Frank Ramsey

Frank Ramsey at eighteen years old

The name of Frank Ramsey is universally known amongst combinatorial mathematicians, but our casual mental picture of him can easily be an unimpressive one – the man who almost stumbled across the theorem that now bears his name, thereby anticipating Erdős and Szekeres, who of course gave the proper proof. Such an idea of Ramsey is entirely false: he was an absolutely brilliant man, who would certainly have become even more famous had he not died so young, and who would surely, it could easily be argued, have made yet further remarkable contributions to philosophy, economics and logic – and to combinatorics.

Frank Plumpton Ramsey was born in Cambridge on 22nd February 1903 and lived there all his life, but this connection with the city was only first generation. His father, A. S. Ramsey, was a Yorkshireman, educated at Batley Grammar School, who became a Fellow of Magdalene College, Cambridge. A. S. Ramsey was an applied mathematician, whose textbooks were extremely successful. There is no doubt that he was a very able man – as Bursar of Magdalene he rescued the college from ruin, and later as Senior Tutor he began a revival in their academic fortunes. He became President of the College and almost became Master, being ultimately denied, he felt, only because of his nonconformist principles. Some people, though, considered him a little dour, and it has been suggested that his young family exerted a civilizing influence on him: one colleague said, 'Ramsey has been converted to a belief in culture by his clever children.' So, perhaps, we glimpse the childhood of Frank Ramsey; clearly he was already sparkling. Frank was the eldest of four children; his only brother, Michael, was to become Archbishop of Canterbury. Michael later recalled how the boys played ball games together during the school holidays, and how they would talk about all sorts of matters:

I was aware that he was far cleverer than I was and knew much more, yet there was such a total lack of uppishness about him that we just conversed in a friendly way and he never made me feel inferior though I was so vastly below par intellectually, and that was the wonderful joy of it.[1]

Sent to Winchester for his schooling, Ramsey continued to pursue knowledge during his vacations in Cambridge. He got to know a couple of young Magdalene dons, C. K. Ogden and I. A. Richards, and having borrowed some abstruse philosophical texts from Ogden he taught himself German in barely a week. It was during this period that he began to think seriously about logic and philosophy for, when he returned to Cambridge in the autumn of 1920 as an undergraduate at Trinity College, his views were already found worth an audience. He was invited to join the *Apostles*, the exclusive Cambridge 'conversation society' with a somewhat homoerotic air, dominated by John Maynard Keynes and Lytton Strachey and comprising the self-proclaimed elite of the University. The Apostles remained of great importance to Ramsey for the rest of his life.

At the time Ramsey arrived back in Cambridge, the local philosophers were pre-occupied with Ludwig Wittgenstein's ground-breaking work *Logisch-Philosophische Abhandlung*, which *inter alia* offered a new interpretation of the logical foundations of mathematics. Wittgenstein had been registered as a research student at Cambridge in 1912, under the supervision of Bertrand Russell, and had often discussed his developing ideas with Russell and with G. E. Moore. He moved out of England after a year, and though he maintained correspondence with the two men, circumstances prevented his return for many years. His thoughts matured during the war, and as a prisoner in Italy in 1919 he had prepared a typescript of them, which was transmitted through the intermediacy of Keynes to Russell.

Russell arranged for the publication of the original German text, and Ogden was keen to publish an English translation. However, neither Russell nor Moore found the

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[1] A significant amount of the information for the present article, including this quote, comes from the fascinating radio programme 'Better than the stars' made for the BBC in 1978 by D. H. Mellor [4]. A transcript of the article, and indeed the audio itself, is available at `http://www.dar.cam.ac.uk/~dhm11/RamseyLect.html`.

work easy to comprehend, and Moore took the view that it was untranslatable. But in a Christmas Eve letter to Wittgenstein in 1921, Russell writes, 'the translation is being done by two young men at Cambridge who know mathematical logic.' One of these was F. P. Ramsey. Who the other man was that Russell intended is now unknown: perhaps he mistakenly assumed that Ramsey would be assisted by his friend (later Professor) R. B. Braithwaite. The magnitude of the task is hinted at by the fact that the translation, *Tractatus Logico-Philosophicus*, was eventually printed in 1922 side by side with the original German. Ogden, the editor, explained that this was 'on account of the obvious difficulties raised by the vocabulary and in view of the peculiar literary character of the whole.' The arrangement also allowed 'a certain latitude … in passages to which objection might otherwise be taken as over-literal.' Ogden expressed his 'indebtedness to Mr F. P. Ramsey, of Trinity College, Cambridge, for assistance … with the translation.'

