may best be illustrated by an exaggeration, *i e*, a helicopter which can just lift itself when the rotor is coned to 35° can never carry more load because further increase in coning angle will not result in any increase in axial lift It should be noted that 35° is not the best coning angle from the viewpoint of carrying maximum useful load—see para 9 , further, the power required to drive a rotor coned at 35° would be excessive

The actual values of $\mathcal{E}\beta_0$ are dependent on the constants k_1, k_2 and k_3, and will vary as between blades of different design It is thought, however, that these constants cannot vary greatly between different designs , also that the general relationship may be studied by assuming the following mean values for these constants —

$$k_1 = 0\ 42$$
$$k_2 = 0\ 56$$
$$k_3 = 0\ 72$$

Using these assumed values for the constants the relationship between $\mathcal{E}\beta_0$ and β_0, as defined in equation 2, are plotted in Fig 3 for various values of (ω R) and radius (R) The depreciation in $\mathcal{E}\beta_0$ as the size of the rotor increases and as ω R and coning angle decrease is apparent from this figure

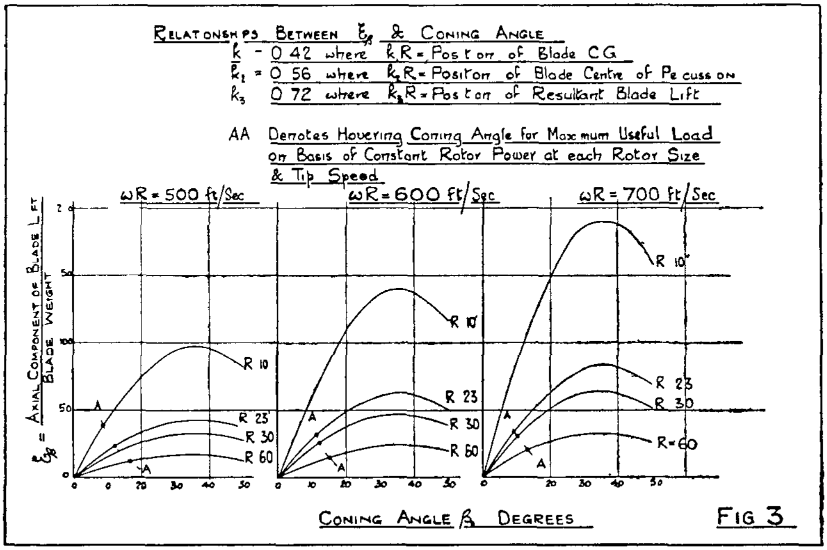


FIG 3

8 RELATIONSHIP BETWEEN THE BLADE WEIGHT/AIRCRAFT WEIGHT RATIO, ω R, RADIUS AND CONING ANGLE

This relationship is derived from equation 2, which is the general equation relating blade weight to axial lift

Since aircraft weight $\propto L_H \cos \beta_H$, it follows that —

$$\frac{Wb}{W} = \frac{k_3}{k_1 \cos^2\beta_H \left[1 + \dfrac{\sin \beta_H \, k_2 \, (\omega R)^2}{gR}\right]} \qquad 3$$

To a close order of approximation

$$\frac{Wb}{W} = \frac{g \, k_3 \, R}{k_1 \, k_2 \cos^2 \beta_H \sin \beta_H \, (\omega R)^2} \qquad 3A$$

Using the previously assumed values for the constants k_1, k_2, k_3, expression 3A becomes

$$\frac{Wb}{W} = \frac{90\ 8\ R}{\cos^2\beta_H \sin \beta_H \, (\omega R)^2} \qquad 3B$$

Expression 3B is plotted in Fig 4 for various values of R, β_H and (ωR) It will be observed from this figure that economy in the $\frac{Wb}{W}$ ratio is achieved by working to large hovering coning angles and high tip speeds

It is of interest to note from Fig 4 that, if it were feasible from constructional considerations to build a blade of any size for a $\frac{Wb}{W}$ ratio of say, 6%, and, if forward speed set a limit of, say, 600 ft /sec to rotational tip speed, then

(*a*) the economical limit of rotor size would be 86 ft when β_H is 11°,

and (*b*) the economical limit of rotor size would be 48 ft when β_H is 6°

The above follows, since larger sizes necessitate an increase in the $\frac{Wb}{W}$ ratio above the 6% figure

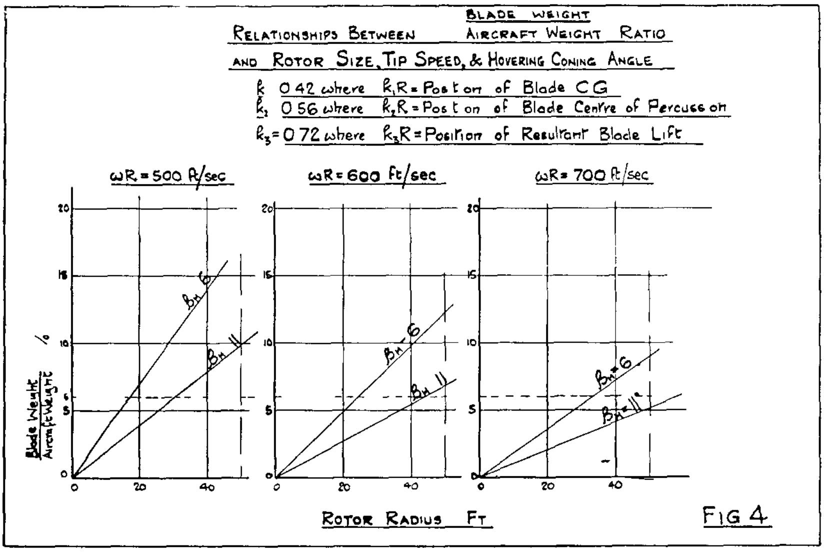


FIG 4

9 BEST HOVERING CONING ANGLE FROM VIEWPOINT OF WEIGHT ECONOMY

While expression 3 enables a study to be made of the effect of change in certain variables on the blade weight/aircraft weight ratio, it does not suggest what the hovering coning angle should be in order to achieve a high useful load If we take any helicopter, and if we lighten the blades, but change nothing else except the payload, it follows that there must be some coning angle at which the payload is a maximum, because —

