ON OBSERVATIONS IN SCHWARZSCHILD BACKGROUND

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ABSTRACT. In the geometrical optics approximation a uniform approach to the description of the influence of strong Schwarzschild field on astronomical observations is given.

Among exact solutions of Einstein field equations the Schwarzschild metric describing the space-time geometry of the body with spherically symmetric mass distribution is of a main importance for astronomical applications. Different effects of this field have been examined in many papers. Usually relativistic corrections are small, and the case of a strong field is significant only when dealing with physical phenomena near objects like neutron stars and black holes. As a rule, one considers mainly the photometric effects due to the field action on the radiation emitted by the gravitating body itself or surrounding shell (see, e.g., [1-3]).

In the present work we investigate the influence of strong Schwarzschild field on following observed parameters : the apparent transverse velocity, apparent proper motion, and luminosity distance. The results obtained here may be of interest for high angular resolution observations of such objects containing probably massive black holes as galaxy cores and quasars. Moreover, they give a uniform approach to various astronomical problems in this space-time. Solving our problem we used a standard form of metric and well-known properties of light rays in the Schwarzschild background (for a complete treatment see [4,5]).

The geometry of the problem is as follows. The point source, observer, and gravitating body are found in a plane of the light ray connecting the source and the observer. Because of gravitational bending of light, this ray, in general, is not an only one so the observer may see several images of the source. Lower we consider one of these images. Let r_s and r_o be Schwarzschild radial coordinates of the source and observer, respectively, and θ_s is the angle between corresponding radial directions. Again, let α be the angle between light ray and radial direction in natural reference frame. The values of this angle at the source and observer we denote as α_s and α_o, respectively. They may be easily computed with the help of light ray equation. Then the position of the source on observer's sky is unambiguously determined by the angle α_o and by azimuth ϕ_o fixing the light ray plane.

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We assume that the observer rests far away from the gravitating body, and the source moves arbitrarily near the body, i.e. $r_o \gg r_s$. With respect to natural frame the source velocity v has following orthogonal components : $v_{\parallel}$ is along the light ray, $v_{\perp}$ is normal to the ray and lies in the ray plane, and v_{α} is normal to the ray plane. Below we take the light speed to be a unit.

If τ_o is the observer's proper time, then the components of the apparent proper motion of the source are $d\alpha_o/d\tau_o$ and $\sin\alpha_o\, d\phi_o/d\tau_o$. Let $v_{t\alpha}$ and $v_{t\phi}$ be the corresponding components of the apparent transverse velocity, obviously these components are orthogonal and

$$v_{t\alpha} = D\,\frac{d\alpha_o}{d\tau_o}\,, \qquad v_{t\phi} = D\sin\alpha_o\,\frac{d\phi_o}{d\tau_o}\,, \tag{1}$$

where D is a distance from the source to observer. Because $r_o \gg r_s$ we put $D = r_o$ (below we ground this equality). Using the solution of the ray equation [4] it can be shown that

$$v_{t\alpha} = K_\alpha\,\frac{v_\perp}{1 - v_\parallel}\,, \qquad v_{t\phi} = K_\phi\,\frac{v_\phi}{1 - v_\parallel}\,. \tag{2}$$

Here we take the following notations

$$K_\alpha = \frac{1}{\cos\alpha_o}\left.\frac{\partial\alpha_s}{\partial\theta_s}\right|_{r_s = \mathrm{const}}\,, \qquad K_\phi = \frac{\sin\alpha_s}{\sin\theta_s}\,. \tag{3}$$

Eqs. (2) are generalization of known expression for the apparent velocity used for the interpretation of superluminal separation in quasars in the relativistic jet model [6] . Dividing (2) by $D = r_o$ we obtain formulae for components of apparent proper motion :

$$\frac{d\alpha_o}{d\tau_o} = \frac{K_\alpha}{1 - v_\parallel}\,\frac{v_\perp}{r_o}\,, \qquad \sin\alpha_o\,\frac{d\phi_o}{d\tau_o} = \frac{K_\phi}{1 - v_\parallel}\,\frac{v_\phi}{r_o}\,. \tag{4}$$

In flat space and in the limit of slow motion these formulae turn into usual ones.

