THE PROBLEM OF THE 1/a-DISTRIBUTION AND COMETARY FADING

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ABSTRACT. The background to the problem of explaining the frequency distribution of cometary 1/a-values is briefly reviewed and it is emphasised that the explanation in terms of the Oort Cloud model relies on an ad hoc fitting function - the fading/disruption probability per revolution. Assuming an underlying steady-state Oort Cloud and the integral equation formalism developed by Oort and Yabushita to predict the 1/a-distribution for an arbitrary fading probability, we have been able to constrain the unknown fading function by comparison with observations. In agreement with previous work we find that the tendency for fading or disruption should be strong at small 1/a-values and weak at large 1/a-values. The mean fading probability per revolution, k(x), is found to lie within a factor roughly of order 2 about $k(x) \approx 0.3(1+(x/4 \times 10^{-3})^2)^{-3/2}$, where x is 1/a in units AU^{-1}. A physical model for fading which might qualitatively account for this behaviour is tentatively proposed. This depends on the thermal shock experienced by a long-period comet nucleus around perihelion passage. It is emphasised that until a viable model for fading has been found, the validity of the steady-state primordial hypothesis remains unresolved.

1. INTRODUCTION

The excess of nearly parabolic orbits amongst the observed comet population has long been a major stumbling block for theory. Thus Laplace (1805, 1816) who could not explain the high eccentricities and near isotropy of aphelion directions within his 'Nebular Hypothesis' for the origin of the Sun and the planets, was forced to adopt an interstellar origin for comets. He showed that this hypothesis naturally explained the parabolic excess, provided that the Solar motion (then in dispute) could be neglected. Despite later arguments (notably by Schiaparelli; see Richter 1963) that this assumption was false and led to erroneous conclusions, the interstellar hypothesis dominated ideas about cometary origins for nearly a century (cf. Newton 1878) and the flaw in Laplace's argument remained generally unrecognised until nearly the end of the period. By the beginning of the present century, however, the interstellar

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hypothesis had been abandoned, and a new consensus, namely that comets were primordial Solar System material which had somehow been formed originally with random inclinations, began to develop. The observed near-parabolic excess could then be explained as a neat example of Darwinian 'survival of the fittest' - the shorter period comets, presumably formed in equal numbers as the long-period group, having long since decayed (Crommelin 1910).

The agreement lasted for less than twenty years. During the second decade of this century the accepted age of the Solar System increased a hundred-fold (into line with its present value), and the decay argument was then found, around 1920, to apply equally strongly to the parabolic group. This dilemma led Bobrovnikoff (1929) to revive the interstellar hypothesis. Following an earlier suggestion by Nölke he proposed that the observed comets had been recently captured from a dense interstellar cloud, and argued that the lack of observed hyperbolic orbits was due to this occurrence having recently ended. Other authors (cf. Öpik 1932), however, continued to advocate a primordial Solar System model. As noted by Russell (1935) the basic difficulty with both ideas at this time was that they each depended on ad hoc assumptions which could not be tested observationally. Thus the interstellar hypothesis had to postulate an extremely dense nearby cometary cloud, while the primordial Solar System hypothesis required an enormous number of unseen comets with large perihelion distances.

This position of uncertainty lasted for a further ten years, after which it was shown (Van Woerkom 1948) that capture of interstellar comets by planetary perturbations was untenable because it would imply an excess of direct orbits (not observed) amongst the long-period group. Thus by about 1950 it appeared inescapable that comets should somehow be primordial Solar System material (Oort 1950), and the development of the now standard model involving a primordial swarm of comets surrounding the Solar System followed.

However it had also been shown by Van Woerkom that planetary perturbations acting on a 1/a-distribution with an initial parabolic excess would cause this distribution to relax quickly to a form having a flat profile. So, although Oort's (1950) theory had shown how stellar perturbations could produce a quasi-steady influx of 'new' nearly parabolic orbits, it did not naturally explain the subsequent sharp fall-off in the 1/a-distribution that was observed. To surmount this difficulty Oort supposed that 'new' comets contained an excess of more volatile material, which made them brighter and more easily discovered than the dynamically 'old' group making second and subsequent passages through the planetary system. These old comets might largely go undetected, thereby providing a possible explanation for the sharp decrease in the number of comets at larger 1/a-values. Thus the 'fading problem' was born: the theory could explain the steady influx of nearly parabolic orbits, but to explain the detailed shape of the 1/a-distribution, it became necessary to invoke arbitrary strong fading (disruption) of the new-comet population. Later work has confirmed this general result (eg. Whipple 1953, 1962; Dobrovol'ski 1972; Shtejns 1972; Weissman 1979; Everhart 1979), and has shown that approximately half of the new comets must physically be able to survive only one perihelion passage as a

