

## Part IV

# Form and spectra of emission beams

Tuesday afternoon. Session Chair: John Seiradakis

- How do average profile properties constrain radio-frequency emission models?
  - ★ Form and spectra of emission beams
    - \* Observations of the form and spectral characteristics of pulsar profiles and profile components.
    - \* Observations of the polarization and frequency variations of pulsar profiles and components.
    - \* Classification of pulsars by their profile characteristics.
    - \* Observations bearing on the physical significance of profile components.
    - \* Observations indicating distinct types or categories of pulsar emission.

*This session was highlighted by two extensive review papers on the structure and classification of pulsar average profiles, one by Joanna Rankin, entitled, An empirical theory of pulsar emission, which is included here, and another by Andrew Lyne and Richard Manchester, presented by Lyne, recently published in *Monthly Notices of the Royal Astronomical Society*, entitled, The shape and evolution of pulsar emission beams (Lyne and Manchester 1988).*

# AN EMPIRICAL THEORY OF PULSAR EMISSION

JOANNA M. RANKIN

Department of Physics, University of Vermont

## Abstract

A system of pulsar profile classification is used as a starting point to study the emission characteristics of pulsars. Two types or mechanisms of pulsar radiation are identified which combine geometrically to produce five major species of profile. The core emission, which forms a pencil beam of radiation, is apparently produced close to the stellar surface throughout the entire polar cap region by low  $\gamma$  particles. The conal emission, which consists of a hollow conical beam, then seems to be emitted at heights of 10 to 20 stellar radii by currents of high  $\gamma$  particles travelling along some of the most peripheral of the "open" field lines.

## Pulsar profiles: Patterns of radio frequency evolution

The system of classification which is the fundamental empirical foundation for this study has been described in a series of published papers (Rankin 1983a, b; 1986). It considers the morphological characteristics of polarized average profiles with particular attention to their formal evolution with radio frequency as well as certain pulse-sequence properties, mode changing, drifting subpulses and pulse nulling.

Classification of pulsar characteristics has long been recognized as a potential source of physical insight into the emission process. Inspired by Radhakrishnan and Cooke's (1969) hollow cone emission model, Huguenin, *et al.* (1971) first proposed a classification scheme for average profiles, and Backer (1976) then greatly extended it. The latest system, summarized in table 1, builds directly upon these early ideas.

On the basis of their morphological and polarization properties, five major categories of pulsar profiles are delineated, the core-single ( $S_t$ ), conal single ( $S_d$ ), double ( $D$ ), triple ( $T$ ), and five-component ( $M$ ). These species in turn are found to entail two distinct types or mechanisms of pulsar radiation, *core* emission and *conal* emission.

One of the species, the five-component ( $M$ ) profile, deserves special mention and is discussed in a separate article in this volume (Rankin 1992). It seems to exhibit the full degree of morphological complexity of which the pulsar emission process is capable. Its outer and inner pairs of components (I and V, and II and IV, respectively) apparently result from a double-conal emission beam, and the central core component (III) from a smaller pencil beam within it.

Summarizing the material in table 1 and anticipating some of the conclusions below, let us com-

pare the characteristics of the core and conal radiation. Core emission is primary in about 70% of all pulsars. It is prominent in core-single ( $S_t$ ), triple ( $T$ ), and five-component ( $M$ ) pulsars, associated with younger pulsars, and emitted at low altitude. Its modulation tends to be steady, and it is frequently marked by sense-reversing (antisymmetric) circular polarization. Finally, it typically has a softer spectrum and thus tends to become less prominent at high frequency.

Conal emission, by contrast, is primary in about 30% of all pulsars. It is most prominent in conal single ( $S_d$ ), double ( $D$ ), and conal triple ( $cT$ ) pulsars, associated with older pulsars, and emitted at altitudes of 10 to 20 stellar radii. Conal emission exhibits complex, quasi-periodic modulation patterns, characteristic linear polarization, and a harder spectrum.

