Part VII: Some General Gas-Dynamical Problems

Examples of Gas Motion and Certain Hypotheses on the Mechanism of Stellar Outbursts

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IN connection with the problem of the explanation of stellar outbursts exact solutions for gas flows with spherical symmetry can be given in the following three cases.

1. Propagation of detonation waves from the interior to the surface of a star, accompanied by an output of nuclear energy on the wave front. Effects of the increase of detonation velocity depending upon the law of density decrease from the center to the outer layer (failure of Chapman-Jourguet's rule) are investigated. As a result of a sufficiently rapid density decrease one obtains a complete dispersion of the detonation products with the formation of a vacuum near the center. Similar solutions are obtained for the spherical problem of the propagation of a rarefaction jump, accompanied by an energy output (a jump of flame front type) through a gas at rest.

2. Perturbed gas motion due to an explosion caused by a sudden large output of energy inside a star. The energy is transferred to the surface together with the shock wave. Exact automodel solutions of the equations for adiabatic time-dependent gas motion, accompanied by the formation of a vacuum when $\gamma=C_p/C_v=4/3$ and without it, are given, gravitation being taken into account. Some solutions for larger values of γ are studied.

3. Examples of dynamically unstable equilibrium states disturbed by an explosion followed by the development of a shock wave, propagating through a gas at rest with density gradient. A motion without energy output develops. The energy of the disturbed motion at any time is equal to the initial energy in the equilibrium state.

The application of these results to the interpretation of observational data requires an investigation of time-dependent effects in stellar photospheres. In addition an investigation of the role of electromagnetic effects in stellar outbursts is needed.

The solutions of the 1st and 2nd types of problems were reported at the International Congress for Applied Mechanics at Brussels, September, 1956. These solutions are given in detail in the author's monograph.[1]

Let us examine now the solutions of the 3rd type and consider the equilibrium state and the time-dependent motion of a perfect gas, taking self-gravitation into account.

We shall give an example of an exact solution of the nonlinear equations of motion, of such a form that the equilibrium distribution of density and pressure in the gas represents the initial state, while the time-dependent motion is of explosive type, arising and developing without energy output. This solution may be considered as describing a phenomenon of mechanical instability, which could be used for the explanation of certain effects observed in variable stars.

We take the equations of one-dimensional time-dependent gas motion with spherical symmetry in the form,

$$\frac{\partial M}{\partial r}=4\pi r^2\rho;\quad \frac{\partial\rho}{\partial t}+\frac{\partial\rho v}{\partial r}+\frac{2\rho v}{r}=0,$$

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{fM}{r^2}=0,$$

$$\frac{\partial(p/\rho^\gamma)}{\partial t}+v\frac{\partial(p/\rho^\gamma)}{\partial r}=0. \tag{1}$$

The symbols have their usual meaning; f is the constant of gravitation. We treat γ as a constant.

When the gas is at rest, the thermodynamic parameters may have the values given by

$$v=0;\quad M_1=\frac{20}{\sqrt{3}}\pi A r^{0.6};\quad \rho_1=\frac{A}{r^{2.4}};\quad P_1=\frac{50\pi}{21}\frac{fA^2}{r^{2.8}}, \tag{2}$$

which represent a solution of Eqs. (1) for any positive value of the constant A. They could also hold for a case where transfer of radiation is present, if we may assume some special values for the coefficients of absorption and/or energy output.

[1] L. I. Sedov, *Methods of Similarity and Dimensions in Mechanics* (Moscow, 1957), 3rd edition 1954, 4th edition 1957.

The solution will be assumed to hold for a region $r<r^*$, while for $r>r^*$ other conditions may be found.

It is easy to verify that another particular, time-dependent exact solution of equations (1) is given by the expressions

$$v=\tfrac{2}{3}rt^{-1},\quad \rho=(6\pi f t^2)^{-1},\quad p=Kf^{-1}t^{-2\gamma},\quad M=2/9r^3f^{-1}t^{-2}, \qquad (3)$$

where K is an arbitrary positive constant.