Wittgenstein   Keynes

That the translation of so important and difficult a work should have been entrusted to the eighteen-year-old Ramsey is remarkable enough, but Ramsey was more than just a translator: he had his own thoughts, and the English version was, as hinted at above, really a reformulation of Wittgenstein's ideas and not a literal translation. Shortly after its publication, Ramsey was commissioned to write a review of the *Tractatus* for the philosophical journal *Mind*, and his review contained not only an excellent exposition of the main lines of the work, but also penetrating criticisms of it. A correspondence had developed between Wittgenstein and Ramsey about the meaning of the more difficult passages, and Wittgenstein invited Ramsey to visit him in the summer of 1923, after Ramsey's graduation. Wittgenstein was living at the time in the Austrian village of Puchberg, where he had taken a job as a schoolteacher, and for about a fortnight he and Ramsey spent several hours a day on the *Tractatus*, going through it line by line. The following spring Ramsey returned to Austria for a longer period, partly to talk to Wittgenstein and partly to submit to psychoanalysis over 'an unhappy passion for a married woman'. While in Vienna he received the extraordinary news that, on 9th May, when he was barely twenty-one years old, he had been elected a Fellow of King's College.

During the autumn of 1924 Ramsey helped Russell with a revision of Russell and Whitehead's *Principia Mathematica*, but Ramsey had serious misgivings about the basis of *Principia*, notably the theory of types and its consequent Axiom of Reducibility. His own first major work, *The Foundations of Mathematics*, published in 1925, avoided these blemishes by incorporating Wittgenstein's notions of tautology. His continued search

for a demonstrably consistent basis for mathematics naturally led him to consider the *Entscheidungsproblem*, the problem of finding a procedure for testing the consistency of logical formulæ. The result was his 1928 paper 'On a problem of formal logic', in the course of which he proved the famous theorem by which we know him. He proved it not, of course, for its own value, but as a lemma on the road to his logical goal.

These two papers are the only strictly mathematical works Ramsey ever published, even though mathematics was what gave him a living, since in 1926 he was appointed University Lecturer in Mathematics. His lectures were generally on the foundations of mathematics, and were very popular with students. This was no doubt due to Ramsey's clarity of presentation, his manifest enthusiasm for the subject, and to his memorable physical presence: he stood six foot three and weighed sixteen stone (Keynes writes of his 'Johnsonian frame'), and he often broke out into 'loud', 'devastating' and 'infectious' laughter.

Ramsey in the Lake District, aged twenty-two

Though a mathematician, philosophy remained Ramsey's first love (his 'vocation', as Braithwaite put it), and a meeting of philosophers in Cambridge in 2003 to mark the centenary of his birth testifies to the respect he still commands in the subject. But Ramsey also did profound work on the foundations of probability theory and in mathematical economics. His 1926 paper 'Truth and probability' exposed deep flaws in the recently propounded Keynesian view that probability is 'partial entailment' (so effectively that Keynes himself dropped the idea), and put forward instead notions of 'subjective probability', rating the strength of a person's beliefs, and 'subjective utility',

rating the strength of their desires. These ideas (and similar ideas of de Finetti) were further developed by Savage in 1954. His two papers in economics, the 1927 'A contribution to the theory of taxation' and the 1928 'A mathematical theory of saving', are seminal achievements: Keynes called the latter paper 'one of the most remarkable contributions to mathematical economics ever made'. Ramsey's ideas of optimal pricing are increasingly being recognized as important and they are in use today by many large companies. Boiteux's refinement of his pricing rule is nowadays known as the Boiteux–Ramsey rule, and Ramsey's theory of optimal control, as restated by Koopmans and Cass, has become the Ramsey–Cass–Koopmans model.

