ON BOUNDARY LAYER FLOWS IN DISK-HALO GALAXIES

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I. INTRODUCTION

The existence of a boundary layer circulation in a system composed of a galactic disk embedded within a surrounding gaseous halo, may be inferred from two physical principles governing the fluid dynamical interactions. The first concerns the presence of a vertical shear in the azimuthal flow of gas, and the localization of this shear to the vicinity of the disk. The second principle concerns the importance of viscous stresses with regard to their influence on the vertical structure of the flow throughout the disk region, and the resultant coupling of the disk to the halo. It is this viscous coupling that gives rise to a meridional circulation extending over several disk scale heights from the galactic plane. In this discussion we shall concentrate on a rather specific model system in order to establish the essential dynamical features of the theory. The details of a somewhat more general analysis are to appear in the Astrophysical Journal in the near future.

Our problem concerns the steady flow of gas in an axisymmetric disk-halo system. The halo is taken to have characteristic dimension $L \sim 10$ kpc in all directions and is assumed to rotate uniformly with angular velocity Ω about the axis of symmetry. The disk is characterized by a vertical scale height $D \sim 100$ pc and a radial scale L. We denote the gas density in the halo by

$$\rho_H = \rho_0 (r/L\,, z/L). \tag{1}$$

In writing the gas density in the disk we choose to decompose it into two parts,

$$\rho_D = \rho_0 (r/L, z/L) + \rho'(r/L\,, z/D). \tag{2}$$

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The first component, ρ_0 , corresponds to the "background" density of the halo. We shall refer to the second component, ρ', as the "density enhancement." It is this density enhancement which is associated with the rapid variations on the short vertical scale D. In this discussion we shall treat the density enhancement as a small perturbation on the background halo, i.e., $\rho'/\rho_0 \ll 1$.

II. PHYSICAL CONSIDERATIONS GOVERNING THE FLOW

For the moment we shall neglect viscosity; thus the system is in a centrifugal and hydrostatic equilibrium with the net gravitational field. Forming the azimuthal component of the inviscid vorticity equation, linearized in the perturbations on the background, we obtain an "inviscid shear equation,"

$$\frac{\partial v'}{\partial z} = \frac{-1}{2\Omega \rho_0^2} \frac{\partial P_0}{\partial r} \frac{\partial \rho'}{\partial z} \left[1 + O(D/L)\right], \qquad (3)$$

where only the dominant terms appear explicitly. In equation (3), v' is the azimuthal velocity of the gas in excess of the uniform rotation Ωr and P_0 is the gas pressure of the background.

Equation (3) implies that the vertical shear in the rotation occurs on the short scale D since this is the scale relevant to the density enhancement. Moreover, equation (3) exhibits the localization of v' to the equatorial plane of the halo where $\rho' = O(1)$. Thus we see that the vertical shear is due primarily to the inclination between the equidensity surfaces of the enhancement and the equipressure surfaces of the background halo gas. That is, the vertical shear is a manifestation of the baroclinicity inherent to disk-halo systems. For halos in which pressure, gravitational, and centrifugal forces are comparable, $\partial P_0/\partial r = O(\rho_0 \Omega^2 L)$. It then follows from equation (3) that

$$\epsilon \equiv O(v'/\Omega L) = O(\rho'/\rho_0), \qquad (4)$$

where ϵ is the Rossby number for the flow, i.e., it indicates the importance of nonlinear advection relative to Coriolis effects in the Navier-Stokes equations. Hence, in the limit of a small density enhancement, equation (4) expresses the equivalence of the Rossby number with the density contrast in the system.

We must now estimate the viscosity of the interstellar gas in order to determine the influence of viscous stresses on the dynamics. The stochastic transport of momentum is dominated by collisions between interstellar gas clouds. Utilizing fundamental notions from the kinetic theory of gases we can estimate the kinematic viscosity for conditions pre-

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vailing in our own galactic disk to be $\nu \sim 10^{26}$ cm^2/sec. This viscosity can be shown to be effective throughout the intercloud gas as well.