(*a*) on the one hand, we are saving blade weight, part of which can appear as increased payload

and (*b*) on the other hand, we are reducing the all-up-weight because we have not materially changed the length of the resultant lift vector, but we have given it a greater inclination and hence a smaller axial component which must result in a decrease in payload

Hence, for constant engine power, increase in hovering coning angle has a twofold effect, namely, blade weight comes down, but so also does the all-up-weight It follows, then, that in order to achieve maximum weight economy the hovering coning angle should be so chosen that the difference between the all-up-weight and the sum of the blade weights is a maximum, *i e*,

$$W_A = W - zW_b \text{ should be a maximum}$$

The coning angle which will satisfy this condition will obviously be dependent on what happens to the length of the resultant lift vector when the coning angle changes, but the rotor power does not Because it would seem a reasonable assumption to make, and in the absence of more reliable information, the above argument is continued on the basis that the length of the resultant lift vector is constant for constant rotor power at all hovering coning angles in which we are likely to be interested

Since,

$$W_A = W - zW_b$$

and $W = zL \cos \beta_H$ and $W_b = \dfrac{Wg\, k_3\, R}{k_1\, k_2 \cos^2\beta_H \sin \beta_H\, (\omega R)^2}$ from 3A

$$W_A = zL \left[\cos \beta_H - \frac{2g\, k_3\, R}{k_1\, k_2\, (\omega R)^2 \sin 2\beta_H}\right] \qquad 4$$

The value of β_H which results in a maximum value of W_A can best be obtained graphically from equation 4 This is done in Fig 5, using the previously assumed values for the constants k_1, k_2, and k_3, and various values of R and (ωR) The optimum hovering angles from the weight viewpoint thus obtained are included in Fig 3, from which it will be observed that —

(*a*) for any given rotor, increase in ωR results in a decrease in the value of best hovering coning angle from the overall weight viewpoint

and (*b*) for any given ωR, increase in rotor size results in an increase in the value of the best hovering coning angle from the overall weight viewpoint

A typical figure for the optimum hovering coning angle is 11° when R is 23 ft and ωR is 600 ft /sec

10 LOSS IN USEFUL LOAD WHEN A HOVERING CONING ANGLE OTHER THAN THE OPTIMUM FROM THE WEIGHT VIEWPOINT IS ADOPTED

Consider the case when the helicopter blades are replaced by ones of like aerodynamic properties and mass distribution, but of different weight Let the engine, transmission system and fuselage be unchanged The power available at the rotor will then be constant, but the coning angle will change because of the change in blade weight

On the grounds of the argument outlined in para 9 there will be a best hovering coning angle from the viewpoint of achieving a maximum value of W_A Since nothing in the fuselage is altered this best hovering angle will result in a maximum value of useful load. Again, if a coning angle other than the best is chosen, W_A will decrease by an amount which must be subtracted from the useful load.

Let β_{H_1} be the best hovering coning angle, *i.e.*, the value of β_H which gives the maximum value of W_A is defined in equation 4.

Let the maximum value of W_A be $W_{A\,(max.)}$

Let the maximum useful load be $Wu\,(max.) = xW_{A\,(max.)}$

Let the hovering coning angle adopted be β_H

Then the ratio of the useful load achieved to the maximum useful load is

$$\frac{W_u}{W_{u\,(max)}} = \frac{xW_{A\,(max)} - (W_{A\,(Max)} - W_A)}{xW_{A\,(max)}}$$

or, from equation 4

$$\frac{W_u}{W_{u\,(max)}} = \frac{1}{x}\left\{\frac{k_1\,k_2(\omega R)^2\cos\beta_H - 2gk_3\,R\,\mathrm{cosec}\,2\beta_H}{k_1\,k_2(\omega R)^2\cos\beta_{H_1} - 2gk_3\,R\,\mathrm{cosec}\,2\beta_{H_1}} + x - 1\right\} \quad 5$$

In order to examine the variation in the $\dfrac{W_u}{W_{u\,(max.)}}$ ratio with hovering coning angle, equation 5 is plotted in Fig. 6 for rotors of 23 ft. and 30 ft. radius running an ωR of 600 ft./sec. The values of the constants k_1, k_2 and k_3 are as previously assumed and the maximum useful load achieved at the optimum hovering coning angle is taken as 0.25 W_A. It will be observed that under these conditions, which approximate to typical modern practice, and when R is 23 ft., the useful load at a 6° hovering angle is 94% of what it would be at the optimum hovering coning angle which is 11°. At a hovering coning angle of 4° $\dfrac{W_u}{W_{u\,(max.)}}$ drops to about 80%.

