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The effect of poloidal magnetic field on type I planetary migration

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Abstract. We study the effect of poloidal magnetic field on type I planetary migration by linear perturbation analysis with the shearing-sheet approximation and the analytic results are compared with numerical calculation. We investigate the cases where magneto-rotational instability (MRI) does not occur: either the disk is two-dimensional, or a very strong field is exerted. We derive formulae for torque exerted on the planet for both cases. We find that two-dimensional torque is suppressed when plasma beta is less than 1 and three-dimensional modes dominate, in contrast to unmagnetized case.

Keywords. MHD — planets and satellites: formation — solar system: formation

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1. Introduction

Type I planetary migration is an important issue in the theory of giant planet formation and there have been a lot of work on this topic. For an unmagnetized disk, Tanaka *et al.* (2002) showed that the protoplanet of $5\mathrm{M}_\oplus$ located at 5AU and embedded in the minimum-mass solar nebula (Hayashi *et al.* 1985) migrates inward in 8×10^5 years, which is shorter than the observed lifetime of a protoplanetary disk [$\sim 10^7$ years, see e.g., Haisch *et al.* 2001].

Magnetic fields are supposed to be present in protoplanetary disks. Significant mass accretion onto the central star requires an effective mechanism for angular momentum transfer, which is most likely driven by magneto-rotational instability, or MRI (Balbus & Hawley 1991).

The property of planetary migration may be totally different if a magnetic field is exerted on the disk. Terquem (2003) performed a linear analysis of the torque for a two-dimensional laminar disk with toroidal magnetic field and showed that when the toroidal magnetic field inside the planet's orbital radius is larger than outside, inward migration may be halted.

In this paper, we investigate the type I planetary migration assuming a poloidal magnetic field, which is a complementary analysis to Terquem (2003). As a first step to understand the nature of migration in a magnetized disk, we perform a shearing-sheet analysis and calculate the one-sided torque. We restrict ourselves to a laminar disk, the case without MRI, and derive analytic formulae of torque. For two-dimensional modes, we derive an analytic formula which generalizes that of Artymowicz (1993). For three-dimensional modes, we use the WKB approximation and derive an analytic torque formula in a strong field limit. We show that two-dimensional modes are suppressed by poloidal magnetic field and three-dimensional modes will dominate the total torque. We then compare the results of the linear analysis with a numerical calculation, and show good agreement between them.

2. Basic Equations and Linear Analysis

The basic equations are ideal MHD equations.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0, \tag{2.1}$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\frac{1}{\rho}\nabla P - \nabla \psi_{\rm eff} - 2\Omega_{\rm p}(\boldsymbol{e}_z \times \boldsymbol{v}) - \frac{1}{4\pi\rho}\boldsymbol{B} \times (\nabla \times \boldsymbol{B}), \tag{2.2}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}), \tag{2.3}$$

where ρ, $\boldsymbol{v}$, P, $\psi_{\rm eff}$, $\Omega_{\rm p}$, $\boldsymbol{e}_z$, and B are the gas density, velocity, gas pressure, effective potential including tidal force and the planet's gravitational potential, Keplerian angular velocity of the protoplanet, a unit vector directed to the z-axis, and the magnetic flux density, respectively. The Keplerian angular velocity of the protoplanet is given by

$$\Omega_{\rm p} = \left(\frac{G M_{\rm c}}{r_{\rm p}^3}\right)^{1/2}, \tag{2.4}$$

where G, $M_{\rm c}$, and $r_{\rm p}$ are the gravitational constant, mass of the central star, and the distance between the protoplanet and the central star, respectively. In the shearing-sheet approximation, effective potential $\psi_{\rm eff}$ is given by

$$\psi_{\rm eff} = -\frac{3}{2}\Omega_p^2 x^2 - \frac{GM_{\rm p}}{r}, \tag{2.5}$$

where M_p is the mass of the planet and $r = (x^2+y^2+z^2)^{1/2}$ is the distance from the planet (see e.g., Narayan *et al.* 1987). The first term includes the gravitational potential of the central star and the centrifugal potential, and higher orders in x, y, and z are neglected. We also neglect the z dependence of the effective potential for simplicity, and neglect vertical stratification. This greatly simplifies the calculation, and it does not seriously affect the results. The second term is the gravitational potential of the protoplanet. We adopt an isothermal equation of state, $P = c^2\rho$, where c is sound speed.

