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Helicon and lower hybrid current drive comparisons in tokamak geometry

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The parallel current driven by applied helicon waves is evaluated in tokamak geometry along with the radio frequency (rf) power absorbed by the passing electrons. The results are compared with the corresponding expressions for lower hybrid current drive. The efficiency of both current drive schemes is found to be the same for any single wave frequency, single mode number limit. The evaluation of the parallel currents is performed using an adjoint technique. Tokamak geometry is retained by using an eigenfunction expansion appropriate for a transit averaged long mean free path treatment of electrons making correlated poloidal passes through the applied rf fields.

Key words: fusion plasma, plasma confinement, plasma waves

1. Introduction

Investigating helicon (or whistler) waves as a means to drive a parallel current in a tokamak is a less studied area of current drive (Pinsker 2015) than the more common mechanism of lower hybrid waves (Fisch 1978; Karney & Fisch 1979, 1985; Fisch & Boozer 1980; Fisch & Karney 1981; Cordey, Edlington & Start 1982; Taguchi 1983; Cohen 1987; Giruzzi 1987; Chiu *et al.* 1989; Ehst & Karney 1991) which is extensively reviewed by Bonoli (2014). Nonetheless, helicon current drive (HCD) remains of considerable interest (Vdovin 2013; Prater *et al.* 2014; Pinsker 2015; Wang *et al.* 2017; Lau *et al.* 2018, 2019; Pinsker *et al.* 2018) because of its many similarities to lower hybrid current drive (LHCD) and its ability to efficiently access higher density regions (Li *et al.* 2020*a*,*b*; Lau *et al.* 2021; Li, Li & Liu 2021; Yin *et al.* 2022). Both HCD and LHCD rely on a Landau resonance and the preferential heating of electrons, although HCD uses a perpendicular component of the applied electric field rather than the parallel component used for LHCD. An early analytic attempt at evaluating the parallel current driven by helicon waves and the associated efficiency appears in de Assis & Busnardo-Neto (1988). They give rough estimates for a model collision operator in a constant magnetic field with some inadequately defined notation. Little else in the way of analytic estimates is available in the literature, even though a great deal of effort has been expended on simulations, planning

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experiments and designing antennas for DIII-D (Prater *et al.* 2014; Lau *et al.* 2018, 2019, 2021; Pinsker *et al.* 2018), KSTAR (Wang *et al.* 2017); EAST (Li *et al.* 2020b); and HL-2M (Li *et al.* 2020a); as well as ITER and DEMO (Vdovin 2013; Lau *et al.* 2018) and CFETR (Li *et al.* 2021). The purpose of the investigation here is to derive analytic expressions for both the parallel current driven by helicon waves and its associated HCD efficiency in a tokamak, and to compare them with the recently derived expressions for LHCD (Catto 2021) that are generalized herein. It turns out that the adjoint procedure (Antonsen & Chu 1982) used for these evaluations have many similarities so the comparisons can be made in a meaningful way. Moreover, these results are derived in tokamak geometry by transit averaging and then using the Cordey (1976) eigenfunctions and associated results (Hsu, Catto & Sigmar 1990; Xiao, Catto & Molvig 2007; Parker & Catto 2012). In addition, since the electrons are only weakly collisional, successive poloidal passes through the applied radio frequency (rf) fields are correlated and the quasilinear (QL) description employed (Catto & Tolman 2021a) accounts for this feature.

The next section introduces the transit averaged QL operator employed for the adjoint evaluations of the parallel current, as well as notation. The adjoint technique is briefly summarized in § 3, where the unlike collision operator is also presented. Section 4 summarizes the solution for the adjoint equation in a tokamak and also gives the like particle collision operator employed. The parallel current driven by helicon waves is derived in § 5, which also presents an improved evaluation of the lower hybrid results. The rf power absorbed by the passing electrons and the current drive efficiencies associated with HCD and LHCD are presented in § 6. Section 7 gives results when HCD and LHCD are both operative. The Appendix presents some cold plasma material that suggests that HCD and LHCD are able to drive comparable currents.

2. Background

In a tokamak the transit averaged QL operator for electrons, when the applied wave field is at a frequency ω much smaller than the electron cyclotron frequency $\Omega_e = eB/m_ec$ and the unperturbed electron distribution function is nearly the Maxwellian f_0, is

$$\bar{Q}\{f_0\} = \sum_{\boldsymbol{k}} \frac{1}{\tau_f} \frac{\partial}{\partial E} \left(\tau_f v^2 D \frac{\partial f_0}{\partial E} \right), \qquad (2.1)$$

with $\tau_f = \oint_f \mathrm{d}\tau$ the time for a full (f) poloidal passing ($\sigma = v_{||}/|v_{||}| = \pm 1$) or trapped ($\sigma = 0$) poloidal circuit, the sum over the applied rf wave vectors $\boldsymbol{k}$, and D the velocity space diffusivity (Catto & Tolman 2021a). Here e is the charge on a proton, c is the speed of light, m_e is the electron mass and $E = v^2/2 - e\Phi/m_e$ is the total energy, with Φ the electrostatic potential, $v = |\boldsymbol{v}|$ the electron speed and $v_{||} = \boldsymbol{n} \cdot \boldsymbol{v}$ the parallel electron velocity along the tokamak magnetic field $\boldsymbol{B} = B\boldsymbol{n} = I\nabla\zeta + \nabla\zeta \times \nabla\psi$. The unit vector along the magnetic field is $\boldsymbol{n}$, ζ is the toroidal angle variable, ψ is the poloidal flux function and the flux function $I(\psi)$ is $I = RB_t$, where B_t is the toroidal magnetic field, R is the major radius and B_p is the poloidal magnetic field in $|\nabla\psi| = RB_p$. The poloidal angle ϑ satisfies $\nabla\zeta \cdot \nabla\psi = 0 = \nabla\zeta \cdot \nabla\vartheta$ and is chosen such that $\boldsymbol{B} \cdot \nabla\vartheta = |I|/qR^2 = q^{-1}|\boldsymbol{B} \cdot \nabla\zeta|$, making the safety factor, $q = q(\psi)$, a flux function. Taking the toroidal current to be in the $\nabla\zeta$ direction makes $B_p > 0$, $\boldsymbol{B} \cdot \nabla\vartheta > 0$ and ϑ increases in the $\boldsymbol{B}_p = \nabla\zeta \times \nabla\psi$ direction. Then the incremental time along a trajectory is $\mathrm{d}\tau = \mathrm{d}\vartheta/v_{||}\boldsymbol{n} \cdot \nabla\vartheta > 0$, with $\mathrm{d}\vartheta$ and $v_{||}$ reversing signs together when a trapped electron reflects, such that $\mathrm{d}\vartheta < 0$ for a passing electron with $v_{||} < 0$.