11 RELATIONSHIP BETWEEN HOVERING CONING ANGLE AND LIFT COEFFICIENT

It will be appreciated that it is essential for the operator to be able to apply lift coefficients greater than the normal hovering lift coefficient for the purpose of catering for growth in all-up-weight, enabling the aircraft to accelerate, and to compensate for decrease in air density with increase in altitude. It is of interest to enquire what the relationship between coning angle and lift coefficient is under conditions when the blade has no acceleration about its flapping hinge, *e.g.*, say, when the all-up-weight changes

This may be done as follows —

From relationship (vi) of para 5 it follows that

$L \cos\beta_0 \propto C_L \cos^3\beta_0$ & $L_H \cos\beta_H \propto C_{L_H} \cos^3\beta_H$

From equation 2

$$L \cos\beta_0 \propto \cos^2\beta_0 \left[1 + \frac{k_2 \sin\beta_0(\omega R)^2}{gR}\right]$$

$$\& \; L_H \cos\beta_H \propto \cos^2\beta_H \left[1 + \frac{k_2 \sin\beta_H(\omega R)^2}{gR}\right]$$

Hence,

$$\frac{C_L}{C_{L_H}} = \frac{\cos\beta_H}{\cos\beta_0}\left(\frac{1 + \frac{k_2 \sin\beta_0(\omega R)^2}{gR}}{1 + \frac{k_2 \sin\beta_H(\omega R)^2}{gR}}\right) \qquad 6$$

Since at normal hovering coning angles the couple resulting from blade weight is small compared with the centrifugal couple, the expression may be simplified to a close order of approximation thus —

$$\frac{C_L}{C_{L_H}} = \frac{\tan\beta_0}{\tan\beta_H} \qquad 6A$$

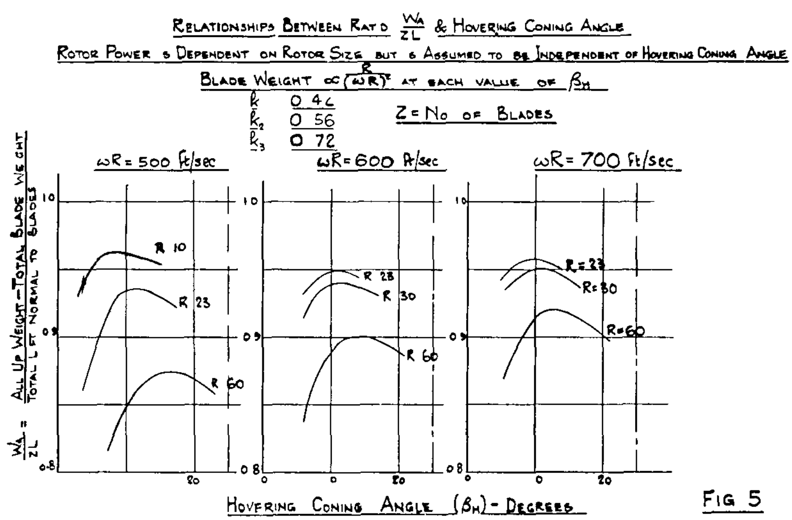


FIG 5

https://doi.org/10.1017/S2753447200000123 Published online by Cambridge University Press

In determining the influence of blade lift on the axial acceleration experienced by the aircraft it is desirable to consider the two conditions mentioned in para 6, *i e*, $\beta_0 = O$ and β_0 real The first condition determines the aircraft axial loading when the blade has no acceleration about the flapping hinge but is displaced to some coning angle β_0 greater than β_H The second condition determines the aircraft axial loading while the blade is being accelerated from β_H to β_0 If the acceleration is of sufficient magnitude the blade will exceed β_0 momentarily by an amount which may be sufficient to give rise to momentary accelerations appreciably in excess of the values determined by considerations of $\beta_0 = O$

In case (*a*) under, the case when $\beta_0 = O$ is considered, in case (*b*) under, the effect of making β_0 real is investigated In both cases it is assumed that there is no loss in ω during the manoeuvre

Case (*a*) when $\beta_0 = O$

Let N_{β_0} = vertical load factor when $\beta_0 = O$

Let $L \cos \beta_0$ = axial component of blade lift

Let $L_H \cos \beta_H$ = axial component of blade lift during hovering

$$N_{\beta_0} = \frac{L \cos \beta_0}{L_H \cos \beta_H} = \frac{C_L \cos^3 \beta_0}{C_{L_H} \cos^3 \beta_H} \text{ from relation (vi) of paragraph 5}$$

$$= \frac{\cos^2 \beta_0 \left[1 + \dfrac{k_2 \sin \beta_0 (\omega R)^2}{gR}\right]}{\cos^2 \beta_H \left[1 + \dfrac{k_2 \sin \beta_H (\omega R)^2}{gR}\right]} \text{ from equn 2} \qquad 7$$

Hence,

$$N_{\beta_0(max)} = \frac{C_{L_M} \cos^3 \beta_M}{C_{L_H} \cos^3 \beta_H} = \frac{\cos^2 \beta_M \left[1 + \dfrac{k_2 \sin \beta_M (\omega R)^2}{gR}\right]}{\cos^2 \beta_H \left[1 + \dfrac{k_2 \sin \beta_H (\omega R)^2}{gR}\right]} \qquad 7A$$

For any given value of β_H the equivalent value of β_M can be found from 6A provided the ratio $\dfrac{C_{L_M}}{C_{L_H}}$ is known Also, the most economical value of β_H can be found from equation 4

When $\omega R = 600$ ft /sec , R = 23 ft and the constants are as before, the most economical hovering coning angle is shown in para 9 to be 11° If $\frac{C_{L_M}}{C_{L_H}}$ is taken as 3, then, from equation 6A, β_M is 30° and, from equation 7A, N_{β_o} (max) = 2 0 If the hovering coning angle is changed from 11° to 6° the equivalent value of N_{β_o} (max) is 2 6 The relationship between N_{β_o} and β_o is shown graphically in Fig 7 for the cases when $\beta_H = 6°$, $\beta_H = 11°$, $\frac{C_{L_M}}{C_{L_H}} = 3$, R = 23ft and $\omega R = 600$ ft /sec

Case (*b*) when β_o is real

When the blade is being accelerated about the flapping hinge it will be seen from Fig 2 that the axial lift component is

$$\left(L \pm \frac{I\beta_o}{k_2 R} \right) \cos \beta_o$$

Hence, the axial load factor N

$$= \frac{\left(L \pm \frac{I\beta_o}{k_2 R} \right)}{W} \cos \beta_o \quad - 8A$$