We can now assume that the motion described by (3) is found inside a sphere with a time-dependent radius r_2, while the state of equilibrium (2) exists outside this sphere. At the shock front, which has the velocity $c=dr_2/dt$, the following conditions must be fulfilled:

$$M_2=M_1,$$
$$\rho_2=\frac{\gamma+1}{\gamma-1}\rho_1\left(1+\frac{2}{\gamma-1}\frac{a_1^2}{c^2}\right)^{-1},$$
$$v_2=\frac{2}{\gamma+1}c\left(1-\frac{a_1^2}{c^2}\right),$$
$$P_2=\frac{2\gamma}{\gamma+1}P_1\frac{c^2}{a_1^2}\left(1-\frac{\gamma-1}{2\gamma}\frac{a_1^2}{c^2}\right), \qquad (4)$$

where $a_1^2=\gamma p_1/\rho_1$. After substitution of the expressions (3) on the left-hand side and of expressions (2) on the right-hand side of (4), we find that these conditions can be satisfied if

$$r_2=(30\pi f A t^2)^{5/12},\quad \gamma=\frac{7}{6},\quad K=\frac{4}{189\pi}(30\pi f A)^{5/6}. \qquad (5)$$

The first expression (5) determines the shock speed. Since Eqs. (1) are exactly satisfied both for $r<r_2$ and for $r>r_2$, at all t, while the interior solution (3) has no singularity at $r=0$, the solution does not imply an additional source of mechanical or thermal energy to keep the motion going.

The solution (3) is a particular form of a more general solution,[2] which leads to nonlinear pulsating motions of a gas sphere. Thus the motion we have obtained here, with its monotonic decrease of the radial velocity, can be considered as a particular case of pulsating motions, typical for Cepheids. The simple solution, however, holds only for $\gamma=7/6$. Nevertheless, the case allows us to suppose that similar types of motion caused by small perturbations may be possible with other initial density distributions and correspondingly different values of γ.

I shall also mention exact solutions of the nonlinear equations of adiabatic motions of gas columns, in which both magnetic and gravitational forces are taken into account and an infinite gas conductivity is supposed. These solutions were investigated in detail by my pupils, A. Kulikovsky[3] and I. Javorskaya.[4]

The equations of gas motion with cylindrical symmetry in Lagrange's form are as follows:

$$\rho\frac{\partial^2 r}{\partial t^2}=-\frac{\partial}{\partial r}\left[P+\frac{1}{8\pi}(H_1^2+H_2^2)\right]-\frac{1}{4\pi}\frac{H_2^2}{r}-\frac{2fM\rho}{r},$$
$$\rho=\rho_0\frac{r_0}{r}\frac{\partial r_0}{\partial r},\quad H_1=H_{10}\frac{r_0}{r}\frac{\partial r_0}{\partial r},$$
$$P=P_0\frac{\rho^\gamma}{\rho_0^\gamma},\quad H_2=H_{20}\frac{\partial r_0}{\partial r}, \qquad (6)$$

where H_1 and H_2 are, respectively, the axial and circumferential components of the vector of the magnetic field strength; ρ_0, P_0, H_{10}, H_{20} are certain functions of the Lagrange coordinate r_0, all other designations being the ordinary ones.

The solution of the system (6) is determined by the following formulas:

$$r=r_0\mu(t),\quad \rho=\rho_0\mu^{-2}(t),\quad H_1^2=H_{10}^2\mu^{-4}(t),$$
$$v=r\frac{\mu'(t)}{\mu(t)},\quad P=P_0\mu^{-2\gamma}(t),\quad H_2^2=H_{20}^2\mu^{-2}(t), \qquad (7)$$

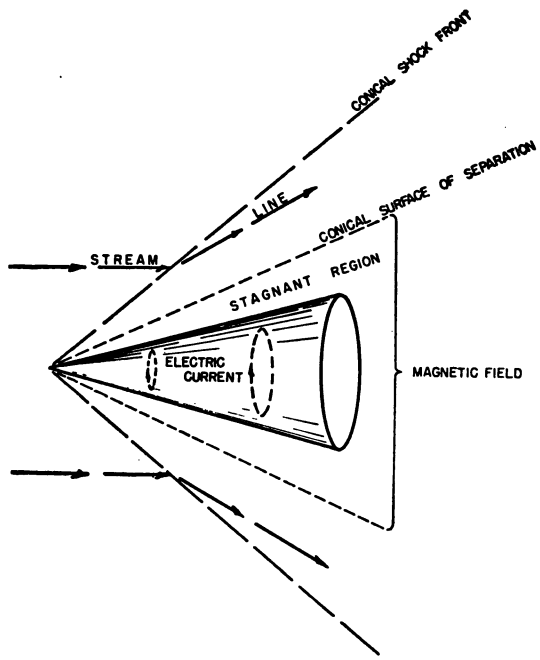

FIG. 1. Flow of an ionized gas around a magnetized conical body.

[2] S. Rosseland, *The Pulsation Theory of Variable Stars* (Oxford, 1949).

[3] A. G. Kulikovsky, Doklady Akad. Nauk S.S.S.R. **114**, 984 (1957).