## Physical basis of speciation

The foregoing classification scheme has obvious physical implications, and these consequences provide clues to the evolutionary significance of the various species. Figure 1 gives the magnetic field as a function of spindown age for some 150 pulsars with well determined classifications; note the tendency of the various species to "clump" in certain regions of the diagram. A somewhat more intelligible plot is shown in figure 2, where the "acceleration potential", proportional to  $B_{12}/P^2$ , is plotted against spindown age. The "linear" form of the plot is expected in that the two quantities are correlated. Note, however, how the  $S_t$  pulsars have  $B_{12}/P^2$  values greater than about 2.5, whereas the  $S_d$ ,  $D$ ,  $cT$  and  $M$  pulsars almost always fall below 2.5.  $T$  pulsars, by contrast, seem to populate the entire range of  $B_{12}/P^2$  values.

Furthermore, the various species differ significantly in age. The  $S_t$  group is by far the youngest

**Table 1**  
**Pulsar Classification System**

Profile Species	TYPE	%	Emission type	PRIMARY IDENTIFICATION CRITERIA	
				Spectral Evolution	Polarization
Core Single	$S_t$	34	Core (and conal at high freqs.)	<ul style="list-style-type: none"> <li>• profile has a SINGLE COMPONENT near 400 MHz and usually develops a CONAL OUTFRIDER PAIR above 1400 MHz. <math>S_t</math> profiles thus evolve to triple or even double forms at very high frequencies, given the steeper spectrum of core emission.</li> </ul>	<ul style="list-style-type: none"> <li>• <u>Circular</u>: moderate in degree, often with an antisymmetric change of hand; moderate symmetric (non-reversing) also observed</li> <li>• Linear: nearly complete to unpolarized</li> <li>• PA traverse: unsystematic; does not appear to follow single-vector (R&amp;C) model.</li> </ul>
Triple	T	25	Central core component plus conal	<ul style="list-style-type: none"> <li>• profile consists of both a CENTRAL CORE COMPONENT and one CONAL OUTFRIDER PAIR. T profiles tend to evolve into double forms at very high frequencies, given the steeper spectrum of core emission.</li> </ul>	<ul style="list-style-type: none"> <li>• <u>Circular</u>: moderate in degree, often with an antisymmetric change of hand; moderate symmetric (non-reversing) also observed; 50-70% circular is observed in some stars</li> <li>• Linear: moderate to virtually unpolarized; polarization-mode depolarization at wings</li> <li>• PA traverse: S-shaped, steep, usually near 180° for M, less for T; closely following the single vector (R&amp;C) model</li> <li>• Corellation: if circular sense change is lh to rh, linear traverse is usually cw (neg.), and vice versa</li> </ul>
Five Component	M	9	component pair(s)	<ul style="list-style-type: none"> <li>• profile consists of both a CENTRAL CORE COMPONENT and TWO CONAL OUTFRIDER PAIRS. M profiles tend to evolve into featureless "boxy" forms at very high frequencies, as the components merge and the core emission subsides.</li> </ul>	
Conal Single	$S_d$	15	c o n a l	<ul style="list-style-type: none"> <li>• profile has a SINGLE COMPONENT near 400 MHz, which BROADENS AND BIFURCATES progressively at lower frequencies, due to the high-altitude spreading of the conal emission beam.</li> </ul> <p style="text-align: center;">↓ closely connected ↓</p>	<ul style="list-style-type: none"> <li>• <u>PA traverse</u>: shallow (<math>\leq 90^\circ</math>), closely following the single vector (R&amp;C) model</li> <li>• Linear: moderate to virtually unpolarized; polarization-mode depolarization at wings</li> <li>• Circular: small and unsystematic</li> </ul> <p style="text-align: center;">↓ closely connected ↓</p>
Conal Double	D	10		<ul style="list-style-type: none"> <li>• profile has a TWO COMPONENTS virtually throughout spectrum, which slowly broaden at lower frequencies, due to the high-altitude spreading of the conal emission beam.</li> </ul>	
OneSided Triple	$T_{\frac{1}{2}}$	3	as above	<ul style="list-style-type: none"> <li>• one-sided "triple" pulsars are similar to triple (T) pulsars in all respects, except that only one conal outrider is apparent. It is not clear whether the outrider is actually</li> </ul>	
Conal Triple	cT	3	conal	<ul style="list-style-type: none"> <li>• pulsars with cT profiles are closely related to those with M profiles, except that the sight line to crosses the polar-cap region at the periphery of the conal emission beams;</li> </ul>	
OneSided Double	$D_{\frac{1}{2}}$	>1	conal	<ul style="list-style-type: none"> <li><i>No pulsars belonging to this species have been identified although there is no a priori why they should not if the excitation of the conal emission beam is "patchy"—and</i></li> </ul>	
Others?	?	1?	?	?	?