From expression 1A

$$\beta_o = \frac{\cos \beta_o \, g}{k_2 R} \left(\frac{k_3 L}{W_b k_1 \cos \beta_o} - 1 - \frac{k_2 \sin \beta_o (\omega R)^2}{gR} \right) \quad - 8B$$

By substituting the value of β_o defined by 8B in equation 8A, it may be shown that

$$\frac{L \cos \beta_o}{W} = \frac{N - \frac{W_b}{W} \frac{k_3}{k_2} \mathcal{E}\beta_o}{1 - k_3/k_2}$$

Since, from equation 2,

$$\frac{W_b \mathcal{E}\beta_o}{W} = \frac{L \cos \beta_o}{W} \quad N_{\beta_o}$$

and since,

$$\left. \begin{array}{l} W \propto L_H \cos \beta_H \propto C_{L_H} \cos^3 \beta_H \\ \& \; L \quad \cos \beta_o \propto C_L \quad \cos^3 \beta_o \end{array} \right\} \text{from relation (vi) of paragraph 5}$$

https://doi.org/10.1017/S2753447200000123 Published online by Cambridge University Press

it follows that

$$\frac{C_L \cos^3 \beta_o}{C_{L_H} \cos^3 \beta_H} = \frac{N - N_{\beta_o} k_3/k_2}{1 - k_3/k_2}$$

i e $$N = \frac{C_L \cos^3 \beta_o}{C_{L_H} \cos^3 \beta_H} - k_3/k_2 \left(\frac{C_L \cos^3 \beta_o}{C_{L_H} \cos^3 \beta_H} - N_{\beta_o} \right) \qquad 8$$

In equation 8 $\frac{C_L}{C_{L_H}}$ denotes any change in lift coefficient which the operator chooses to select instantaneously at any coning angle β_o and is not necessarily the value, defined by equation 6, required for static equilibrium Also, in equation 8, N_{β_o} is the load factor imposed on the aircraft by virtue of coning angle alone, *i e*, as defined by equation 7 The above may be followed more clearly by assuming that during hovering C_{L_H} is suddenly increased to C_{L_M} Then the instantaneous value of N becomes

$$N = \frac{C_{L_M}}{C_{L_H}} - k_3/k_2 \left(\frac{C_{L_M}}{C_{L_H}} - 1 \right)$$

However, as the coning angle increases to some angle β_o greater than β_H, because of the blade's upward acceleration about the flapping hinge, the axial load factor becomes —

$$N = \frac{C_{L_M} \cos^3 \beta_o}{C_{L_H} \cos^3 \beta_H} - k_3/k_2 \left(\frac{C_{L_M} \cos^3 \beta_o,}{C_{L_H} \cos^3 \beta_H} - N_{\beta_o} \right)$$

where N_{β_o} is as defined by equation 7

Using our usual assumed values for the constants and a $\frac{C_{L_M}}{C_{L_H}}$ ratio of 3, the relationship between N and β_o, as defined by equation 8, is plotted on Fig 7

Also plotted on Fig 7 is the relationship between N_{β_o} and β_o, as defined by equation 7 It will be seen from Fig 7 that at coning angles less than β_M sudden increase in blade lift may give momentary relief to the aircraft as compared with the case when the increase in blade lift is made slowly The extent of the relief is dependent on the relative positions of the blade centre of pressure and centre of percussion $\frac{(k_3)}{(k_2)}$ When the blade overshoots the equilibrium position and is being retarded there may be an increase in axial loading caused by the reversed sense of the inertia term In order to

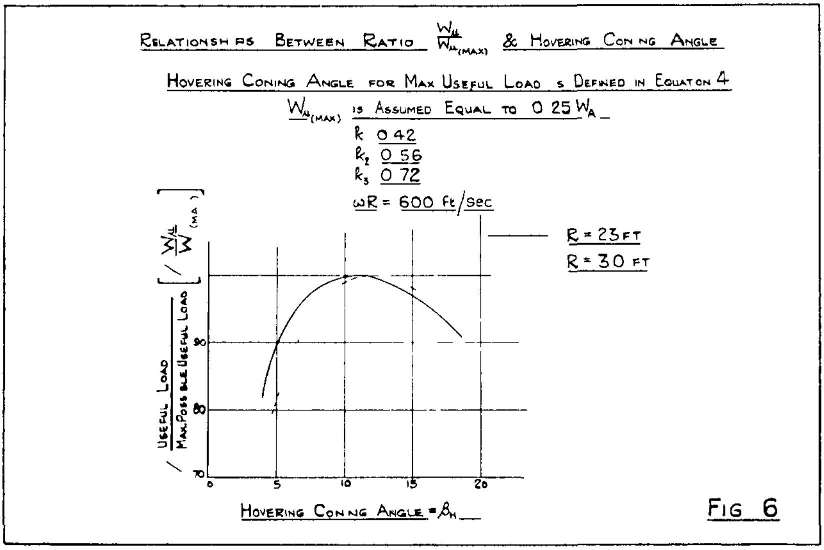


FIG 6

determine the maximum value of N it is necessary, then, to determine if the expression 8 reaches a maximum before or after the blade has reached its maximum angle of flap resulting from acceleration

The maximum coning angle resulting from acceleration may be obtained by equating the gain in kinetic energy of the blade during acceleration about the flapping hinge to its loss in kinetic energy during retardation

In general terms the gain in K E is $\frac{1}{2}I\beta_o{}^2$

From expression 1A

$$\beta_o = \frac{g}{k_2 R}\left(\frac{k_3 \, LW}{k_1 \, W_b W} - \cos\beta_o - \frac{k_2 \cos\beta_o \sin\beta_o (\;R)^2}{gR}\right)$$