The QL diffusivity D in tokamak geometry for successive correlated interactions through the applied rf (Catto & Tolman 2021) is

$$D = \frac{\pi e^2}{2m_e^2 v^2 \tau_f} \sum_\ell \delta(\oint_f d\tau \chi - 2\pi \ell) \left| \oint_f d\tau \boldsymbol{e}_k \cdot \left[\boldsymbol{n} v_{||} J_0(\eta) - i\boldsymbol{n} \times \boldsymbol{k} \frac{v_\perp}{k_\perp} J_1(\eta) \right] e^{-i \int_{\tau_0}^\tau d\tau' \chi(\tau')} \right|^2,$$
(2.2)

with the transit averaged resonance condition defined by

$$\oint_f d\tau \chi = \omega \tau_f - 2\pi \sigma (qn - m),$$
(2.3)

and the exponential phase factor given by

$$\int_{\tau_0}^\tau d\tau' \chi(\tau') = \omega(\tau - \tau_0) - \sigma(qn - m)\vartheta(\tau),$$
(2.4)

where τ_0 is taken to be the trajectory time at the equatorial plane crossing where B is a minimum. In the preceding and what follows $\boldsymbol{e}_k$ is the Fourier amplitude of the applied electric field of wave vector $\boldsymbol{k} = \boldsymbol{k}_\perp + k_{||}\boldsymbol{n}$ having $\boldsymbol{k}_\perp = k_\perp(\boldsymbol{\psi} \cos \varsigma + \boldsymbol{p} \sin \varsigma)$, $k_{||} = (qn - m)/qR$ and $\eta = k_\perp v_\perp/\Omega_e$, with $\boldsymbol{\psi} = \nabla\psi/|\nabla\psi| = \nabla\psi/RB_p$, and $\boldsymbol{p} = \nabla\zeta \times \nabla\psi/|\nabla\zeta \times \nabla\psi| = \boldsymbol{B}_p/B_p$. The orthonormal unit vectors satisfy $\boldsymbol{\psi} \times \boldsymbol{p} = \boldsymbol{n}$, n and m are the toroidal and poloidal mode numbers, respectively, and large aspect ratio is assumed to write $\boldsymbol{n} \cdot \nabla\vartheta \approx 1/qR$. The integer ℓ denotes the resonant path in velocity space as electrons can experience other (usually less important) resonances besides $\ell = 0$ in toroidal geometry as their velocity changes along a field line as they cannot remain in resonance indefinitely in an inhomogeneous magnetic field. Indeed, they can have other resonant interactions by crossing the various resonant paths or curves in velocity space defined by $\oint_f d\tau \chi = 2\pi \ell$.

For LHCD the applied rf fields are tailored to make the first or $v_{||}J_0$ term dominate. For HCD the rf fields must be applied in a manner that makes the second or $v_\perp J_1$ term dominate (Prater *et al.* 2014; Pinsker 2015; Pinsker *et al.* 2018; Yin *et al.* 2022), so J_1 will not be expanded by assuming its argument is small. Of course, both HCD and LHCD rely on there being a Landau resonance. Here the goal is to evaluate HCD by an adjoint technique (Antonsen & Chu 1982) very similar to the one used recently to evaluate LHCD in full toroidal geometry to find the aspect ratio modifications to the driven current and efficiency (Catto 2021). Unlike for LHCD, HCD does not seem to have an accepted analytic expression for the driven current or efficiency even without toroidal effects retained.

3. Adjoint technique summary

For an adjoint evaluation of HCD the preceding QL operator is all that is required along with the perturbed electron kinetic equation

$$v_{||}\boldsymbol{n} \cdot \nabla f_1 = C\{f_1\} + Q\{f_0\},$$
(3.1)

and its adjoint equation for the adjoint function h associated with f_1

$$v_{||}\boldsymbol{n} \cdot \nabla h + C\{h\} = -\left(\frac{B}{I} - \frac{m_e v_u}{T_e x^3} V_{||} \right) v_{||} f_0,$$
(3.2)

where the small term containing the parallel mean ion velocity $V_{||}$ is needed to account for the non-self-adjoint term in the electron-ion collision operator and f_0 is the Maxwellian

$$f_0 = f_0(\psi, E) = n_e(\psi) \left(\frac{m_e}{2\pi T_e} \right)^{3/2} e^{-[m_e E + e\Phi(\psi)]/T_e(\psi)}. \tag{3.3}$$

The potential may be assumed to be a flux function to lowest order. In the electron kinetic equation f_1 is the perturbed electron distribution function with $f = f_0 + f_1$, $Q\{f_0\}$ is the QL operator prior to transit averaging, and $C\{f_1\} = C_{ee}\{f_1\} + C_{ei}\{f_1\}$ is the sum of the self-adjoint linearized electron-electron collision operator plus the electron-ion collision operator

$$C_{ei}\{f_1\} = \frac{\nu_u}{x^3} L \left\{ f_1 - \frac{m_e}{T_e} V_{||} v_{||} f_0 \right\}, \tag{3.4}$$

where $L\{h\}$ is the self-adjoint Lorentz or pitch angle scattering operator

$$L\{h\} = \frac{2B_0}{B} \xi \frac{\partial}{\partial \lambda} \left(\lambda \xi \frac{\partial h}{\partial \lambda} \right), \tag{3.5}$$

with $x = v/v_e$ and $v_e = (2T_e/m_e)^{1/2}$ the electron thermal speed. The pitch angle is defined as $\lambda = 2\mu B_0/Bv^2 = B_0 v_\perp^2/Bv^2$ and $\xi = v_{||}/v$, with B_0 a normalizing flux function to be defined shortly. The unlike collision frequency ν_u is defined as

$$\nu_u = 3\sqrt{\pi} \nu_{ei}/4, \tag{3.6}$$

where $\nu_{ei} = 4\sqrt{2\pi} Z^2 e^4 n_i \ell n \Lambda_C / 3 m_e^{1/2} T_e^{3/2} = Z\nu_{ee}$ for a quasineutral plasma with the ion, n_i, and electron, n_e, densities satisfying $Zn_i = n_e$, Z the ion charge number, ν_{ee} the electron-electron collision frequency and $\ell n \Lambda_C$ the Coulomb logarithm.

Defining the flux surface average of any quantity A by

$$\langle A \rangle = \left(\oint d\vartheta A / \boldsymbol{B} \cdot \nabla \vartheta \right) \Big/ \left(\oint d\vartheta / \boldsymbol{B} \cdot \nabla \vartheta \right), \tag{3.7}$$

using $\langle \boldsymbol{B} \cdot \nabla (B^{-1} \int d^3 v v_{||} h f_1 / f_0) \rangle = 0$, and the self-adjointness of both C_{ee} (that requires f_0 to be Maxwellian) and L to combine the equations leads to the convenient adjoint relation

$$\left\langle B \int d^3 v v_{||} f_1 \right\rangle = I \left(\int d^3 v \frac{v_{||} \bar{h}}{B f_0} \tau_f \bar{Q}\{f_0\} \right) \Big/ \left(\oint d\vartheta / \boldsymbol{B} \cdot \nabla \vartheta \right), \tag{3.8}$$

where the $V_{||} \sim v_i \rho_{pi}/a$ term, with v_i the ion thermal speed, ρ_{pi} the poloidal ion gyroradius and a the minor radius, is negligible for the rf amplitudes of interest. Catto (2021) can be consulted for more details on this approximation and the treatment to follow. To write the final form of $\langle B \int d^3 v v_{||} f_1 \rangle$ the lowest-order result $\boldsymbol{n} \cdot \nabla \bar{h} = 0$ is employed along with the transit average definition $\bar{A} = \oint d\tau A / \tau_f$. It is convenient to let $B_0^2 = \langle B^2 \rangle$.

Recall the Ohmic current is in the positive toroidal direction. Therefore, the helicon waves must drive the current in the same direction, requiring $\langle B \int d^3 v v_{||} f_1 \rangle < 0$.