with a mean log age of 6.1, whereas the  $S_d$ , D, cT, and M pulsars are all old with mean log age values of about 7.5. The T pulsars again fall in between with a value of 6.6. These differences in age are reiterated in figure 3 which gives the galactic z-height distribution for  $S_t$  pulsars in contrast with the combined group of  $S_d$  and D stars. Note that the  $S_t$  pulsars have a scale height ( $|z|$ ) of only about 160 parsecs, whereas the other group has a value almost twice as great.

### Interpulsars

Twelve pulsars are now known which exhibit core emission in their main-pulse and/or interpulse profiles. These pulsars are very important because only for these few stars do we have any direct information about the angle  $\alpha$  between their rotation and magnetic axes. Study of these interpulsars having core emission has resulted in the conclusion that most, but not all, have magnetic axes which are nearly orthogonal to their rotation axes (Rankin 1990).

Six of the pulsars have core components whose width (i.e., full width at half maximum) can be

Pulsar Classification System (cont'd)

Geometry	Beam size (°)	Acceler. Potential $B_{12}/P^2$	Profile Mode Changes	Sub-pulse Modulation	Null-ing	Age $\langle \log_{10} t \rangle$ (10 <sup>6</sup> years) $\langle  z  \rangle$
Uncertain: the linear angle traverse apparently provides little reliable information about where the sight line crosses the emission beam	$\frac{2.45^\circ}{P^{1/2} \sin \alpha}$ (FWHM)	large, $\geq 2.5$ ; typically 10	not observed	little; steady, flat fluctuation spectrum	none	young 6.1 160 pc
The sight line crosses the polar-cap region somewhat peripherally—i.e., away from the center of the conal emission beam	• core beam diam. 2.45° • conal beam radii 4.3° and 5.9°, discrete values for M	full range; large to small	yes	stationary subpulse modulation, associated with the conal components	yes	middle age to old 6.6 210 pc
The sight line crosses the polar-cap region near the magnetic axis—i.e., near the center of the conal emission beam	• both scale as $P^{-1/2}$ and $\sin^{-1} \alpha$	small, generally less than 2.5	(always?)			
The sight line crosses the polar-cap region near the periphery of the conal emission beam $\updownarrow$ closely connected $\updownarrow$	not well known	small, $\leq 2.5$ ; lowest known value 0.16	maybe; difficult	yes, orderly <b>drifting</b> subpulses		old 7.6 335 pc
The sight line crosses the polar-cap region near the magnetic axis—i.e., near the center of the conal emission beam			to discern	little or no drifting; stationary subpulse modulation		old 7.7 260 pc
missing or whether it is simply merged or overlain with the core component.						as with triple
thus, the the inner conal beam appears as a central conal component.		$\leq 2.5$	yes	<b>drifting</b> subpulses	yes	old 7.7 $\leq 400$ pc
there are many indications that the illumination is at least non-uniform.						presumably very much like double (D) pulsars
?	?	?	?	?	?	?

measured with reasonable accuracy and interpolated to 1-GHz. When fitted against period, these values exhibit an accurate power-law relation. A least-squares fit to these values yields the result that

$$W = 2.45^\circ P^{-0.50}$$

The core components of most other interpulsars also have widths comparable with those given by the above relation, but cannot be so accurately determined.

This core width-period relationship has a very simple interpretation in terms of the magnetic field structure of the pulsar. Assuming a dipolar field, the angle  $\rho$  between the field-line tangent and the

magnetic axis can easily be evaluated. Taking the 1-GHz width of the core component as twice this value

$$W_{\text{core}} = 2\rho \approx 2.49^\circ (r/R)^{1/2} P^{-1/2}$$

where  $r$  is the emission height, measured from the star's center, and  $R$  ( $\equiv 10$  km) the stellar radius.