In September 1925 Ramsey married Lettice Baker, who went on to bear him two daughters. Frank first met Lettice when he was a freshman, she being a third-year philosopher, but no romance ensued. However, she returned to Cambridge three years later to work in the psychology laboratory and, as she afterwards said, 'Frank remembered that he'd met me and invited me round to tea and then we got to know each other and that was how it happened really.' They did not discuss their work with each other but together they greatly enjoyed music and concert-going. They were also very fond of hill walking, both in England and elsewhere: the best-known photograph of Ramsey is one of him walking on Red Pike in the Lake District, the picture being taken by Lettice. She later owned the photography shop Ramsey and Muspratt that was a Cambridge institution, recording generations of begowned graduands and triumphant sporting teams.

Wittgenstein returned to Cambridge in January 1929. Because of his absence from the university he had never received a degree, but a means had now been found whereby he might graduate, following a few months residence and the submission of a dissertation. Hence he was re-enrolled as a research student, with Ramsey as his research supervisor, and for a fortnight he lived with Frank and Lettice. Wittgenstein went on to become a professor at the university.

But Ramsey's career was over. Towards the end of 1929 he fell seriously ill. He was admitted to Guy's Hospital, where an operation revealed that he was suffering from jaundice. Shortly afterwards, his condition became critical. Wittgenstein visited him in the evening of 18th January, and, contrary to his normal behaviour, he tried desperately to cheer his friend up. Ramsey died a few hours later, at 3 a.m. on 19th January 1930, aged twenty-six.

The advanced nature of many of Ramsey's ideas, the clarity of his perception which often left others behind, his easy-going and unassertive manner in life, and his early death, meant that much of what he did was not taken up by the rest of the world until much later. It is easy but natural to speculate on what might have happened had he lived longer. For example, in economics, surely he would have received the Nobel prize! In philosophy, it seems quite plausible that the reverence for Wittgenstein which pervaded Cambridge in the 1930s would have been moderated, had Ramsey been around to provide a counterbalance. In logic, he would certainly have been thrilled by the appearance in 1931 of Gödel's work, showing that the *Entscheidungsproblem* has no solution, and it is a safe bet that he would, in consequence, have devoted a lot of his subsequent energy to mathematics. So what of combinatorics?

Ramsey at age twenty-five

In 1934 Erdős visited Cambridge, where he met Rado. For most of the next four years Erdős was in England, often in Cambridge. Obviously, Ramsey and Erdős would have talked about Ramsey's theorem. The ideas of transfinite Ramsey theory (the partition calculus) were already developing in conversations between Erdős and Rado. Surely Ramsey, who had proved the first theorem and who was so interested in the mathematics of set theory, would have joined in? And what of finite Ramsey theory? It is not always recognized that Ramsey was actually interested in bounds for Ramsey numbers. In his original paper he wrote, 'we should at the same time like to have information as to how large' the Ramsey numbers actually are. The simplest argument for the existence of the Ramsey number $R(k)$ gives the upper bound 2^{2k-3}, only slightly larger than Erdős and Szekeres's refinement $\binom{2k-2}{k-1}$. On the other hand, Ramsey's general argument, when specialized to the two-colour graph case, gives only $2^{\binom{k}{2}}$. But he is unhappy with this, and supplies a separate argument for this case, giving the bound $k!$. He even observes that this can be reduced further because regular graphs of odd order and odd degree don't exist! Finally he adds that 'this value is, I think, still much too high.' He clearly gave quite some thought to the matter and he clearly had a gift for combinatorial thought.

Surely, then, Ramsey and Erdős would have discussed Ramsey numbers together. And would it not have excited Ramsey when Erdős discovered the connection between these numbers and probability? It seems highly likely that, for one reason or another, these two exceptional minds would have combined in at least one joint paper. We can only wonder at what we have missed.

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