From the fundamental relationship between velocity, distance, and acceleration (i e $\beta_o{}^2 = 2\beta_o\beta_o$),

$$\beta_o 2 = 2 \int_{\beta 1}^{\beta 2} \beta_o \, d\beta_o \qquad \text{(a)}$$

Obviously, $\beta_o = O$ when $\beta_o = \beta_H$ and β_o is a maximum when $\beta_o = \beta_M$

Hence, since $W \propto C_{L_H} \cos^3 \beta_H$

$$KE_{(max)} = W_b k_1 R \int_{\beta H}^{\beta M} \left(\frac{k_3 C_L \cos^2 \beta_o \, W}{k_1 C_{L_H} \cos^3 \beta_H \, W_b} - \cos\beta_o - \frac{k_2 \cos\beta_o \sin\beta_o (\;R)^2}{gR} \right) d\beta_o$$

Let the blade reach a maximum coning angle during retardation of β_F which is greater than β_M. Then the loss in blade K E

$$= \tfrac{1}{2} I \int_{\beta_M}^{\beta_F} - \beta_0 \, d\,\beta_0$$

Equating the energy gain to the energy loss it follows that

$$\frac{k_3 W C_L \left(\beta_F - \beta_H + \tfrac{1}{2}(\sin 2\beta_F - \sin 2\beta_H)\right) + \sin\beta_H - \sin\beta_F}{2\, k_1\, W_b C_{L_H} \cos^3 \beta_H}$$

$$+ \frac{k_2(\omega R)^2 \left(\cos 2\beta_F - \cos 2\beta_H\right)}{4gR} = 0 \qquad 9$$

When $\frac{C_L}{C_{L_H}} = 3$, $\beta_{H_1} = 11°$, $\omega R = 600$ ft /sec, and $R = 23$ ft, it follows from equation 3 that $W/W_b = 33$, and equation 9 becomes

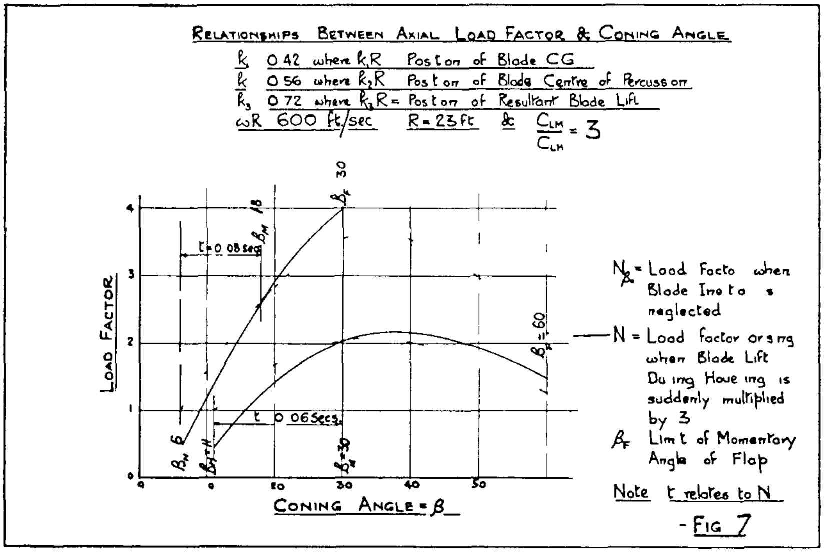
$$\beta_F + \tfrac{1}{2}\sin 2\beta_F + 765 \cos 2\beta_F = 1\ 089, \qquad — \quad 9A$$

which is satisfied when $\beta_F = 60°$

When $\frac{C_L}{C_{L_H}} = 3$, $\beta_H = 6°$, $\omega R = 600$ ft /sec, and $R = 23$ ft, it follows from equation 3 that $\frac{W}{W_b} = 17$, and equation 9 becomes

$$\beta_F + \tfrac{1}{2}\sin 2\beta_F + 1\ 57 \cos 2\beta_F = 1\ 739 \qquad — \quad 9B$$

which is satisfied when $\beta_F = 30°$



- FIG 7

https://doi.org/10.1017/S2753447200000123 Published online by Cambridge University Press

By comparing the above values of β_F with Fig 7 it will be seen that the maximum values of N are 2 2 and 4 when β_H is 11° and 6° respectively Thus, it can be concluded that the effect of not working to the optimum blade weight is twofold since, in addition to loss in useful load, the aircraft is liable to experience higher normal accelerations for the same value of the $\frac{C_L}{C_{L_H}}$ ratio

13 RELATIONSHIP BETWEEN TIME, ROTOR ANGULAR DISPLACEMENT AND ANGLE OF FLAP

It is of interest to know the order of the time taken for the blade to move from its equilibrium position at one value of C_L to its equilibrium position at another

The fundamental relationship is

$$\beta_o = \beta_x t$$

where β_x is the average acceleration over the angular displacement from the blade's position of rest to the position when the velocity is β_o

Hence,

$$\beta_x = \frac{\int_{\beta_1}^{\beta_2} \beta_o \, d\beta_o}{\beta_2 - \beta_1}$$

and using relation (*a*), on page 20,

$$t = \frac{\sqrt{\int_{\beta_1}^{\beta_2} \beta_o \, d\beta_o \, (\beta_2 - \beta_1{}^2)}}{\int_{\beta_1}^{\beta_2} \beta_o \, d\beta_o}$$

i e,

$$t = \frac{\sqrt{2}\,(\beta_2 - \beta_1)}{\sqrt{\int_{\beta_1}^{\beta_2} \beta_o \, d\beta_o}}$$