4. Solution of the adjoint equation

The advantage of the adjoint method is that only the simpler adjoint equation

$$v_{||}\boldsymbol{n} \cdot \nabla h + C_{ee}\{h\} + v_u x^{-3} L\{h\} = -I^{-1} B v_{||} f_0, \tag{4.1}$$

need be solved instead of the more complicated electron kinetic equation. To solve the preceding equation, it is adequate to approximate the electron-electron collision operator by its standard high speed ($v^2 \gg v_e^2$) expansion. Then its self-adjoint form becomes

$$C_{ee}\{h\} = \frac{2v_\ell B_0 \xi}{B} \frac{\partial}{\partial \lambda}\left(\lambda \xi \frac{\partial h}{\partial \lambda}\right) + \frac{v_\ell T_e}{m_e v^2}\frac{\partial}{\partial v}\left[\frac{v^2 f_0}{x^3}\frac{\partial}{\partial v}\left(\frac{h}{f_0}\right)\right], \tag{4.2}$$

where the like collision frequency is defined as

$$\nu_\ell = 3\sqrt{\pi}\nu_{ee}/4, \tag{4.3}$$

with $\nu_{ee} = 4\sqrt{2\pi}e^4 n_e \ell n \Lambda_C / 3m_e^{1/2}T_e^{3/2}$. This like particle operator is the usual non-momentum conserving, high speed expansion of the Rosenbluth potentials for collisions with a Maxwellian used by Karney & Fisch (1979, 1985). Recent estimates (Catto 2021; Catto & Tolman 2021*a,b*) indicate f_0 must be nearly Maxwellian for QL theory to remain valid.

The adjoint equation is solved using the Cordey eigenfunctions (Cordey 1976; Hsu *et al.* 1990; Xiao *et al.* 2007; Parker & Catto 2012) by writing $h = \bar{h} + \tilde{h}$ with $\partial \bar{h}/\partial \vartheta = 0$ to obtain, upon annihilating $v_{||}\boldsymbol{n} \cdot \nabla \tilde{h}$ the term, the transit average equation

$$\bar{C}_{ee}\{\bar{h}\} + v_u x^{-3} L\{\bar{h}\} = -I^{-1}\overline{Bv_{||}}f_0. \tag{4.4}$$

Integration over a full trapped (t) bounce gives $\overline{Bv_{||}} = 0$ implying that $\bar{h}_t = 0$. For the passing (p) electrons flux surface averages are used to rewrite the adjoint equation as

$$2(Z+1)\frac{\partial}{\partial \lambda}\left[\lambda\langle\xi\rangle\frac{\partial}{\partial\lambda}\left(\frac{\bar{h}_p}{f_0}\right)\right] + \frac{T_e x^3 \langle B/v_{||}\rangle}{m_e B_0 v^2 f_0}\frac{\partial}{\partial v}\left[\frac{v^2 f_0}{x^3}\frac{\partial}{\partial v}\left(\frac{\bar{h}_p}{f_0}\right)\right] = -\frac{\langle B^2\rangle v x^3}{IB_0 v_\ell}, \tag{4.5}$$

where $\overline{Bv_{||}}\oint_p \mathrm{d}\tau = \langle B^2\rangle \oint \mathrm{d}\vartheta/\boldsymbol{B}\cdot\nabla\vartheta$.

For the purposes here, the recent solution of (4.5) by Catto (2021) is adequate and convenient. Ignoring order $\varepsilon = r/R \ll 1$ terms it is given by

$$\begin{aligned}
\frac{\bar{h}_p}{f_0} &\approx \frac{v x^3}{R v_\ell}\left\{\frac{(1+0.62\sqrt{\varepsilon})\Lambda_1(\lambda) - 1.02[(Z+5)/(7Z+11)]\sqrt{\varepsilon}\Lambda_2(\lambda)}{[(Z+1)(1+1.46\sqrt{2\varepsilon})+4]}\right\} \\
&\equiv \frac{v x^3}{R v_\ell}\frac{\Lambda_{1+2}(\sqrt{\varepsilon}, Z, \lambda)}{\left[(Z+1)(1+1.46\sqrt{2\varepsilon})+4\right]},
\end{aligned} \tag{4.6}$$

where the Cordey (1976) eigenfunctions Λ_j with eigenvalues κ_j satisfy the Sturm–Liouville differential equation obtained by transit averaging,

$$\frac{\partial}{\partial\lambda}\left(\lambda\langle\xi\rangle\frac{\partial\Lambda_j}{\partial\lambda}\right) = \kappa_j\frac{\partial\langle\xi\rangle}{\partial\lambda}\Lambda_j. \tag{4.7}$$

In the preceding, $\partial\langle\xi\rangle/\partial\lambda = -\langle B/2B_0\xi\rangle$, $\Lambda_j(\lambda = 0) = 1$,

$$\Lambda_{1+2}(\sqrt{\varepsilon}, Z, \lambda = 0) = 1 + [0.62 - 1.02(Z + 5)/(7Z + 11)]\sqrt{\varepsilon}, \tag{4.8}$$

$$\langle\xi\rangle = 2\sqrt{2\varepsilon}E(k)/\pi\sqrt{(1 - \varepsilon)k^2 + 2\varepsilon}, \tag{4.9}$$

where $E(k)$ is the elliptic integral of the second kind, $k^2 = 2\varepsilon\lambda/[1 - (1 - \varepsilon)\lambda]$ and $\Lambda_j = 0$ at $k = 1$. The response $\bar{h}_p$ increases with speed because of the v dependence of $C\{h\}$. For the passing

$$\tau_p = \oint_p \mathrm{d}\tau \approx 4qR\sqrt{(1 - \varepsilon)k^2 + 2\varepsilon}K(k)/v\sqrt{2\varepsilon}, \tag{4.10}$$

with $K(k)$ the elliptic integral of the first kind. The Λ_2 term has a somewhat small numerical coefficient. It was ignored in Catto (2021), but is retained here as it modestly alters the $\sqrt{\varepsilon}$ correction of interest (order ε corrections are ignored). More eigenfunction details are available in Catto (2021), Hsu *et al.* (1990), Xiao *et al.* (2007) and Parker & Catto (2012).

5. Helicon driven current in a tokamak compared with LHCD

Only the passing electrons contribute to helicon drive current, giving

$$\left\langle B \int \mathrm{d}^3 v\, v_{||}f_1 \right\rangle = \frac{\langle B^2\rangle}{2\pi q} \int \mathrm{d}^3 v \frac{v_{||}\bar{h}_p}{Bf_0}\tau_p \bar{Q}\{f_0\} = \frac{\langle B^2\rangle}{2\pi q}\sum_k \int \mathrm{d}^3 v \frac{v_{||}\bar{h}_p}{Bvf_0}\left.\frac{\partial}{\partial v}\right|_\mu \left(\tau_p vD \left.\frac{\partial f_0}{\partial v}\right|_\mu\right), \tag{5.1}$$

at large aspect ratio, with the QL diffusivity for helicon waves simplifying to

$$D = \frac{\pi e^2|e_k \cdot \mathbf{n} \times \mathbf{k}|^2\tau_p v_\perp^2 J_1^2(\eta)}{8m_e^2 k_\perp^2 v^2}\sum_\ell \delta[\omega\tau_p - 2\pi\sigma(qn - m) - 2\pi\ell]\Theta(v, \lambda, n, m). \tag{5.2}$$

Here,

$$\Theta = \frac{1}{\tau_p^2}|\oint_p \mathrm{d}\tau\, \mathrm{e}^{-\mathrm{i}[\omega(\tau - \tau_0) - \sigma(qn - m)\vartheta(\tau)]}|^2 = \frac{q^2 R^2}{\tau_p^2 v^2}\left|\int_{-\pi}^{\pi}\frac{\mathrm{d}\vartheta}{\xi}\mathrm{e}^{-\mathrm{i}\left[\omega qR\int_0^\vartheta \mathrm{d}\vartheta/v_{||} - \sigma(qn - m)\vartheta\right]}\right|^2 \leq 1, \tag{5.3}$$

and $\mathrm{d}^3 v \to 2\pi B\, \mathrm{d}E\, \mathrm{d}\mu/v_{||} \to 2\pi Bv^3\, \mathrm{d}v\, \mathrm{d}\lambda/B_0 v_{||}$. Integrating by parts in E, μ variables, yields

$$\left\langle B \int \mathrm{d}^3 v\, v_{||}f_1 \right\rangle = \frac{m_e\langle B^2\rangle}{2\pi q T_e}\sum_k \int \mathrm{d}^3 v \frac{v_{||}}{B}\tau_p vDf_0\left.\frac{\partial}{\partial v}\right|_\mu \left(\frac{\bar{h}_p}{f_0}\right). \tag{5.4}$$

Catto (2021) has an extra v^2 multiplying $\bar{h}_p$ in his (4.7), implying the factors of v in his (4.27) for the lower hybrid driven parallel current need to be corrected, as will be found shortly.