Comparing the empirical relationship for  $W$  with the geometrically-derived expression above, both equations have a  $P^{-1/2}$  term, and thus it appears that the period dependence of the core width is geometrical in origin. However, the two expressions can only be reconciled numerically if the ratio of the emission height  $r$  to the stellar radius  $R$  is

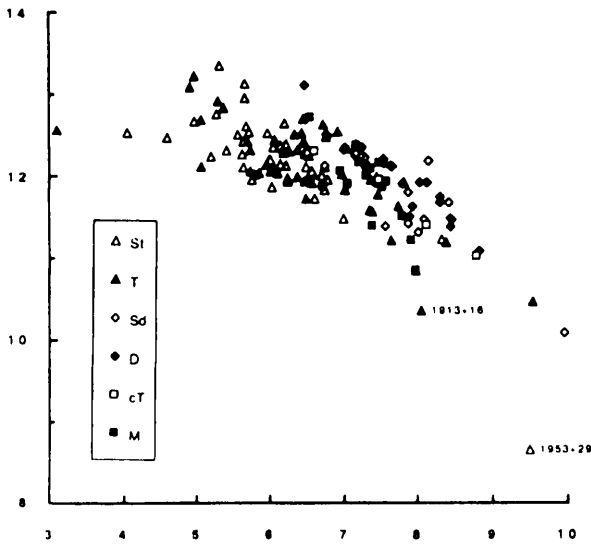


Figure 1 Log of the inferred magnetic field (Gauss) vs. spindown age for 150 pulsars.

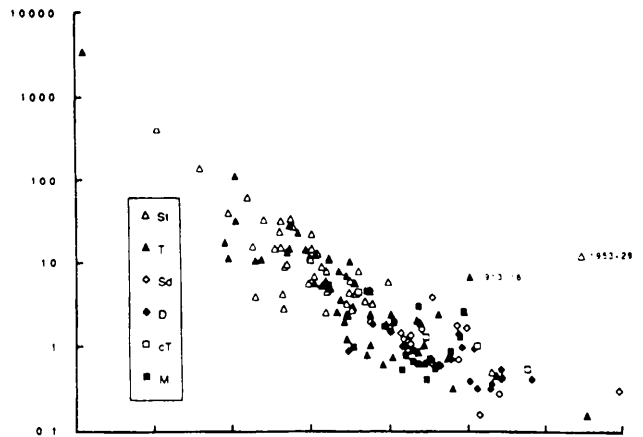


Figure 2 "Acceleration parameter"  $B_{12}/P^2$  vs. spindown age for 150 pulsars.

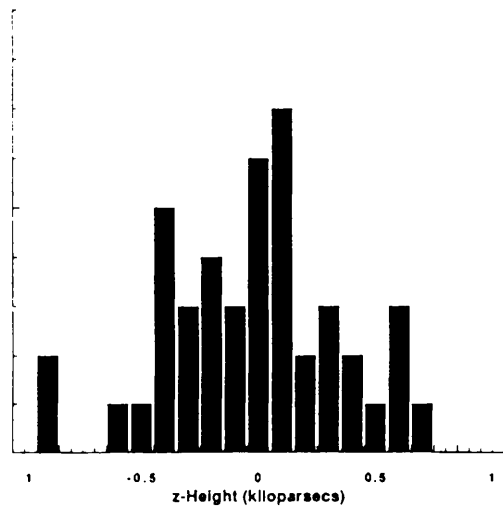
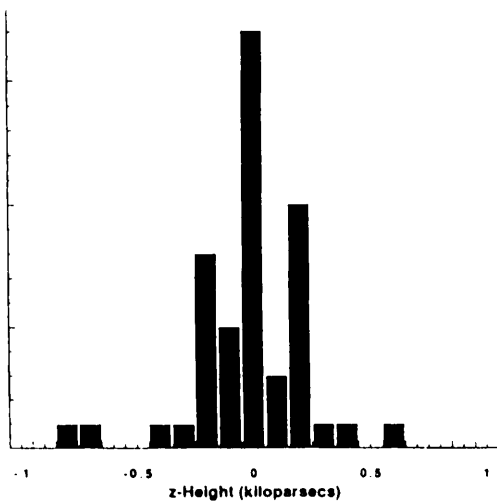


Figure 3 Histogram of the z-distribution of a) pulsars with core-single ( $S_c$ ) profiles, and b) pulsars with conal-single ( $S_d$ ) and double (D) profiles.

about unity. This in turn indicates that the core emission comes from very near the stellar surface.