Hence, in general terms, the time taken for the blade to move from coning angle β_1, to coning angle β_2 under acceleration β_o as defined by equation 8B is

$$t = \frac{\sqrt{2}\,(\beta_2 - \beta_1)}{\sqrt{\frac{g}{k_2 R}\left[\frac{k_3 W C_L(\beta_2 - \beta_1 + \frac{1}{2}(\sin 2\beta_2 - \sin 2\beta_1))}{2 k_1 W_b C_{L_H} \cos^3 \beta_H} - \sin\beta_2 + \sin\beta_1 + \frac{k_2(\omega R)^2(\cos 2\beta_2 - \cos 2\beta_1)}{4gR}\right]}} \quad - \quad 10$$

If the number of revolutions made by the rotor in time t = n, then

$$n = \frac{\omega t}{2\pi} \qquad 11$$

where t is as defined in equation 10

Using relations 10 and 11 it follows that when $\beta_1 = \beta_H = 11°$, when $\beta_2 = \beta_M = 30°$, when $\frac{C_{L_M}}{C_{L_H}} = 3$, $\omega R = 600$ ft /sec , and R = 23 ft ,

the time taken for the blade to move from the hovering coning angle to β_M is about 0 06 sec , and the corresponding angular rotor displacement is about $\frac{1}{4}$ REV The equivalent figures when $\beta_1 = 6°$ and $\beta_2 = 18$ are 0 08 sec and $\frac{1}{3}$ REV respectively The effect of air damping is neglected in the calculations and this will lead to a slight underestimate of $\bar{t}$ and $\bar{n}$

PART 2

14 PROBABLE RELATIONSHIPS BETWEEN ROTOR SIZE, MAXIMUM USEFUL LOAD AND PERCENTAGE USEFUL LOAD

It is obvious that the total load which any blade can support is the vertical component of its axial lift, *i e* , L cos β_0

In the hovering case the vertical component of rotor axial lift $zL_H \cos \beta_H$) must overcome the weights of the various parts which constitute the aircraft, *i e* , the blades, transmission, engine and power plant, airframe, tail rotor (if any), crew weight and useful load If we knew how all these aircraft parts except useful load varied with rotor size we could equate them to $L_H \cos \beta_H$ and study the variation in useful load with rotor size Since we do not know exactly how these quantities vary with rotor size it is necessary to assume possible ways in which they might vary and in the following treatment this is done in the belief that, even should the assumptions be wrong, the overall method adopted is correct and therefore capable of application when reliable data about component weight variation with rotor size are available

Let us now consider what the various major aircraft components are and how they might vary with rotor size if, say, we keep the disc loading and tip speed constant

https://doi.org/10.1017/S2753447200000123 Published online by Cambridge University Press

MAJOR COMPONENT	ASSUMPTION
W_{T_1} = That part of the transmission the weight of which must be dependent on rotor torque	Part of the transmission must run at rotor speed, while the speed of the remainder of the transmission can be varied by suitable gearing to counter torque and weight increase with growth in rotor size The manner in which the weight of the transmission running at rotor speed varies with varying rotor radius is not very clear, however, there seems to be an indication that $\frac{W_{T_1}}{W}$ varies as the square of the rotor size In the absence of more complete data this law will be assumed, but the limitations of the basis of the assumption should be borne in mind when examining the results which the expressions to be derived will show for useful load and percentage useful load
W_{T_2} = that part of the transmission the weight of which can be made independent of rotor torque	Part of the transmission need not run at rotor speed and hence need not be subjected to rotor torque because angular speed variation can be used to counter growth in torque and weight In the absence of more accurate data, $\frac{W_{T_2}}{W}$ is assumed to be constant for all values of rotor size
W_E = engine and power plant	W_E is dependent on the power requirements of the rotor and the power/weight ratio of the engine For constant tip speed and disc loading the power requirements of the rotor may be shown to be proportional to the aircraft weight Hence, since the power/weight ratio of conventional engines is more or less constant, $\frac{W_E}{W}$ can be taken as constant for rotors of different size, but having the same tip speed and disc loading $\frac{W_E}{W}$ is then assumed to be constant when the tip speed and the disc loading are constant

https://doi.org/10.1017/S2753447200000123 Published online by Cambridge University Press

	MAJOR COMPONENTS	ASSUMPTIONS
W_F	= airframe, including landing gear, furnishings, etc	For any given configuration, W_F is likely to be proportioned to the all-up-weight $\frac{W_F}{W}$ is then assumed to be constant
W_b	= Main rotor blades	W_b has been studied in Part 1 of this paper and it is shown in equation 3A that $\frac{W_b}{W}$ is proportional to $\frac{R}{(\omega R)^2}$ for constant coning angle
W_{b_T}	= Tail rotor blades	W_{b_T} will vary with R because the lift required from the tail rotor blades will vary with the main rotor torque in order to achieve balance Exact treatment would show $\frac{W_{b_T}}{W}$ to obey a law similar to equation 3 However, in practice, the tail rotor usually operates at such a low coning angle that any weight increase could be offset by a slight increase in coning angle Hence, it is assumed that $\frac{W_{b_T}}{W}$ can be regarded as constant for all values of R in which we are likely to be interested
W_c	= Crew weight	W_c is charged to the aircraft tare weight because this weight cost must be met before the aircraft can be operated and, hence, it is as much a part of the aircraft as any other essential component W_c will not vary with rotor size, but will depend only on the number of crew members carried Hence, W_c can be taken as constant if we assume, say, only one crew member $\frac{W_c}{W}$, however, is not constant
W_u	= Useful load including fuel, oil, freight, passengers, but excluding crew	$W_u = W - W_{T_1} - W_{T_2} - W_E - W_F - W_b - W_{b_T} - W_c$