To sustain or enhance the poloidal magnetic field the parallel current driven by the helicon waves must be positive, requiring $\langle B \int \mathrm{d}^3 v\, v_{||}f_1\rangle < 0$. Therefore, passing electrons with $v_{||} < 0$ must drive the current implying $\sigma = -1$ and $\ell + m - qn > 0$ in the argument

of the delta function (requiring $k_\parallel < 0$ for $\ell = 0$). As a result, writing it in a form allowing the speed integral to be performed gives

$$\delta[\omega\tau_p - 2\pi(\ell + m - qn)] = v_{\omega/k_\parallel}^2 \delta(v - v_{\omega/k_\parallel})/\omega v\tau_p, \tag{5.5}$$

with

$$v_{\omega/k_\parallel} = \omega v\tau_p/2\pi(\ell + m - qn) > 0. \tag{5.6}$$

Then the exponential factor in the Maxwellian becomes

$$x_{\omega/k_\parallel}^2 = m_e v_{\omega/k_\parallel}^2/2T_e, \tag{5.7}$$

where recall a high speed expansion of the like collision operator is used so $x_{\omega/k_\parallel}^2 \gg 1$ in f_0. In addition, the barely passing do not contribute significantly to the current since $v\tau_p/4qR \underset{k\to 1}{\to} (2\varepsilon)^{-1/2}\ln(4/\sqrt{1-k^2}) \gg 1$. Therefore, because of the exponential from f_0, the freely passing ($k^2 \lesssim \varepsilon \ll 1$) electrons make the dominant contribution. As a result, only $\ell = 0$ need be retained in

$$D \approx \frac{\pi e^2 |e_k \cdot n \times k|^2 \lambda J_1^2(\eta)}{8m_e^2 k_\perp^2 |k_\parallel|} \delta(v - v_{\omega/k_\parallel}) \equiv D_{\bar{k}} \lambda J_1^2(\eta)\delta(v - v_{\omega/k_\parallel}). \tag{5.8}$$

Defining the exponential factor of f_0 for $\ell = 0$ as $X^2(\lambda) \equiv x_{\omega/k_\parallel}^2(\ell = 0)$, gives

$$X^2(\lambda) \approx (\omega^2/k_\parallel^2 v_e^2)(1 + k^2/2\varepsilon) \approx (\omega^2/k_\parallel^2 v_e^2)[\lambda/(1-\lambda)], \tag{5.9}$$

as $v\tau_p \underset{k^2 \ll 1}{\to} 2\pi qR\sqrt{1 + k^2/2\varepsilon}$, $v_{\omega/k_\parallel} \underset{\ell=0}{\to} \omega\sqrt{1 + k^2/2\varepsilon}/k_\parallel$ and

$$\Theta \underset{\varepsilon \ll 1}{\to} \left| \int_{-\pi}^{\pi} \frac{d\vartheta}{2\pi} e^{-i\ell\vartheta} \right|^2 = \delta_{0\ell} = \begin{cases} 1 & \ell = 0 \\ 0 & \ell \neq 0 \end{cases}. \tag{5.10}$$

Then, integrating over $v_\parallel < 0$ only, as D vanishes for $v_\parallel > 0$, leads to

$$\left\langle \frac{B}{B_0} \int d^3 v v_\parallel f_1 \right\rangle = -4\pi R \sum_k^{k_\parallel < 0} \frac{\omega^2}{k_\parallel^2 v_e^2} D_k \int_0^{\lambda < 1 - \varepsilon} d\lambda \lambda J_1^2(\eta) \int_0^{\infty} dv \delta(v - v_{\omega/k_\parallel}) v f_0 \frac{\partial}{\partial v}\bigg|_\mu \left(\frac{\bar{h}_p}{f_0} \right). \tag{5.11}$$

In v, λ variables

$$v f_0 \frac{\partial}{\partial v}\bigg|_\mu \left(\frac{\bar{h}_p}{f_0} \right) = v f_0 \left[\frac{\partial}{\partial v}\bigg|_\lambda \left(\frac{\bar{h}_p}{f_0} \right) + \frac{\partial\lambda}{\partial v}\bigg|_\mu \frac{\partial}{\partial\lambda}\bigg|_v \left(\frac{\bar{h}_p}{f_0} \right) \right] = \left(4 - \frac{2\lambda}{\Lambda_{1+2}} \frac{\partial\Lambda_{1+2}}{\partial\lambda} \right) \bar{h}_p, \tag{5.12}$$

since the lowest-order solution is $\bar{h}_p/f_0 \propto v^4 \Lambda_{1+2}(\lambda)$. The exponential in the Maxwellian makes the evaluation of the λ integral insensitive to its upper limit ($\lambda < 1 - \varepsilon$) as will be shown shortly. The pitch angle derivative of Λ_{1+2} is finite at $\lambda = 0$ making $\Lambda_{1+2}^{-1}\partial\Lambda_{1+2}/\partial\lambda \sim 1$.

Only the $\lambda = 0$ limit of this term will matter when the pitch angle integral is performed in (5.18) by integrating by parts, leaving just

$$\left\langle \frac{B}{B_0} \int d^3 v v_{\parallel} f_1 \right\rangle = -16\pi R \sum_k^{k_{\parallel} < 0} \frac{\omega^2}{k_{\parallel}^2 v_e^2} D_k \int_0^{\lambda < 1 - \varepsilon} d\lambda \lambda J_1^2(\eta) \bar{h}_p, \tag{5.13}$$

to be evaluated, where at resonance $\eta = \sqrt{\lambda} k_{\perp} \omega / k_{\parallel} \Omega_e$ and

$$\bar{h}_p = \frac{4 n_e \omega^4 \, \mathrm{e}^{-\omega^2/k_{\parallel}^2 v_e^2} \Lambda_{1+2}(\sqrt{\varepsilon}, Z, \lambda) \, \mathrm{e}^{-X^2(\lambda)}}{3\pi^2 R \nu_{ee} k_{\parallel}^4 v_e^6 [(Z+1)(1 + 1.46\sqrt{2\varepsilon}) + 4]}. \tag{5.14}$$

Inserting $\bar{h}_p$ leads to the expression for the parallel current driven by helicon waves to be

$$J_{\parallel}^H = -e\left\langle \frac{B}{B_0} \int \mathrm{d}^3 v v_{\parallel} f_1 \right\rangle = \frac{8 e n_e}{(Z+1)(1 + 1.46\sqrt{2\varepsilon}) + 4}$$

$$\times \sum_k^{k_{\parallel} < 0} \frac{e^2 |\boldsymbol{e}_k \cdot \boldsymbol{n} \times \boldsymbol{k}|^2}{3 m_e^2 k_{\perp}^2} \frac{\omega^6 \, \mathrm{e}^{-\omega^2/k_{\parallel}^2 v_e^2}}{\nu_{ee} |k_{\parallel}^7| v_e^8} \int_0^{\lambda < 1 - \varepsilon} \mathrm{d}\lambda \lambda J_1^2(\eta) \Lambda_{1+2}(\lambda) \, \mathrm{e}^{-X^2(\lambda)}, \tag{5.15}$$

where $\eta \sim \sqrt{\lambda} k_{\perp} v_e / \Omega_e = \sqrt{\lambda} k_{\perp} \rho_e$. As a result, $X^2(\lambda)$ provides a cut-off before $\lambda \to 1 - \varepsilon$. However, the procedure is more complex in the HCD case than in the LHCD case because of the Bessel function behaviour due the helicon wave interacting with gyromotion of the electron, while the lower hybrid wave only interacts with its parallel streaming.