Most pulsars do not have interpulses, and we must consider how an emission beam of angular radius  $\rho$  about the magnetic axis projects onto the sight-line direction. Simple geometrical arguments (see Rankin 1990) suggest that the above relation can be generalized to any pulsar with a core component as

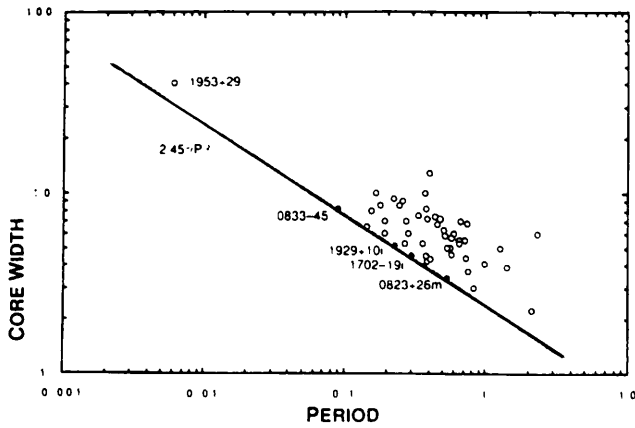
$$W_{\text{core}} = 2.45^\circ P^{-1/2} \sin \alpha. \quad (1)$$

Apparently, this simple relationship describes the angular width of core emission beams at 1 GHz. The relationship depends only on the pulsar period, which determines the height of the velocity-of-light cylinder and thus the polar-cap radius, and the an-

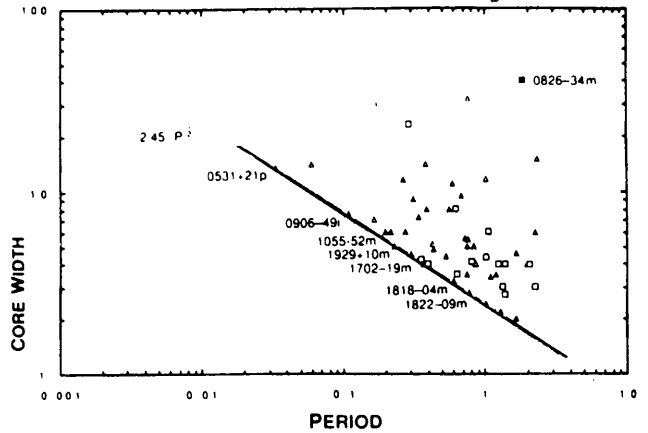
gle  $\alpha$ , which enters in considering how much of a rotation cycle the core beam occupies.

### Geometry of core emission

The 1-GHz, half-power width of pulsars with core-single ( $S_c$ ) profiles are plotted as a function of period in figure 4. The points corresponding to the interpulsars (filled symbols) as well as several other stars are labelled, and a line showing the fitted widths of the interpulsars indicated. Note the minimum width defined by the four interpulsars; several other pulsars—most notably the 6-ms pulsar 1953+29—have core widths just in excess of the interpulsar minimum, but none have smaller widths.



**Figure 4** The half-power profile width of core-single ( $S_t$ ) pulsars as a function of period. The symbols of a few prominent pulsars are labelled, and those with interpulses indicated by a filled symbol. (The suffixes “i”, “m”, and “p” refer to the interpulse, main pulse and postcursor, respectively). The indicated curve is  $2.45^\circ P^{-1/2}$  (see text).



**Figure 5** The half-power core-component widths of triple (T) pulsars (triangle symbols) and five-component (M) pulsars (square symbols) as a function of period as in figure 4.

The widths of the central core components of pulsars with triple (T) and five-component (M) profiles are plotted in figure 5. Pulsars with triple profiles are indicated by triangular symbols and the M stars with squares. Filled symbols are used to indicate the interpulsars which, along with several other prominent stars, are explicitly identified. A curve indicating the fitted widths of the interpulsars is again superposed. The core width values of the seven triple interpulsars (filled triangles) all fall accurately on this curve within their measurement or estimation errors. (By contrast, the core width value for PSR 0826–34m, which has an aligned geometry, falls furthest from the curve.)