Now, $W = W_{T_1} + W_{T_2} + W_E + W_F + W_b + W_{b_T} + W_c + W_u$ —— 12

On the basis of the above assumptions, equation 12 may be re-written thus —

$$\frac{W_u}{W} = 1 - \frac{(W_{T_2}+W_E+W_F+W_{b_T})}{W} - C_1R^2 - \frac{C_2R}{(\omega R)^2} - \frac{W_c}{\pi R^2 DL} \quad - 12A$$

$$\text{and } W_u = \pi R^2 DL\left[1 - \frac{(W_{T_2}+W_E+W_F+W_{b_T})}{W} - C_1R^2 - \frac{C_2R}{(\omega R)^2} - \frac{W_c}{\pi R^2 DL}\right] \quad - 12B$$

where C_1 and C_2 are constants

From 12A, it will be observed that the ratio $\frac{W_u}{W}$ has a turning value which can best be obtained by using the appropriate values of the constants and plotting the ratio against the variable R

From equation 12B, it can be shown that for constant ωR and DL, W_u is a maximum when —

$$R = \frac{-\frac{3C_2}{(\omega R)^2} + \sqrt{\left[\frac{3C_2}{(\omega R)^2}\right]^2 + 32C_1\left[1 - \frac{W_{T_2}+W_E+W_F+W_{b_T})}{W}\right]}}{8C_1} \quad - 12C$$

In considering equations 12A, B and C, in the light of the assumptions made above, it will be observed that $\frac{W_E}{W}$ is only constant when the tip speed and disc loading are constant , also, in deriving the expression β_H has been assumed constant Hence, general deduction by the application of a set of numerical values for the constants in these equations is not possible However, for a given set of values for tip speed, disc loading and hovering coning angle $\frac{W_u}{W}$ and W_u can be calculated This has been done for an ordinary commercial type helicopter designed to achieve a top speed of about 150 m p h and having a disc loading of 3 lbs /ft ^{2}, a tip speed of 600 ft /sec , and hovering coning angles of 6° and 11° The results are shown on Fig 8 and are based on the following values for the constants —

$\frac{W_{T_1}}{W} = 8\%$ when R is 23 ft , which results in a value of $C_1 = \frac{1\ 54}{10^4}$

$\frac{W_{T_2} + W_E + W_F + W_{b_T}}{W} = 0\ 6$

$\frac{W_b}{W} = 6\%$ when ωR = 600 ft /sec and R = 23 ft , which leads to a value of C_2 = 945 when β_H = 6° and 657 when β_H = 11°

W_c = 200 lbs

https://doi.org/10.1017/S2753447200000123 Published online by Cambridge University Press

It will be observed that the following results are obtained for our commercial type helicopters having one crew member, a disc loading of 3 lbs /ft ^{2} and an ωR of 600 ft /sec —

(*a*)	(i) Rotor size for maximum useful weight lifted when $C_2 = 945$ ($\beta_H = 6°$)	=	60 ft dia
	(ii) Equivalent useful load	=	1,350 lbs
	(iii) Equivalent aircraft weight	=	8,460 lbs
	(iv) Equivalent percentage useful load	=	15 9%
(*b*)	(i) Rotor size for maximum useful weight lifted when $C_2 = 657$ ($\beta_H = 11°$)	=	64 ft dia
	(ii) Equivalent useful load	=	1,560 lbs
	(iii) Equivalent aircraft weight	=	9,700 lbs
	(iv) Equivalent percentage useful load	=	16 2%
(*c*)	(i) Rotor size for maximum percentage useful load when $C_2 = 945$ ($\beta_H = 6°$)	=	35 ft dia
	(ii) Equivalent percentage useful load	=	24%
	(iii) Equivalent aircraft weight	=	2,900 lbs
(*d*)	(i) Rotor size for maximum percentage useful load when $C_2 = 657$ ($\beta_H = 11°$)	=	36 ft dia
	(ii) Equivalent percentage useful load	=	25%
	(iii) Equivalent aircraft weight	=	3,100 lbs

It should be noted that the useful load relationships (W_u) shown in Fig 8 are largely dependent on the value of the constant C_1 Since this constant reflects the weight penalty in the transmission, the curves are only meant to be applicable to machines of conventional design and are not applicable to aircraft of the type in which power is supplied from jets at the blade tips One would expect the maximum useful load to be obtained with a larger diameter rotor with tip jet blades because of the simplified and lighter transmission

It should be noted also that the maximum value of the percentage useful load relationship is dependent on the crew weight charge , so also is the equivalent value of rotor size

While the above assumed quantities influence the numerical results obtained they do not influence the method used which presumably will be capable of greater accuracy when more design information on the constants becomes available

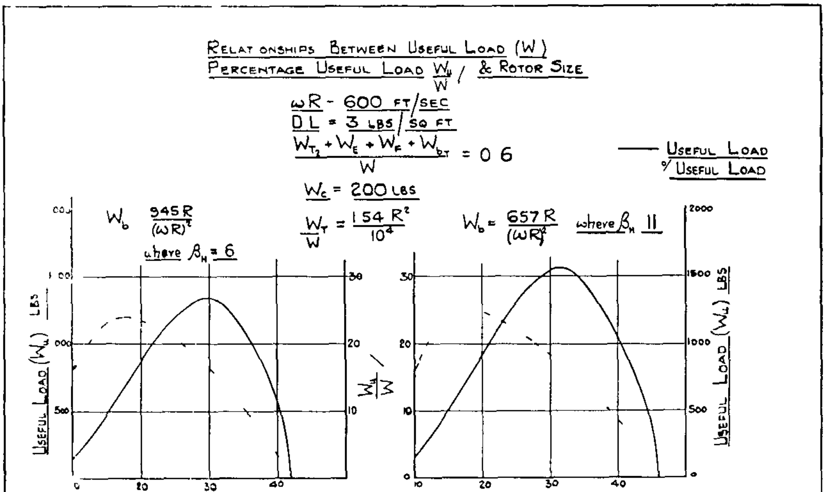


Fig 8

15 Conclusions

The broad conclusions arrived at and outlined under are based on the assumptions made and given in paragraphs 4 and 14