To understand the procedure and generalize the LHCD results, it is helpful to first consider the LHCD case in more detail than was given in Catto (2021). To obtain the parallel current driven by the LH waves, $J_{\parallel}^{\mathrm{LH}}$, the preceding steps are repeated starting with the replacement $k_{\perp}^{-2} |\boldsymbol{e}_k \cdot \boldsymbol{n} \times \boldsymbol{k}|^2 v_{\perp}^2 J_1^2(\eta) \to |\boldsymbol{e}_k \cdot \boldsymbol{n}|^2 v_{\parallel}^2 J_0^2(\eta)$ and then using $v_{\perp}^2 \approx \lambda v^2 \approx \lambda \omega^2 / k_{\parallel}^2 \to v_{\parallel}^2 \approx \omega^2 / k_{\parallel}^2$. As a result, the driven parallel LH current is

$$J_{\parallel}^{\mathrm{LH}} = \frac{8 e n_e}{(Z+1)(1 + 1.46\sqrt{2\varepsilon}) + 4} \sum_k^{k_{\parallel} < 0} \frac{e^2 |\boldsymbol{e}_k \cdot \boldsymbol{n}|^2}{3 m_e^2} \frac{\omega^6 \, \mathrm{e}^{-\omega^2/k_{\parallel}^2 v_e^2}}{\nu_{ee} |k_{\parallel}^7| v_e^8} \int_0^{\lambda < 1 - \varepsilon} \mathrm{d}\lambda J_0^2(\eta) \Lambda_{1+2}(\lambda) \, \mathrm{e}^{-X^2(\lambda)}, \tag{5.16}$$

where again $\eta = \sqrt{\lambda} k_{\perp} \omega / k_{\parallel} \Omega_e$. Then, an integration by parts is performed by using $\mathrm{e}^{-X^2(\lambda)} = -[(1 - \lambda)^2 / z] \partial \, \mathrm{e}^{-X^2(\lambda)} / \partial \lambda$ to obtain an asymptotic expansion in inverse powers of $z = \omega^2 / k_{\parallel}^2 v_e^2 \gg 1$ to find

$$J_{\parallel}^{\mathrm{LH}} \approx \frac{8 e n_e \Lambda_{1+2}(\sqrt{\varepsilon}, Z, 0)}{(Z+1)(1 + 1.46\sqrt{2\varepsilon}) + 4} \sum_k^{k_{\parallel} < 0} \frac{e^2 |\boldsymbol{e}_k \cdot \boldsymbol{n}|^2}{3 m_e^2} \frac{\omega^4 \, \mathrm{e}^{-\omega^2/k_{\parallel}^2 v_e^2}}{\nu_{ee} |k_{\parallel}^5| v_e^6}, \tag{5.17}$$

as $\int_0^{\lambda < 1 - \varepsilon} \mathrm{d}\lambda J_0^2 \Lambda_{1+2} \, \mathrm{e}^{-X^2} \approx \Lambda_{1+2}(\lambda = 0) / z$ since $\Lambda_j(0) = 1 = J_0^2(0)$. This result is 2/3 of the value given by Catto (2021) when Λ_2 is neglected. The difference is due to the extra power of v^2 multiplying $\bar{h}_p$ inside the v derivative beginning of his (4.7). Notice that only terms that remain finite at $\lambda = 0$ will contribute, as assumed in (5.13).

In the helicon case three integrations by parts in λ are required. Again, only terms finite at $\lambda = 0$ contribute. Ignoring exponentially small terms

$$
\int_0^{\lambda<1-\varepsilon} d\lambda \lambda J_1^2 \Lambda_{1+2}\, e^{-X^2} \approx \frac{1}{z^2} \int_0^{\lambda<1-\varepsilon} d\lambda\, e^{-X^2} \frac{\partial}{\partial \lambda} \left\{ (1-\lambda)^2 \frac{\partial}{\partial \lambda}[(1-\lambda)^2 \lambda J_1^2 \Lambda_{1+2}] \right\}
$$

$$
\approx \frac{1}{z^3} \frac{\partial}{\partial \lambda} \left\{ (1-\lambda)^2 \frac{\partial}{\partial \lambda}[(1-\lambda)^2 \lambda J_1^2 \Lambda_{1+2}] \right\} \bigg|_{\lambda=0} = \frac{1}{z^3} \frac{\partial J_1^2}{\partial \lambda} \bigg|_{\lambda=0} = \frac{k_\perp^2 \rho_e^2}{2z^2}.
$$

(5.18)

Therefore, for HCD

$$
J_\parallel^H = \frac{4en_e \Lambda_{1+2}(\sqrt{\varepsilon}, Z, 0)}{(Z+1)(1+1.46\sqrt{2\varepsilon})+4} \sum_k^{k_\parallel<0} \frac{e^2 |e_k \cdot n \times k|^2}{3m_e^2 k_\perp^2} \frac{\omega^2\, e^{-\omega^2/k_\parallel^2 v_e^2}}{v_{ee}|k_\parallel^3|v_e^4} k_\perp^2 \rho_e^2.
$$

(5.19)

The stronger T_e dependence of $J_\parallel^H$ compared with $J_\parallel^{\mathrm{LH}}$ was previously noted by Li *et al.* (2020*a*, 2021). Helicon waves are also sensitive to the density. If n_e increases substantially, (A14) indicates $k_\parallel^2$ will increase, causing a less effective rf to interaction with the bulk electrons.

Letting $z = \omega^2/k_\parallel^2 v_e^2 = c^2/v_e^2 n_\parallel^2$, then HCD and LHCD depend on $z^{3/2}e^{-z}$ and $z^{5/2}e^{-z}$, respectively, and are maximized at $z = 3/2$ and $z = 5/2$, consistent with assuming $v^2 > v_e^2$ in C_{ee}. Consequently, a slightly higher parallel refractive index is desirable for HCD compared with LHCD, as noted by Prater *et al.* (2014). In addition to penetrating into the central core plasma, the current driven by helicon waves can be comparable to or larger than that driven by lower hybrid waves for the same $\omega^2/k_\parallel^2 v_e^2$ based on the estimate of (A15) and simulations (Vdovin 2013; Prater *et al.* 2014; Lau *et al.* 2018, 2019; Li *et al.* 2020*a*,*b*, 2021).