We can interpret the core-width/period relationship in figures 5 and 6 as deriving primarily from the polar-cap geometry, and as such it should be well described by eq.(1) above. Only two factors apparently determine the 1-GHz width of a core component: a) the angular radius of the polar-cap emission region at the stellar surface, which goes as  $2.45^\circ P^{-1/2}$ , and b) the angle between the magnetic axis and the rotation axis  $\alpha$ .

### Estimation of $\alpha$ values

Assuming that the angular width of all core emission is determined by the angular extent of the open field lines at the stellar surface, then eq.(1) can be inverted to estimate the magnetic orientation angle  $\alpha$ . Histograms of the  $\alpha$  values are then given in figure 6. a) The angles for the  $S_t$  stars range between some  $15^\circ$  and  $90^\circ$  and exhibit a median value of about  $35^\circ$ . b) The  $\alpha$  values for the T (filled bars)

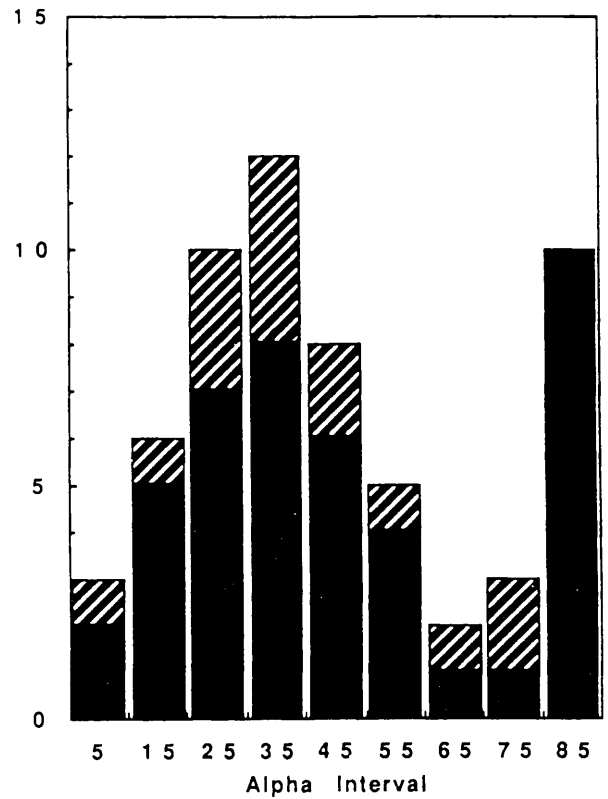
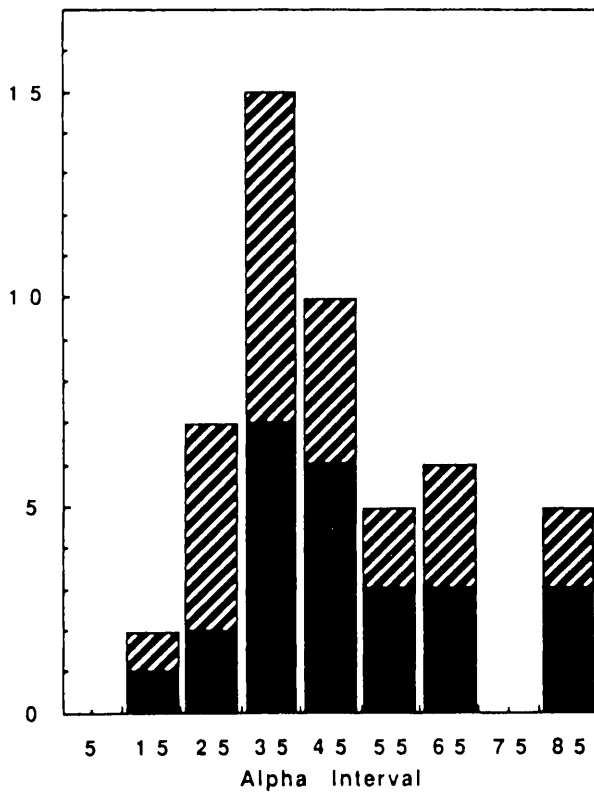
and M (striped bars) stars range more widely than for the  $S_t$  stars. The median value is still about  $35^\circ$ , but the distribution is flatter and a larger fraction are very nearly aligned.