We solve these equations by linear perturbation analysis by the following steps:

(a) We assume that the background disk has no planet, constant density ρ_0, constant poloidal magnetic field $B_0\boldsymbol{e}_z$ and constant Keplerian shear $\boldsymbol{v}_0 = -(3/2)\Omega_{\rm p} x \boldsymbol{e}_y$

(b) We assume the steady state, $\partial/\partial t = 0$, and perturbations are Fourier decomposed in y- and z- directions.

(c) For each Fourier mode, we solve the resulting ordinary differential equation under outgoing boundary condition.

(d) For each Fourier mode, the torque exerted on the disk by the planet is calculated by

$$T_{k_y,k_z} = -2L_y L_z \rho_0 r_p k_y \int dx {\rm Im}\left(\frac{\delta\rho_{k_y,k_z}(x)}{\rho_0}\right)\psi_{\rm p k_y k_z}(x), \tag{2.6}$$

where k_y and k_z are wavenumber of y- and z- directions and L_y and L_z are box size of the searing-sheet. Note that the box size L_y (L_z) and the wavenumber are related by $k_y = 2\pi n_y/L_y$ ($k_z = 2\pi n_z/L_z$) where n_y (n_z) is an integer. The total torque is the sum of all the Fourier modes.

Two-dimensional mode

For two-dimensional modes, $k_z = 0$, magnetic pressure changes the effective sound speed by

$$c \rightarrow \sqrt{c^2 + v_A^2} \tag{2.7}$$

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where $v_A = B_0/\sqrt{4\pi\rho_0}$ is Alfvén speed. The position of effective Lindblad resonances is given by

$$\sigma^2(x_{\rm eff}) - \Omega_p^2 - c^2 k_y^2 (1+\beta^{-1}) = 0, \tag{2.8}$$

where $\beta = c^2/v_A^2$ and $\sigma(x) = (3/2)\Omega_{\rm p} k_y x$ (it is analyzed by Artymowicz (1993) for an unmagnetized disk). Waves are excited at the effective Lindblad resonances and propagate away from the planet, carrying angular momentum away.

We find a formula for the two-dimensional mode torque exerted at the effective Lindblad resonances, which extends the formula given by Artymowicz (1993). It reads,

$$T_{\rm 2D} = \frac{2\pi}{3} r_p \rho_0 L_y L_z \frac{\Omega_p}{\Omega_p^2 + 4c^2 k_y^2 (1+\beta^{-1})} \frac{1}{\sqrt{\Omega_p^2 + c^2 k_y^2 (1+\beta^{-1})}} \Psi_{\rm eff}^2, \tag{2.9}$$

where

$$\Psi_{\rm eff} = \frac{d\psi_p}{dx}(x_{\rm eff}) - 2k_y \frac{\sqrt{\Omega_p^2 + c^2 k_y^2 (1+\beta^{-1})}}{\Omega_p} \psi_p(x_{\rm eff}) \tag{2.10}$$

and the subscript "eff" denotes the value at the effective Lindblad resonances.

Three-dimensional mode

For three-dimensional modes, $k_z \neq 0$, we find that magnetic resonances, which is found by Terquem (2003) in toroidal field case, also appears in poloidal case. This becomes important when magnetic field is strong. The location of the magnetic resonances is given by

$$\sigma^2 = \frac{c^2 v_A^2 k_z^2}{c^2 + v_A^2}. \tag{2.11}$$

We find that the perturbed surface density is singular at the location of the magnetic resonances and strong point-like torque is exerted.