6. The rf power and current drive efficiency

To form the HCD efficiency requires evaluating the rf power absorbed by the passing electrons. Integrating over $v_\parallel < 0$ and ignoring the negligible power into the barely passing leads to

$$
P_{\mathrm{cd}}^H = \frac{m_e}{2} \left\langle \int d^3v\, v^2 Q \right\rangle \approx \frac{m_e B_0}{4\pi q R} \int d^3v \frac{v_\parallel}{B} v^2 \tau_p \bar{Q}
$$

$$
= \frac{m_e B_0}{4\pi q R} \int d^3v \frac{v_\parallel}{B} v \frac{\partial}{\partial v}\bigg|_\mu \left(\tau_p v D \frac{\partial f_0}{\partial v}\bigg|_\mu \right) = \frac{m_e^2}{q R T_e} \int_0^{\lambda<1-\varepsilon} d\lambda \int_0^\infty dv\, \tau_p v^5 D f_0
$$

(6.1)

$$
= \frac{\pi^{1/2}}{2} m_e n_e \sum_k^{k_\parallel<0} \frac{e^2 |e_k \cdot n \times k|^2 \omega^4}{m_e^2 k_\perp^2\, |k_\parallel^5| v_e^5}\, e^{-\omega^2/k_\parallel^2 v_e^2} \int_0^{\lambda<1-\varepsilon} d\lambda \lambda J_1^2(\eta)\, e^{-X^2(\lambda)}.
$$

Similarly, fixing the v^2 and the numerical factor in Catto (2021) for LHCD, gives

$$
P_{\mathrm{cd}}^{\mathrm{LH}} = \frac{\pi^{1/2}}{2} m_e n_e \sum_k^{k_\parallel<0} \frac{e^2 |e_k \cdot n|^2 \omega^4}{m_e^2 |k_\parallel^5| v_e^5}\, e^{-\omega^2/k_\parallel^2 v_e^2} \int_0^{1-\varepsilon} d\lambda J_0^2(\eta)\, e^{-X^2(\lambda)}.
$$

(6.2)

Again using $e^{-X^2(\lambda)}$ to integrate by parts gives

$$P_{\text{cd}}^H = \frac{\pi^{1/2}}{4} m_e n_e \sum_{\boldsymbol{k}}^{k_{||}<0} \frac{e^2 |\boldsymbol{e}_k \cdot \boldsymbol{n} \times \boldsymbol{k}|^2}{m_e^2 k_\perp^2 \, |k_{||}| v_e} k_\perp^2 \rho_e^2 \, e^{-\omega^2/k_{||}^2 v_e^2}, \tag{6.3}$$

and

$$P_{\text{cd}}^{\text{LH}} = \frac{\pi^{1/2}}{2} m_e n_e \sum_{\vec{k}}^{k_{||}<0} \frac{e^2 |\boldsymbol{e}_k \cdot \boldsymbol{n}|^2 \omega^2}{m_e^2 |k_{||}|^3 v_e^3} \, e^{-\omega^2/k_{||}^2 v_e^2}. \tag{6.4}$$

The numerical coefficient of $P_{\text{cd}}^{\text{LH}}$ is half that of (5.7) in Catto (2021).

The current drive efficiency is defined by the ratio $J_{||}/P_{\text{cd}}$. Consequently

$$\frac{J_{||}^H}{P_{\text{cd}}^H} = \frac{16e\{1 + [0.62 - 1.02(Z+5)/(7Z+11)]\sqrt{\varepsilon}\} \sum_{\boldsymbol{k}}^{k_{||}<0} \frac{e^2 |\boldsymbol{e}_k \cdot \boldsymbol{n} \times \boldsymbol{k}|^2 \omega^2}{\nu_{\text{ee}} m_e^2 k_\perp^2 \, |k_{||}|^3 v_e^4} k_\perp^2 \rho_e^2 \, e^{-\omega^2/k_{||}^2 v_e^2}}{3\pi^{1/2} m_e [(Z+1)(1+1.46\sqrt{2\varepsilon})+4] \sum_{\boldsymbol{k}}^{k_{||}<0} \frac{e^2 |\boldsymbol{e}_k \cdot \boldsymbol{n} \times \boldsymbol{k}|^2}{m_e^2 k_\perp^2 \, |k_{||}| v_e} k_\perp^2 \rho_e^2 \, e^{-\omega^2/k_{||}^2 v_e^2}}, \tag{6.5}$$

while

$$\frac{J_{||}^{\text{LH}}}{P_{\text{cd}}^{\text{LH}}} = \frac{16e\{1 + [0.62 - 1.02(Z+5)/(7Z+11)]\sqrt{\varepsilon}\} \sum_{\boldsymbol{k}}^{k_{||}<0} \frac{e^2 |\boldsymbol{e}_k \cdot \boldsymbol{n}|^2 \omega^4}{\nu_{\text{ee}} m_e^2 |k_{||}|^5 v_e^6} \, e^{-\omega^2/k_{||}^2 v_e^2}}{3\pi^{1/2} m_e [(Z+1)(1+1.46\sqrt{2\varepsilon})+4] \sum_{\boldsymbol{k}}^{k_{||}<0} \frac{e^2 |\boldsymbol{e}_k \cdot \boldsymbol{n}|^2 \omega^2}{m_e^2 |k_{||}|^3 v_e^3} \, e^{-\omega^2/k_{||}^2 v_e^2}}. \tag{6.6}$$

In the terms in the sums in these two general forms and in (5.17), (5.19), (6.1) and (6.2), the Fourier mode amplitudes $|\boldsymbol{e}_k \cdot \boldsymbol{n} \times \boldsymbol{k}|^2$ and $|\boldsymbol{e}_k \cdot \boldsymbol{n}|^2$ and the wavenumbers $k_\perp$ and $k_{||}$ are to be obtained from a full wave code for precise evaluations.

Remarkably, for any (not necessarily the same) single ω and $\boldsymbol{k}$ the preceding forms are identical

$$\frac{J_{||}^H/en_e v_e}{P_{\text{cd}}^H/n_e m_e v_e^2 \nu_{\text{ee}}} = \frac{16\{1 + [0.62 - 1.02(Z+5)/(7Z+11)]\sqrt{\varepsilon}\}\omega^2}{3\pi^{1/2}[(Z+1)(1+1.46\sqrt{2\varepsilon})+4]k_{||}^2 v_e^2} = \frac{J_{||}^{\text{LH}}/en_e v_e}{P_{\text{cd}}^{\text{LH}}/n_e m_e v_e^2 \nu_{\text{ee}}}. \tag{6.7}$$

Approximation (6.7) suggests that comparable current drive efficiencies are possible with helicon and lower hybrid waves, with HCD providing core access as well as profile control. In addition, based on the cold plasma estimate (A15) of the Appendix, it seems possible to drive comparable parallel currents with helicon and lower hybrid waves, as suggested by the simulations of Prater *et al.* (2014) and Lau *et al.* (2018). The $\varepsilon = 0$ form of (6.7) is essentially the same as the non-relativistic, $z = v_p^2 \gg 1$ limit (31) in Karney & Fisch (1985).