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detectable comet. Those that survive once, however, should then go on to make tens or thousands of revolutions in order to explain the extended tail of the 1/a-distribution towards relatively large 1/a-values.

2. RECENT WORK

In recent discussions Everhart (1982), Yabushita (1983) and Bailey (1984) have drawn attention to this problem and have emphasised that the conventional Oort Cloud model does not account naturally for the sharp peak in the 1/a-distribution. The validity or otherwise of the model therefore depends crucially on the assumption made about fading (cf. Yabushita 1983; Bailey 1985).

In order to quantify the degree of fading necessary on the steady-state hypothesis, the author (Bailey 1985) has re-worked the problem of the 1/a-distribution within the integral equation formalism developed and used by Oort (1950) and Yabushita (1983). This work includes, in an approximate way, the two principal effects of stellar perturbations: injection of new comets, and removal of out-going comets with small 1/a-values into unobservable orbits of large perihelion distances. Inclusion of the latter effect is important in that it (a) 'mimics' fading, and (b) allows the integral equation method to be compared directly with Monte-Carlo studies (Weissman 1979; Everhart 1979). The equation which is solved is

$$\nu(x,q) = \nu_{inj}(x,q) + (1-P_{rem}(x,q,q_{obs})) \int_{y_L}^{y_U} (1-k(y,q))\nu(y,q)\phi_{pl}(q,x-y)dy \tag{1}$$

where $\nu(x,q)dx\,dq$ is the number of comets passing perihelion per unit time with 1/a-values, x, in the range (x, x+dx), and perihelia, q, in the range (q,q+dq), ν_{inj} is the injection spectrum of new comets from the Oort Cloud, P_{rem} is the probability that a stellar perturbation will deflect an out-going comet into an unobservable orbit with $q > q_{obs} \sim 2$ AU, ϕ_{pl} (q, Δ) is the probability that planetary perturbations will change a comet's x-value by Δ and k(x,q) is the unknown fading/disruption probability per revolution. Here P_{rem} is calculated in a simple way by assuming that the random increments in angular momentum per revolution due to stellar perturbations are governed by a two-dimensional Maxwellian distribution with dispersion $\sigma_J(a) \simeq 4 \times 10^{15}$ $(a/10^4\ \mathrm{AU})^2$ $\mathrm{m^2\ s^{-1}}$. A graph of P_{rem} is given by Bailey (1985, Fig. 4); for q in the observable region it is a function which decreases from order unity at $x \lesssim 5 \times 10^{-5}$ $\mathrm{AU^{-1}}$ to zero at $x \gtrsim 10^{-4}$ $\mathrm{AU^{-1}}$. The expression for the planetary perturbation function ϕ_{pl} is assumed to be Gaussian with dispersion $\sigma(a) \simeq 10^{-3} \exp(-q/5.2\ \mathrm{AU})$ $\mathrm{AU^{-1}}$, which gives reasonable agreement with values for ϕ_{pl} determined numerically by several authors (eg. Fernandez 1981). The lower and upper limits of the integration are taken to be $y_L \simeq 10^{-5}$ $\mathrm{AU^{-1}}$ (corresponding to a cloud with outer radius of order 10^5 AU) and $y_U \simeq 10^{-1}$ $\mathrm{AU^{-1}}$. The results are insensitive to the precise value of y_U provided $(y_U-x_{max})/\sigma >> 1$, where σ is the dispersion of the planetary perturbation function and x_{max} is the largest 1/a-value for which the

1/a-distribution is required. Comparison of numerical solutions of eqn.(1) with the observed 1/a-distribution then in principle enables the fading probability distribution $k(x,q)$ to be approximately constrained.