In the strong field limit, $\beta \to 0$, we find an analytic expression of the torque at the magnetic resonances. This reads

$$T_{\rm MR} = \frac{2\pi}{3} L_y L_z \frac{\rho_0 r_p k_z}{\Omega_p c} \psi_{p,{\rm MR}}^2, \tag{2.12}$$

where subscript MR denotes the value at the magnetic resonances. Note that this torque expression does not depend on β and hence strength of magnetic field. This indicates that three-dimensional torque converges to one value when magnetic field is very strong. For details of the analyses, we refer the readers to our full paper (Muto *et al.* 2007).

3. Numerical Method

We have performed numerical calculations in order to investigate how well the equations (2.9) and (2.12) agree with the realistic values of the torque. We have done two sets of runs. One is for a two-dimensional disk. The other is for a three-dimensional thick disk. We adopt the nested grid method (see, e.g., Machida *et al.* 2005, Matsumoto & Hanawa 2003) to obtain high spatial resolution near the planet. Each level of rectangular grid has the same number of cells ($= 64 \times 256$) for 2D run, while ($= 64 \times 256 \times 16$) for 3D run. The cell width $\Delta s(l)$ depends on the grid level l. The cell width is divided by two in each direction with increasing grid level ($l \to l+1$). We use four grid levels (l =1,2 $\cdots$ 4) for 2D run and five levels for 3D run. We normalize time by Kepler time of the planet $\Omega_{\rm p}$ and velocity by sound speed c and therefore, length scale is normalized by scale hight $h = c/\Omega_{\rm p}$. The box size of the coarsest grid $l = 1$ is chosen so that $(L_x, L_y) = (64h, 256h)$

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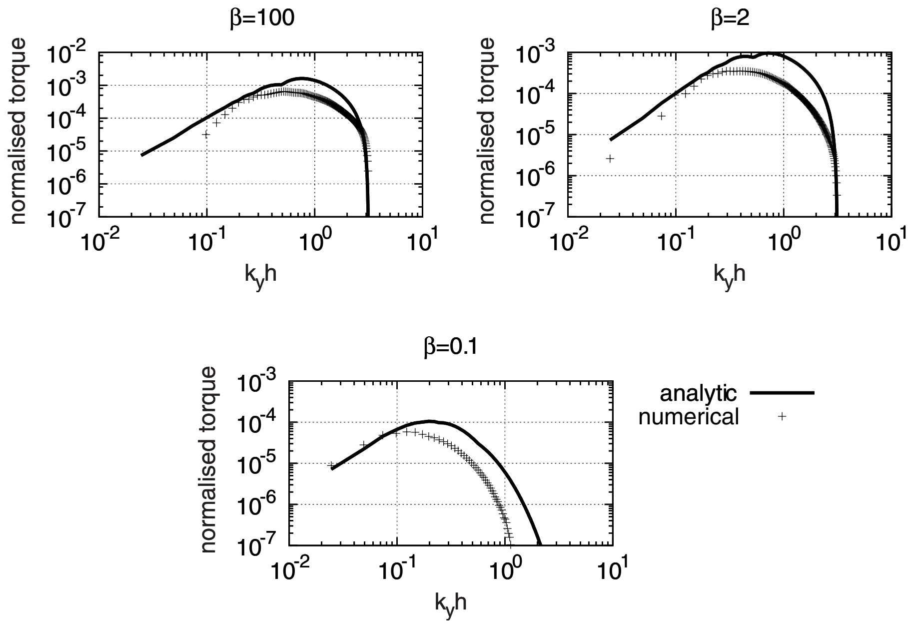

Figure 1. Comparison of the torque obtained by the two-dimensional numerical calculation (plus) and the linear analysis (line), the equation (2.9) for $\beta = 100$ (top left), $\beta = 2$ (top right), and $\beta = 0.1$ (bottom). Horizontal axis is the azimuthal mode number and the vertical axis is normalized torque.