7. Combined helicon and lower hybrid

The preceding idealized analytic evaluations of the currents driven and the form of the QL operator suggest it is possible to combine HCD with LHCD for the same applied resonant wave frequency (Yin *et al.* 2022). Helicon and lower hybrid waves then drive current by simultaneously acting on the perpendicular and parallel electron distribution

function, respectively. Indeed, an antenna used for LHCD can drive some level of helicon waves and vice versa. Retaining only $\ell = 0$, using the approximation $\mathrm{e}^{-\mathrm{i}\int_{\tau_0}^{\tau} \mathrm{d}\tau'\chi(\tau')} \approx 1$ and inserting the delta function, the combined QL diffusivity for the passing electrons becomes

$$D = \frac{\pi e^2}{2m_e^2 v^2 \tau_p} \left| \oint_p \mathrm{d}\tau \, \boldsymbol{e}_k \cdot \left[\boldsymbol{n} v_{\|} J_0(\eta) - \mathrm{i}\boldsymbol{n} \times \boldsymbol{k} \frac{v_\perp}{k_\perp} J_1(\eta) \right] \right|^2 \frac{v_{\omega/k_\|}^2 \delta(v - v_{\omega/k_\|})}{\omega v \tau_p}, \tag{7.1}$$

with

$$v_{\omega/k_\|} = \omega v \tau_p / 2\pi(m - qn) > 0. \tag{7.2}$$

The combined parallel current that can be driven is therefore

$$
\begin{aligned}
J_{\|}^{H+LH} =\ & \frac{8e^3 n_e \{1 + [0.62 - 1.02(Z+5)/(7Z+11)]\sqrt{\varepsilon}\}}{3m_e^2 \nu_{\mathrm{ee}}[(Z+1)(1+1.46\sqrt{2\varepsilon})+4]} \sum_{\boldsymbol{k}}^{k_\|<0} \frac{\omega^6 \, \mathrm{e}^{-\omega^2/k_\|^2 v_e^2}}{|k_\||^7 v_e^8} \left\{ |\boldsymbol{e}_k \cdot \boldsymbol{n}|^2 \right. \\
& \left. + \mathrm{i}\frac{|k_\|| v_e \rho_e}{2\omega}[(\boldsymbol{e}_k \cdot \boldsymbol{n})(\boldsymbol{e}_k^* \cdot \boldsymbol{n} \times \boldsymbol{k}) - (\boldsymbol{e}_k^* \cdot \boldsymbol{n})(\boldsymbol{e}_k \cdot \boldsymbol{n} \times \boldsymbol{k})] + \frac{k_\|^2 v_e^2 \rho_e^2}{2\omega^2}|\boldsymbol{e}_k \cdot \boldsymbol{n} \times \boldsymbol{k}|^2 \right\},
\end{aligned} \tag{7.3}
$$

where the integrals are performed by integration by parts as before, but with one new integral appearing in the cross terms, namely

$$\int_0^{\lambda<1-\varepsilon} \mathrm{d}\lambda \, \lambda^{1/2} J_1(\eta) J_0(\eta) \Lambda_{1+2}(\lambda) \, \mathrm{e}^{-X^2(\lambda)} \approx \frac{1}{z} \frac{\partial(\lambda^{1/2} J_1)}{\partial \lambda}\bigg|_{\lambda=0} = \frac{k_\perp \rho_e}{2z^{3/2}} \Lambda_{1+2}(\sqrt{\varepsilon}, Z, \lambda = 0). \tag{7.4}$$

The rf power absorbed by passing electrons is

$$
\begin{aligned}
P_{\mathrm{cd}}^{H+LH} =\ & \frac{\pi^{1/2} e^2 n_e}{2m_e} \sum_{\boldsymbol{k}}^{k_\|<0} \frac{\omega^2}{|k_\||^3 v_e^3} \, \mathrm{e}^{-\omega^2/k_\|^2 v_e^2} \left\{ |\boldsymbol{e}_k \cdot \boldsymbol{n}|^2 \right. \\
& \left. + \mathrm{i}\frac{|k_\|| v_e \rho_e}{2\omega}[(\boldsymbol{e}_k \cdot \boldsymbol{n})(\boldsymbol{e}_k^* \cdot \boldsymbol{n} \times \boldsymbol{k}) - (\boldsymbol{e}_k^* \cdot \boldsymbol{n})(\boldsymbol{e}_k \cdot \boldsymbol{n} \times \boldsymbol{k})] + \frac{k_\|^2 v_e^2 \rho_e^2}{2\omega^2}|\boldsymbol{e}_k \cdot \boldsymbol{n} \times \boldsymbol{k}|^2 \right\}.
\end{aligned} \tag{7.5}
$$

As a result, the efficiency is the ratio $J_{\|}^{H+LH}/P_{\mathrm{cd}}^{H+LH}$, which for a single frequency and wavenumber recovers the same expression as given at the end of § 6. Moreover, the cross term may allow more current to be driven and more power to be absorbed, perhaps as observed in the EAST numerical simulations of Yin *et al.* (2022). Synergy is possible, but not assured, since helicon and lower hybrid waves independently act on perpendicular and parallel motion, respectively, to make the electrons less collisional.

8. Summary

The new results are the analytic expressions for the parallel current that can be driven in a tokamak by a helicon wave, (5.19), and the associated efficiency of HCD, (6.5) and (6.7). In addition, the numerical coefficients of the corresponding tokamak expressions for LHCD (Catto 2021), (5.17), (6.4), (6.6) and (6.7), are generalized as well as corrected, by using a more systematic derivation. Interestingly, for any single applied frequency and

wave vector the efficiency of HCD and LHCD are shown to be the same, (6.7). In addition, HCD and LHCD can be combined for the same applied frequency as shown in (7.3) and (7.5) of § 7, and, of course, recover the same single wave vector efficiency, (6.7). If a combination of HCD and LHCD is possible from the same antenna and rf source, more current than the sum might be driven and steady state tokamak operation might be slightly more feasible.

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Appendix A. Helicon and lower hybrid waves in a cold plasma

To highlight the differences between helicon and lower hybrid waves in the simplest fashion, cold plasma theory can be employed. Helicon current drive employs a perpendicular component of the applied electric field, while the parallel component is used for LHCD.

In a cold plasma, Maxwell's equations lead to the need for the Fourier transform of the applied electric field, $\boldsymbol{e}_k$, to satisfy

$$[\boldsymbol{\varepsilon} - n^2(\boldsymbol{I} - k^2\boldsymbol{kk})] \cdot \boldsymbol{e}_k = 0, \tag{A1}$$

with $\boldsymbol{k}$ a wave vector, $k = |\boldsymbol{k}|$, $\boldsymbol{I}$ the unit dyad and $n = kc/\omega$ the index of refraction, and the dielectric tensor written as

$$\boldsymbol{\varepsilon} = \varepsilon_\perp(\boldsymbol{I} - \boldsymbol{nn}) + \varepsilon_{||}\boldsymbol{nn} - i\varepsilon_\times \boldsymbol{n} \times \boldsymbol{I}, \tag{A2}$$

where the magnetic field is $\boldsymbol{B} = B\boldsymbol{n}$. The components of the dielectric tensor are defined as

$$\varepsilon_\perp = 1 - \sum_s \frac{\omega_{\text{ps}}^2}{\omega^2 - \Omega_s^2} \approx 1 + \frac{\omega_{\text{pe}}^2}{\Omega_e^2} - \frac{\omega_{\text{pi}}^2}{\omega^2} = 1 + \frac{\omega_{\text{pe}}^2}{\Omega_e^2}\left(1 - \frac{\Omega_i\Omega_e}{\omega^2}\right), \tag{A3}$$

$$\varepsilon_{||} = 1 - \sum_s \frac{\omega_{\text{ps}}^2}{\omega^2} \approx -\frac{\omega_{\text{pe}}^2}{\omega^2}, \tag{A4}$$

$$\varepsilon_\times = -\sum_s \frac{\omega_{\text{ps}}^2\Omega_s}{\omega(\omega^2 - \Omega_s^2)} \approx -\frac{\omega_{\text{pe}}^2}{\omega\Omega_e} - \frac{\omega_{\text{pi}}^2\Omega_i}{\omega^3} \approx -\frac{\omega_{\text{pe}}^2}{\omega\Omega_e}, \tag{A5}$$

with $\omega_{\text{pe}}^2\Omega_i = \omega_{\text{pi}}^2\Omega_e$, where the approximate forms are valid for $\Omega_e \gg \omega \gg \Omega_i = Z_ieB/m_ic$, which is the frequency range of interest here. The species (s) plasma frequency

is defined as $\omega_{\text{ps}}^2 = 4\pi Z_s^2 e^2 n_s/m_s$, and $\beta_e = v_e^2 \omega_{\text{pe}}^2 /\Omega_e^2 c^2$. The orderings give $\varepsilon_\times^2/\varepsilon_\parallel \sim \omega_{pe}^2/\Omega_e^2 \sim 1$ and $\varepsilon_\parallel \gg \varepsilon_\times \gg \varepsilon_\perp \sim 1$. Moreover, for a resonant Landau interaction $\omega/k_\parallel v_e > 1$ suggesting the ordering $n_\parallel = k_\parallel c/\omega \gtrsim 1$.