[4] I. M. Javorskaya, Doklady Akad. Nauk S.S.S.R., **114**, 988 (1957).

where the function $\mu(t)$ satisfies the simple equation

$$\left(\frac{d\mu}{dt}\right)^2=\frac{A}{\gamma-1}\mu^{2(1-\gamma)}-2(B+2\pi f\rho_0)\ln\mu-D\mu^{-2}+C. \quad (8)$$

The quantities P_0, H_{10}, and H_{20} are given by

$$P_0=A\int_0^{r_0}\rho_0 r_0 dr_0+N,$$

$$H_{10}^2=8\pi D\int_0^{r_0}\rho_0 r_0 dr_0+L,$$

$$H_{20}^2=\frac{8\pi B}{r_0^2}\int_0^{r_0}\rho_0 r_0^3 dr_0+\frac{M}{r_0^2}, \quad (9)$$

A, N, D, L, B, M, and C being arbitrary constants at our disposal.

If the gravitational forces are not taken into account and the gravitational constant f in Eqs. (6) and (8) is supposed to be zero, the function $\rho_0(r_0)$ can be arbitrary in Eqs. (7) and (9). If $f\neq 0$, Eqs. (7), (8), and (9) will determine an exact solution of the system (6) if $\rho_0=$constant.

Various forms can be obtained for the function $\mu(t)$ determined by Eq. (8), depending upon the magnitude of the constants A, B, D, C (and ρ_0 if $f\neq 0$). In particular nonlinear periodic oscillatory motions are possible (cf. Kulikovsky and Javorskaya[3,4]).

The astrophysical application of analogous motions has been studied by S. Chandrasekhar and E. Fermi[5] for the case of linearized equations.

I would also mention a paper by A. Kulikovsky,[6] in which a formulation is given of the problem of the flow of a conducting gas around a magnetized body, together with the main qualitative conclusions that can be drawn. It is of interest to note that there can appear a region in which the flow does not penetrate, while a strong magnetic field gives a pressure deflecting the fluid away from this region. The particular case of a conical field has been indicated in Fig. 1. There is a magnetic field inside the conical body and in the conical "stagnant" region around it; beyond that region there is flow, separated by a conical shock wave from the original undisturbed flow field. Outside the surface bounding the "stagnant" region the magnetic field is zero: the lines of magnetic force have all been driven together into the "stagnant" region.

[5] S. Chandrasekhar and E. Fermi, Astrophys. J. **118**, 1 (1953).
[6] A. G. Kulikovsky, Doklady Akad. Nauk S.S.S.R. **117**, 199 (1957).

DISCUSSION

(Since part of the discussion was formed by requests for explanations which have been given in the present text of the paper, much of the discussion has been left out. Some further questions referred to the case of conical flow represented in the diagram.—*Editors*.)

H. W. Liepmann, *California Institute of Technology, Pasadena, California:* What happens if you break off the cone somewhere?

L. I. Sedov, *Academy of Sciences of the U.S.S.R., Moscow, U.S.S.R.:* Near the cone point, the motion remains conical.

J. M. Burgers, *University of Maryland, College Park, Maryland:* My own work on the penetration of a shock wave into a magnetic field had brought me to the question whether in the case of an infinitely conducting medium, containing a concentrated magnetic field, say a dipole field, a stationary solution might be possible in which a certain region would be impenetrable to the flow. I think that the case mentioned by Sedov is an example of such a flow field. There is a body of conical form, around which there appears a region in which there is no flow. This region is bounded on the outside by a concentrated current sheet, and beyond that sheet there is no magnetic field: all the lines of force have been blown away. Inside the current sheet there is a magnetic field, determined both by the set of currents assumed within the body and by the effect of the concentrated current sheet. Further outside there is, of course, a conical shock wave, as is necessary in any case of supersonic flow.

I believe that when you have a magnetic dipole, in a supersonic field of flow which at large distance from the dipole has a constant velocity in a given direction, there will appear a more or less parabolic surface of separation, again carrying a concentrated electric current. Outside of this surface, the whole magnetic field would be blown away (again, a shock wave is needed somewhere outside this surface of separation); while inside the surface of separation, the magnetic dipole field would be supplemented by a field due to the current sheet. This inside field presumably will be irrotational, that is, there will be no currents in the region inside the surface of separation. The magnetic pressure at the surface of separation must balance the gas pressure on the outside. In the case of the conical field this is the condition determining the angle at the vertex of the conical sheet of separation; in the case of the flow around a dipole, there would arise a much more complicated equation from which the form of the surface of separation should be obtained.

[**L. I. Sedov**, in his concluding remarks, remarked that other solutions (apparently of similar stationary fields) have been obtained, and he particularly mentioned the case of a dipole.]

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