To calculate the distance we assume that the source has the form of an infinitesimal luminous screen normal to the ray. If dA is the proper area of the sreen, and $d\Omega$ is the measured solid angle subtended by the screen then the corrected (for a spectral shift) luminosity distance D is defined by the relation

$$dA = D^2\,d\Omega. \tag{5}$$

We use the term of Kristian and Sachs [7] . D is independent of the source velocity and that simplifies significantly the computation. The final result is

$$D^2 = \frac{1 - r_g/r_s}{|K_\alpha \; K\phi|} \, r_o^2 \quad , \tag{6}$$

where $r_g = 2GM/c^2$ is the Schwarzschild radius. Note that usual luminosity distance D_L is defined so that the bolometric flux should satisfy the inverse-square law. The equality holds [7,8]

$$D_L = (1 + z)^2 \, D, \tag{7}$$

where z is the spectral shift. In our case

$$1 + z = \frac{1 - v_{\|}}{(1 - v^2)^{1/2} \, (1 - r_g/r_s)^{1/2}} \, . \tag{8}$$

From Eqs. (2), (4), and (6) it follows that for the description of the Schwarzschild background influence on observations it is sufficient to introduce two correction factors K_α and K_ϕ. According to (3) these factors are due to light ray geometry and their definition is formally independent on whether the gravitation is present or not. In flat space

$$K_\alpha = K_\phi = r_o \, (r_o^2 + r_s^2 - 2r_o r_s \cos \theta_s)^{-1/2} \xrightarrow[r_o \gg r_s]{} 1. \tag{9}$$

Considering appropriate rays it may be shown that omitting the additional factor $(1 - r_g/r_s)^{-1/2}$ K_α and K_ϕ are angular magnifications of Schwarzschild gravitational lens in α_o - and ϕ_o - direction, respectively. Because of distortion of light rays $K_\alpha \neq K_\phi$ except for the case when the source is just between the gravitating body and observer ($\theta_s = 0$). Then due to axial symmetry both factors are equal and exactly

$$K_\alpha = K_\phi = \frac{(1 - r_g/r_s)^{1/2}}{1 - r_s/r_o} \, . \tag{10}$$

In accordance with (6) in this case we have

$$D = r_o - r_s . \tag{11}$$

Eq. (11) has been obtained earlier in [9,10] . For $r_o \gg r_s$ $D = r_o$ that justify our choice of observer's radial coordinate as the distance measure in Eqs. (1), (2), and (4).

The elementary consideration based on Eqs. (3) and the solution of light ray equation [4] shows that the factor K_α is positive continuous function. The behaviour of the factor K_ϕ is more complicated. If $\theta_s \to 180°n$ ($n = 1, 2, \ldots$) then $K_\phi \to \infty$ because the light rays are focused by the field on the axis passing through the gravitating body and the observer, and the geometrical optics approximation is invalid. Every time as the ray crosses this axis K_ϕ not only tends to infinity but also changes its sign. Geometrically condition $K_\phi < 0$ means that the source and its image are of opposite orientation. Hence, relativistic corrections have essential values if the source is near the gravitating body or the focusing axis. In latter case, for example, the apparent velocity may exceed greatly the light speed. Note that one of the explanations of superluminal separation in quasars is based on similar property of galaxy gravitational lenses [11] .

In conclusion we emphasize that for given r_o, r_s, and θ_s the values of the factors K_α and K_ϕ may be calculated without any difficulty by means of computers. As illustration, Figure 1 displays the behaviour of $\log K_\alpha$ and $\log |K_\phi|$ as functions of the angle θ_s between radial directions to the source and the observer.
Radial coordinate of the source r_s was be taken to be constant and equal to four Schwarzschild radii while the observer's radial coordinate r_o went into infinity.

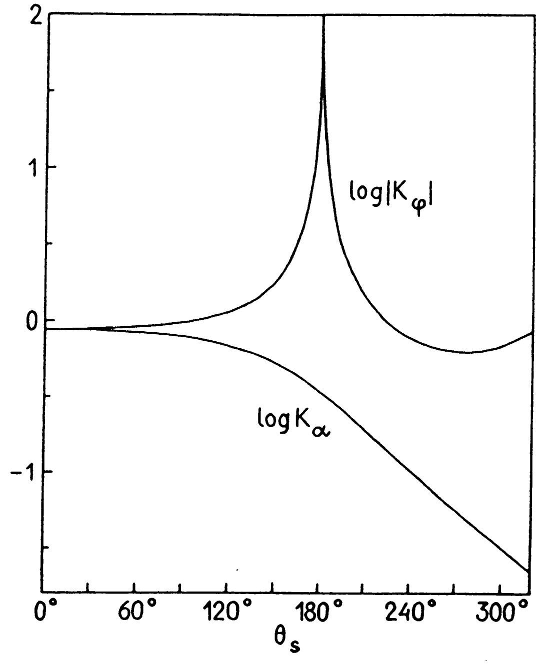

Figure 1. Illustration of behaviour of the correction factors K_α and K_ϕ. Curves of $\log K_\alpha$ and $\log |K_\phi|$ versus the angle θ_s for $r_s = 4r_g$ and $r_o \to \infty$.

Thus, we have shown that the influence of spherically symmetric background on observations is described by two correction factors which in the geometrical optics approximation characterize completely Schwarzschild gravitational lenses.

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DISCUSSION

Kreinovich : what increase in precision is necessary to test your model of apparent super-luminal velocities ?

Khmil : it is very difficult to tell because it depends on the structure of quasars. Perharps $0\overset{''}{.}001$ could be sufficient.

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