(*a*) The coning angle at which the component of axial lift is a maximum is approximately 35° and, within all practical considerations, is independent of rotor diameter and tip speed

(*b*) To a first order of approximation the ratio of blade weight to aircraft weight is proportional to the rotor radius, inversely proportional to the tip speed squared and inversely proportional to the product of the sine and cosine squared of the hovering coning angle While maximum economy in blade weight is obtained by adopting the largest possible coning angle it does not follow that very large hovering coning angles result in maximum useful load

(*c*) For any given rotor to achieve maximum useful load, a compromise must be struck between the saving in blade weight and the loss in the total axial component of lift (aircraft weight) as the coning angle increases On the basis that, for constant rotor power the length of the resultant lift vector is independent of hovering coning angle, optimum hovering coning angles from the weight viewpoint can be calculated for all rotors

These angles

(i) will decrease if ωR is increased and size is kept constant ,

(ii) will increase if the size of the rotor is increased and ωR is kept constant

On the above basis, a typical figure for the optimum hovering coning angle is 11° when R is 23 feet and ωR is 600 ft /sec (*d*) The values of maximum aircraft axial acceleration imposed by an articulated rotor system are dependent on the relative positions of the resultant blade lift vector and the centre of percussion of the blade, and are a minimum when the centre of lift is as far inside the centre of percussion as possible If the optimum coning angle for useful load is chosen (see (*c*) above), a sudden increase in blade lift coefficient of 200% results, for blades having the assumed positions of centre of gravity, centre of percussion and centre of resultant lift, in momentary maximum axial accelerations of about 2 g when ΩR is 600 ft /sec and R is 23 ft The maximum momentary angle of flap is about 60° and the blade equilibrium position at maximum C_L is 30°

If a weight penalty is paid in the blades and a coning angle of, say, 6° adopted during hovering the equivalent momentary acceleration is 4 0g The maximum momentary angle of flap is 30° and the blade equilibrium position is 18° at maximum C_L It follows that the effect of not adopting the optimum coning angle is twofold since, in addition to losing useful load, the aircraft is liable to experience a higher axial acceleration for the same increase in lift coefficient The heavier rotor will, however, make the aircraft more manoeuverable

(*e*) The time taken for the blades to move from one position of equilibrium to another when the blade lift is changed is very short For rotors of the type with which we are familiar the time taken for displacement from the hovering coning angle to the coning angle corresponding to maximum C_L is shown to be less than 1/10th sec if the effect of air damping on the blade is ignored

(*f*) There would seem to be limits to the sizes of rotors and these limits are dependent on the tip speed, hovering coning angle, disc loading and the structural efficiency of the aircraft design On the basis of very limited experience and on the assumptions that the tip speed is 600 ft /sec , the disc loading is 3 lbs /ft ^{2} and 200 lbs of crew weight is charged to each rotor,

(i) the rotor size for maximum percentage useful load appears to be about 35 ft dia and the equivalent values of percentage useful load and aircraft weight are about 24% and 2,900 lbs respectively ,

(ii) the rotor size for maximum useful load appears to be about 60 ft dia and the equivalent values of percentage useful load and aircraft weight are about 16% and 8,500 lbs respectively

The above values assume a single rotor configuration and part of the ratio of the transmission weight to the aircraft weight to vary as the square of the rotor radius Since the latter assumption can at best only approximate to the truth, the values quoted above should be regarded only as indicating the possible order of things, rather than absolute quantities Further, they are not relevant to jet driven blades

https://doi.org/10.1017/S2753447200000123 Published online by Cambridge University Press

Mr O L L Fitzwilliams' Vote of Thanks to Mr McClements

Mr Chairman, Fellow Members and Guests,—I have accepted with pleasure our chairman's invitation to propose a vote of thanks to Mr McClements for the lecture he has just given us I had expected to refer to him as reading his paper but I would like to call your attention to the rather extraordinary fact that this, the most difficult paper to which we have listened, is the first which has been presented without actually being read

In any case it is obvious that Mr McClements' presentation of his lecture is the culmination of a long and painstaking effort, and for this he is certainly entitled to our fullest thanks But he is also entitled to the thanks of everybody else interested in rotating wings, because he has presented a subject of fundamental importance in a manner which ensures that the major part of his paper will be included in all future text books on the design of rotating-wing aircraft

Moreover Mr McClements has today played a star part in an occasion of great significance in the development of our Association

For one thing, we have today listened for the first time to a paper of a specifically research nature, and by this I mean an original essay in pure knowledge, conceived and executed for the purpose of study, as distinct from the more usual kind of lecture which is generally an account of past thoughts and actions, mostly undertaken to overcome practical difficulties It is hardly necessary for us to be reminded that the influence and prestige of a professional Association such as our own, must depend at least partly, on the ability of its members to produce, to understand and to use the essays of this kind

Secondly, our Association is not only a convenient meeting place for old friends, it is also a sounding board for the knowledge and perhaps more important, the personalities of its members

Mr McClements, like most of us, is relatively unknown by comparison with our previous lecturers, all of whom had world-wide reputations even before the War In speaking this afternoon, he has fulfilled an important object of our Association in introducing himself to us and, through our Journal, to the world, as a new figure in the field of rotating wing aircraft development and also as an encouraging example of the persistance and ability upon which we base our confidence in the future of rotating wings in Great Britain

Mac has done us a great favour and I know that I have your support in offering him our thanks

https://doi.org/10.1017/S2753447200000123 Published online by Cambridge University Press