for 2D run and $(L_x, L_y, L_z/2) = (64h, 256h, 16h)$ for 3D run. Note that in z-direction, the simulation box extends from midplane to $z = L_z/2$. The box size of the finest grid is $(x, y) = (2h, 8h)$ for 2D run and $(x, y, z) = (2h, 8h, h)$ for 3D run. The cell width of the coarsest grid is $\Delta s(1) = h$, while that of the finest grid has $\Delta s(4) = 0.125h$ for 2D run and $\Delta s(5) = 0.0625h$ for 3D run. We use a fixed boundary condition in the x-direction and a periodic boundary condition in the y-direction. In the z-direction, we impose a periodic boundary condition between $z = -L_z/2$ and $z = L_z/2$. The planet-to-primary mass ratio is $q = 9 \times 10^{-6}$ and the disk aspect ratio is $h/r = c/\Omega_{\rm p} r_{\rm p} = 0.05$, and the strength of magnetic field is varied.

4. Results

We compare the results of numerical calculation with the analytic torque formula (2.9) for two-dimensional calculations. Also, we compare the formula (2.12) for three-dimensional calculations in which we do not observe MRI.

For two-dimensional calculation, we find that the larger B_0, the larger the effective sound speed [see Eq. (2.7)], the more Lindblad resonances are shifted away from the orbit, and the more the one-sided Lindblad torque decreases. We show in figure 1 the comparison between the results of numerical calculation and linear analysis [the equation (2.9)]. They show reasonably good agreement, to within an order of magnitude, even though the equation (2.9) estimates the torque by the value of density perturbation only at the position of effective Lindblad resonances. Therefore, the equation (2.9) is useful to estimate two-dimensional torque when poloidal magnetic field is exerted on the disk.

Figure 2 compares the torque obtained from the three-dimensional numerical calculations and that calculated from linear analysis of $k_z = 2\pi/L_z$ ($n_z = 1$) modes [the equation (2.12)]. The formula (2.12) shows a very good agreement.

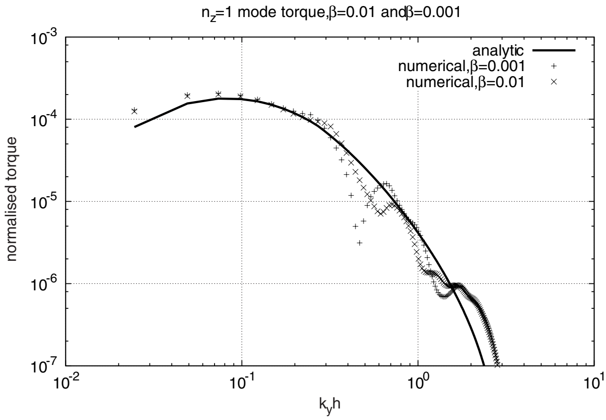

Figure 2. The comparison of $k_z = 2\pi/L_z$ ($n_z = 1$) mode torque between three-dimensional numerical calculation (symbols) and the linear analysis (line) for $\beta = 0.001$ (plus) and $\beta = 0.01$ (cross). The horizontal axis shows the azimuthal mode number and the vertical axis shows the normalized torque.

From linear analysis and numerical calculation, it is indicated that the stronger the magnetic field, the weaker the two-dimensional mode. For $\beta < 0.01$, three-dimensional modes dominate the total torque, in contrast to unmagnetized case where three-dimensional modes are always subdominant (see e.g., Tanaka *et al.* 2002).

5. Summary

We performed a linear perturbation analysis to calculate the torque exerted on a low-mass planet by a disk with poloidal magnetic field. We derived torque expressions in the shearing-sheet approximation, in two-dimensions [Eq. (2.9)], and in three dimensions [in the strong field limit, Eq. (2.12)]. Our torque analytic expressions are in good agreement with the results of numerical simulations.

Since we have been working on the shearing-sheet approximation and derived the torque formulae in some restricted cases, the analysis of more general cases and other resonances is necessary (T. Muto and S. Inutsuka 2008, in preparation). We also need more quantitative analysis of the differential torque, which gives the actual value of the torque exerted on the planet.

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