The viscous forces in this system are of the order of $\nu v'/L^2$ for velocity variations on the long scale, and $\nu v'/D^2$ for variations on the short scale. Taking the ratio of viscous to Coriolis forces (the characteristic force in the problem) we obtain the dimensionless quantities $\nu/\Omega L^2 \equiv E$ and $\nu/\Omega D^2 = EL^2/D^2$, where E so defined is the Ekman number. With ν as given above, L=10 kpc, and Ω =25 km/sec/kpc we can estimate the Galactic Ekman number, $E \sim 10^{-4}$. Thus when considering velocity variations on the long scale L we may neglect viscous forces altogether. However, with D=100 pc we find $EL^2/D^2 \sim 1$. We conclude that viscous forces are essential only in determining the vertical structure of the flow in the vicinity of the disk. In short, we have shown that the problem admits of a boundary layer simplification.

We now have the necessary elements to establish the existence of a boundary layer circulation in disk-halo systems, namely, a vertical shear whose scale height necessitates the inclusion of frictional forces. Now the drag of fluid on overlying fluid rotating more slowly tends to act as a centrifugal fan throwing the overlying fluid radially outward; similarly, the underlying fluid looses angular momentum and flows radially inward. Mass conservation requires a weak vertical flow to feed the radial flow and, by symmetry across z=0, this flow must be upward in the inner regions of the disk and downward beyond $r \approx$ L. What we have described here is a meridional circulation that should exist over several disk scale heights on either side of the galactic plane.

III. REPRESENTATIVE CIRCULATION PATTERNS

I have solved the Navier-Stokes equations, linearized in the perturbations on the background, by the method of matched asymptotic expansions for the case of a Maclaurin Spheroid model halo. I have chosen to explore density enhancements with Gaussian vertical structure and two types of radial structure; in this sense the approach is a diagnostic one.

In Figure 1 we display the streamlines in the meridional plane for two representative circulation patterns. The pattern on the left corresponds to a density enhancement whose radial structure is constant for $r \leq$ L and then decreases monotonically to zero by $r \approx 1.5$ L. If we extrapolate our solution to $\epsilon = O(1)$ then the radial flows are of the order of 20 km/sec while the vertical flows are of the order of 0.5 km/sec. We see that this circulation implies a thorough mixing of the metal-enriched disk gas with the overlying halo gas on a timescale of about a billion years. Note that for L somewhat less than 10 kpc, the Sun would be immersed in a region of downflowing gas. It is tempting to identify this flow with the low velocity "galactic infall" observed in the solar neighborhood.

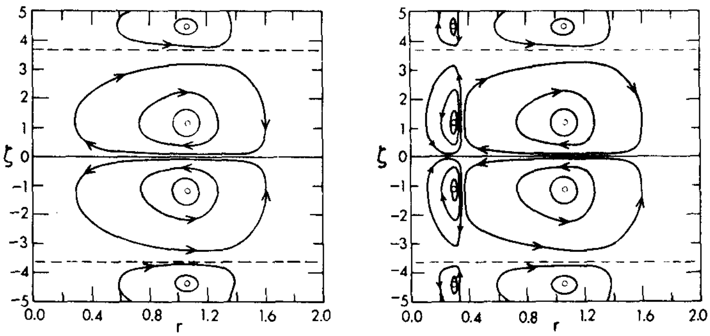

Figure 1

Fig. 1.-Two examples of galactic boundary layer flows in the meridional plane. (ζ is in units of $E^{1/2}L \approx 100$ pc and r is in units of L $\approx$ 10 kpc.)

The pattern on the right-hand side of Figure 1 corresponds to a density enhancement which simulates a decrease of density below that of the overlying halo within $r \approx 4$ kpc. We see that the circulation cells have each split into two. The region of intensified upward flow and the boundary between inner and outer circulation cells are both new features that may be suggestive of a "3-kpc arm."

IV. CONCLUDING REMARKS

A fundamental problem that remains to be investigated is the stability of this galactic boundary layer flow, in particular to large scale spiral waves. Thus, we point out the presence of points of inflection in the vertical structure of the flow (i.e. vorticity extrema) and note that these are often signposts of instability. However, any definitive statement concerning the stability of this flow must await a thorough stability analysis which I hope to complete in the near future.

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