Unfortunately an accurate observed 1/a-distribution as a function of q is not available, due to the increase of important selection effects as q increases and the small numbers of comets actually observed. We have therefore adopted an average 1/a-distribution $\nu(x)$ using the 225 comets in the catalogues of Marsden, Sekanina and Everhart (1978) and Everhart and Marsden (1983). Solving eqn.(1) with $q = 0$ then enables the mean fading probability distribution $k(x,0)$ to be determined for these observed comets.

The principle results of this work are two-fold:

(1) The initial very sharp fall-off in the 1/a-distribution (the 'parabolic excess', with $1/a < 10^{-4}$ AU^{-1}) can be attributed to the injection spectrum of new comets from the Oort Cloud (cf. Bailey 1983). For such small 1/a-values removal of comets from the observable region by stellar perturbations occurs with relatively high probability, and from (1) we see that the appropriate solution is then indeed $\nu(x, q) \simeq \nu_{inj}(x, q)$. The precise form of the injection spectrum depends on the detailed model of the Oort Cloud and on the form of the velocity distribution function within the loss cone.

(2) The subsequent continuing decrease in the 1/a-distribution towards larger 1/a-values ($\nu(x, q) \overset{\propto}{\sim} x^{-1}$; Yabushita 1983) can only be understood in this model by invoking strong fading. Assuming a Gaussian distribution for ϕ_{pl} with dispersion $\sigma \simeq 10^{-3}$ AU^{-1} (appropriate to setting $q = 0$), and adopting an Oort Cloud model with energy spectral index $\gamma = 3/2$ (cf. Bailey 1983), it is found that the fading probability distribution is constrained to lie within a factor of order 2 about $k(x) \simeq 0.3\,(1 + (x/4 \times 10^{-3})^2)^{-3/2}$. Changing the spectral index of the Oort Cloud model or allowing the planetary perturbation function to realistically have a non-Gaussian tail does change this result, but not by a large amount (cf. Bailey 1985).

Thus we conclude, in agreement with previous Monte-Carlo studies, that in order to explain the observed 1/a-distribution the required fading probability per revolution has to be high at small 1/a-values and low at large 1/a-values. The detailed shape derived for $k(x)$ does however suggest that the initial loss of volatiles may not be the most important effect (cf. Whipple 1962; Weissman 1979), as strong fading appears to be necessary both for 'new' comets ($a \gtrsim 10^4$ AU) and for dynamically old comets having a-values $\gtrsim$ few hundred AU.

3. A THERMAL SHOCK MODEL FOR FADING

Since it is not our aim to explain the 1/a-distribution purely by assumption, it is important to develop physical models to account for the required fading probability distribution. In the past a number of qualitative suggestions have been made to account for the required fading behaviour of comets. These include loss of volatiles during the first significant warming of the comet, the formation of an inert surface layer or crust, and physical disruption of comets (see Weissman 1980 and

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references therein). Implicit in some of these explanations is that the degree of fading experienced by a comet depends on an ageing process measured, for example, in the number of orbital revolutions. On the other hand, splitting events or major outbursts might allow an 'old' comet on this scheme to rejuvenate (for example the crust might be removed), thereby complicating the relationship between degree of fading and orbit number. A third possibility is that the amount of fading per revolution is unconnected with orbit number, but instead correlates simply with the semi-major axis, as for example could be implied by a straightforward interpretation of the fading probability distribution $k(x)$. Since it is important to investigate all possibilities for fading, we here present a 'thermal shock' model for fading of this third type.

We assume that fading or disruption is related fundamentally to the detailed temperature and physical structure of the nucleus, appealing to a physical process for fading similar to that which causes the cometary outburst phenomenon (eg. thermal stress, release of volatiles, low-temperature phase transitions, release of the energy of frozen-in radicals etc.). It is thus plausible to assume that when the temperature of part of the nucleus reaches some critical value (which may, in fact, be quite low; cf. Greenberg 1982) this part is somehow broken away from the main body, leading either to disruption of the nucleus or strong physical fading. In this way one might expect the amount of fading to correlate with the proportion of the nucleus which is significantly affected by the heat pulse occurring around perihelion passage.