Letting

$$\boldsymbol{e}_k = e_\parallel \boldsymbol{n} + e_\perp(\boldsymbol{k}_\perp + \Upsilon \boldsymbol{n} \times \boldsymbol{k})/k_\perp\sqrt{1 + |\Upsilon|^2}, \tag{A6}$$

the three components of the cold plasma equation are related by

$$e_\parallel/e_\perp = n_\parallel n_\perp/[(n_\perp^2 - \varepsilon_\parallel)\sqrt{1 + |\Upsilon|^2}], \tag{A7}$$

and

$$\Upsilon = i\varepsilon_\times/(\varepsilon_\perp - n^2), \tag{A8}$$

and the cold plasma dispersion relation is

$$n_\parallel^2 \varepsilon_\parallel/(\varepsilon_\parallel - n_\perp^2) = \varepsilon_\perp - \varepsilon_\times^2/(\varepsilon_\perp - n^2), \tag{A9}$$

which when rewritten in powers of $n_\perp^2$ yields

$$\varepsilon_\perp n_\perp^4 + [(\varepsilon_\parallel + \varepsilon_\perp)(n_\parallel^2 - \varepsilon_\perp) + \varepsilon_\times^2]n_\perp^2 + \varepsilon_\parallel[(n_\parallel^2 - \varepsilon_\perp)^2 - \varepsilon_\times^2] = 0. \tag{A10}$$

The two distinct branches satisfying accessibility ($n_\perp^2 > 0$) are found by assuming $[(\varepsilon_\parallel + \varepsilon_\perp)(n_\parallel^2 - \varepsilon_\perp) + \varepsilon_\times^2]^2 \gg 4\varepsilon_\perp\varepsilon_\parallel[(n_\parallel^2 - \varepsilon_\perp)^2 - \varepsilon_\times^2]$ to find

$$n_\perp^2 \approx \begin{cases} -[(\varepsilon_\parallel + \varepsilon_\perp)(n_\parallel^2 - \varepsilon_\perp) + \varepsilon_\times^2]/\varepsilon_\perp & \text{lower hybrid/slow} \\ -\varepsilon_\parallel[(n_\parallel^2 - \varepsilon_\perp)^2 - \varepsilon_\times^2]/[(\varepsilon_\parallel + \varepsilon_\perp)(n_\parallel^2 - \varepsilon_\perp) + \varepsilon_\times^2] & \text{helicon/fast} \end{cases}. \tag{A11}$$

For the lower hybrid branch $\varepsilon_\times^2 + \varepsilon_\perp n_\perp^2 + \varepsilon_\parallel(n_\parallel^2 - \varepsilon_\perp) \approx 0$. Keeping $|\Upsilon|^2 \ll 1$, requires $n^2 \gg \varepsilon_\times \gg \varepsilon_\perp$, leading to

$$\omega^2 = \frac{\Omega_i\Omega_e(1 + \omega_{\text{pe}}^2/k_\perp^2 c^2 + k_\parallel^2\Omega_e/k_\perp^2\Omega_i)}{(1 + \Omega_e^2/\omega_{\text{pe}}^2)(1 + \omega_{\text{pe}}^2/k_\perp^2 c^2) + \omega_{\text{pe}}^2/k_\perp^2 c^2} \approx \frac{\Omega_i\Omega_e(1 + k_\parallel^2\Omega_e/k_\perp^2\Omega_i)}{1 + \Omega_e^2/\omega_{\text{pe}}^2}, \tag{A12}$$

where the last form assumes $k_\perp^2 c^2 \gg \omega_{\text{pe}}^2$ or $n_\perp^2 \gg \varepsilon_\parallel$ to further increase the size $e_\parallel$ to allow $e_\parallel/e_\perp \approx k_\parallel/k_\perp$. Lower hybrid accessibility requires $n_\parallel^2 > 1 + \omega_{\text{pe}}^2/\Omega_e^2$ (Golant 1972), which does not allow it to penetrate to the core in reactor relevant tokamaks.

The helicon or whistler branch can be approximated by

$$n_\perp^2 = [\varepsilon_\times^2 - (n_\parallel^2 - \varepsilon_\perp)^2]/[(n_\parallel^2 - \varepsilon_\perp) + \varepsilon_\times^2/\varepsilon_\parallel]. \tag{A13}$$

Substituting in the components of the dielectric tensor and rewriting gives

$$\omega^2 = \frac{\Omega_i\Omega_e(1 + k_\parallel^2 c^2/\omega_{\text{pi}}^2)}{1 + k_\perp^2/k^2 + \Omega_e^2/\omega_{\text{pe}}^2 + \omega_{\text{pe}}^2/k^2 c^2} \rightarrow \Omega_i\Omega_e\left(\frac{k_\parallel^2 c^2}{\omega_{\text{pi}}^2}\right)\left(\frac{k^2 c^2}{\omega_{\text{pe}}^2}\right), \tag{A14}$$

where the first form allows $k_\perp^2 c^2 \sim \omega_{\text{pe}}^2$, while the last assumes $\omega_{\text{pe}}^2/k^2 c^2 \gg 2 + \Omega_e^2/\omega_{\text{pe}}^2$ and $k_\parallel^2 c^2/\omega_{\text{pi}}^2 \gg 1$ to recover the form in Preinhaelter & Vaclavik (1967) and used in de Assis & Busnardo-Neto (1988). The last form is sometimes referred to as an oblique whistler wave. Moreover, helicon waves can propagate in the high harmonic fast

wave (HHFW) range of frequencies (Ono 1995; Lashmore-Davies *et al.* 1998) and are sometimes referred to as HHFWs (Yin *et al.* 2022).

For the same value of $\omega^2/k_\parallel^2 v_e^2$ the ratio of the parallel currents driven by helicon and lower hybrid waves (or the power absorbed) is roughly

$$
\begin{aligned}
\frac{J_\parallel^H}{J_\parallel^{\mathrm{LH}}} &\sim \frac{\rho_e^2 |\boldsymbol{e}_k \cdot \boldsymbol{n} \times \boldsymbol{k}|^2}{|\boldsymbol{e}_k \cdot \boldsymbol{n}|^2} \sim \frac{k_\perp^2 \rho_e^2 |\varUpsilon| \, e_\perp^2}{(1 + |\varUpsilon|^2) e_\parallel^2} \sim k_\perp^2 \rho_e^2 \frac{\varepsilon_\times (n_\perp^2 - \varepsilon_\parallel)^2}{n_\perp^2 n_\parallel^2 (n^2 - \varepsilon_\perp)} \\
&\sim k_\perp^2 \rho_e^2 \frac{\omega \omega_{\mathrm{pe}}^2 (\omega_{\mathrm{pe}}^2 + k_\perp^2 c^2)^2}{\Omega_e k_\parallel^2 \, c^2 k_\perp^2 \, c^2 k^2 c^2},
\end{aligned}
\tag{A15}
$$

which can be order unity.

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