The characteristic skin depth of penetration of a heat pulse of duration τ is typically of order $(k_D\tau)^{\frac{1}{2}}$, where $k_D = \kappa/\rho C$ is the thermal diffusivity of the material, κ is the thermal conductivity, ρ the density and C the specific heat. Assuming the nucleus is made primarily of crystalline water ice, we have at low temperatures the approximate relations $\kappa \simeq 30\ T^{-1.4}$ W cm^{-1} °K^{-1} (Klinger 1975, Fig.1) and $C \simeq 2.43 \times 10^{-5}\ T^{-2.83}$ Jg^{-1} °K^{-1} (Giauque and Stout 1936). With $\rho = 0.94$ g cm^{-3} this gives $k_D \simeq 1.3 \times 10^6\ T^{-4.23}$ cm^2 s^{-1} $\underset{\sim}{\propto} T^{-4}$ ($10 \lesssim T \lesssim 22$ K). If the mean temperature of a comet nucleus in an orbit of semi-major axis a is assumed to be $(L_{\odot}/16\pi\sigma a^2)^{1/4} \propto a^{-\frac{1}{2}}$, we thus have that the depth of significant penetration of the heat pulse is $\delta \gtrsim (k_D\tau)^{\frac{1}{2}} \gtrsim 0.1$ (a/200 AU) km, where we have taken $\tau \gtrsim 1$ yr. The longest period comets, with nuclear radii of order of a few km, may therefore be substantially affected by the heat pulse associated with perihelion passage, while the comets of a shorter period may only be affected in a thin surface layer. This argument therefore suggests a possible qualitative explanation for the fading probability distribution required by observations. We emphasise, however, that whatever the true explanation for 'fading' (and a combination of factors seems probable) the net effect must be to produce the $k(x)$-distribution found here. The a priori probability of this being the case leads some authors to reject the steady-state assumption!

4. CONCLUSIONS

The fading probability per revolution required in order to explain the

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observed 1/a-distribution in the context of a steady-state Oort Cloud type of model is given approximately by $k(x) \sim 0.3\ (1+(x/4\mathrm{x}10^{-3})^2)^{-3/2}$. A qualitative physical explanation for fading has been presented, but until this or some other model has been shown to work quantitatively the validity of the primordial Solar System hypothesis remains unresolved.

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DISCUSSION

A. H. Delsemme: (1) I want to describe a possible mechanism for the fading problem. If comets have stayed for 5 billion years in the Oort Cloud they will have been irradiated by cosmic rays sufficiently that the first 1 - 3 metres of the nucleus will have been considerably modified. This modification yields highly reactive molecules and radicals that remain frozen in the icy matrix until the comet comes closer to the Sun. Such a layer may be completely vapourised in less than one passage for small perihelion distances ($q < 0.2$ or 0.3 AU), or in a small number of passages if the perihelion distance is larger (the layer removed per passage varies as q^{-2}).

(2) Statistically speaking (and whatever the reason) it is well known that the absolute magnitude of new comets (first passage with $q < 7$ AU) is 3 - 4 magnitudes brighter than that of periodic comets. Is this what your fading model would predict?

(3) I have published (In 'Dynamics of the Solar System' p.265, ed. R. L. Duncombe, Reidel, IAU Symp., 81, 265, 1979) a distribution of the absolute magnitudes of new comets. It is (surprisingly) bimodal; 82% have a peak near $H_0 = 5.5$ (true new comets?), whereas 18% show a peak near $H_0 = 10.0$ (fragmented comets). Fragmentation is therefore well documented and could certainly be one of the contributing factors to the fading of new comets.

P. R. Weissman: I agree with you that there is a fading factor that varies with cometary age. In my work I modelled it as most comets having a 10% disruption probability, and only 15% having a zero disruption probability. But at the same time I agree very strongly with Delsemme that new comets fade strongly after their first return due to this loss of a surface layer of extremely volatile materials. That is an effect that we should try to include in our models in the future.

M. E. Bailey: (In answer to this and Professor Delsemme's first point.) Yes. I agree that it does seem probable that comets coming in for the first time should be a little brighter, due to loss of volatiles etc, than the others. My point is that it is important that modelling of this process should be put on a quantitative physical basis.

To take Delsemme's other two points in reverse order: Yes; and unfortunately my fading 'model' is too qualitative to make definite predictions. But it does 'predict' that long-period comets should be more prone to disruption than the comets of shorter period if one accepts the thermal shock hypothesis. This should correlate with the absolute magnitude distribution!