The population of  $S_t$  pulsars has been divided into two subgroups according to their spindown age. The younger group (solid bars) has a mean log age of 5.6, whereas the older one (striped bars) has a mean log spindown age of 6.7. The older group shows a weak tendency toward alignment, which is probably not significant at this level of analysis. The  $\alpha$  distributions for the T and M pulsars are very similar despite differences in mean log age (6.6 and 7.3, respectively). These results are summarized in figure 7; clearly, we find no support here for the proposition that the magnetic axes of pulsars align with age.

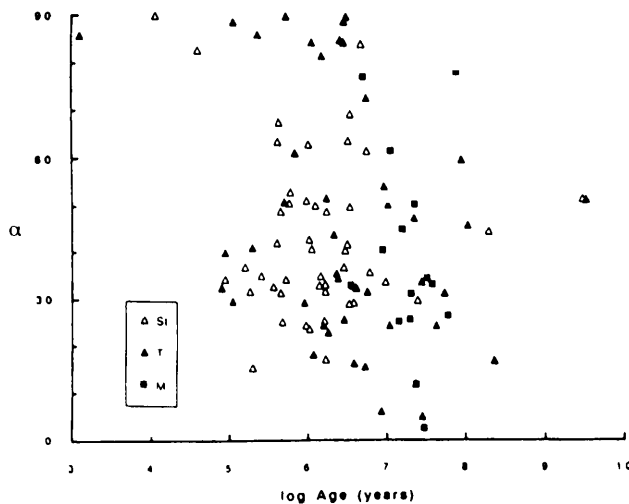
### Circular polarization and the figure of the polar-cap region

A phenomenological study of the circular polarization associated with pulsar emission has shown that most is observed in core components—that is, in core-single ( $S_t$ ) profiles and in the central components of triple (T) and five-component (M) profiles (Radhakrishnan and Rankin 1990). Two types of circular signature are identified in the observations: a) an antisymmetric type wherein the circular polarization changes sense in mid-pulse, and b) a symmetric type wherein it is predominantly of one sense.

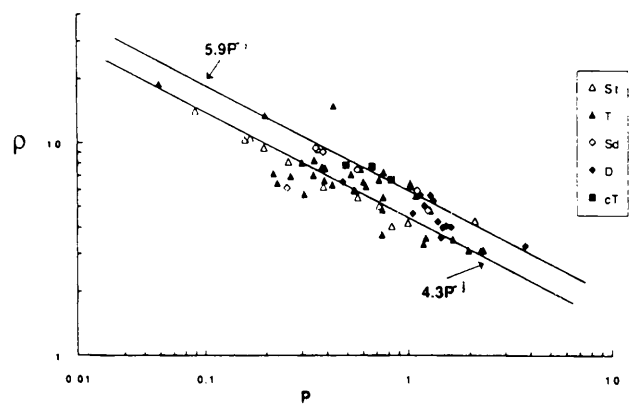
In pulsars with triple (T) and five-component (M) profiles, the antisymmetric type is usually cor-



**Figure 6** Histograms of inferred  $\alpha$  values for a) core-single ( $S_t$ ) pulsars, and b) triple ( $T$ ) and multiple ( $M$ ) pulsars. The  $S_t$  population is divided into two equal groups, a younger group (solid bars) whose mean log spindown age is 5.6, and an older group (striped bars) whose mean log age is 6.7. Similarly, the  $T$  (solid bars) and  $M$  (striped bars) pulsars have mean log ages of 6.6 and 7.3, respectively.



**Figure 7** Magnetic inclination angle  $\alpha$  values plotted against spindown age for populations of  $S_t$ ,  $T$ , and  $M$  pulsars



**Figure 8** Conal emission radius  $\rho$  as a function of period for populations of  $S_t$ , and  $T$  pulsars. These values are computed from the full conal analysis as described in the text.  $\rho$  values are also shown for a small group of  $S_d$ ,  $D$ , and  $cT$  stars; here the  $\alpha$  values and other observational data are taken from the analysis of Lyne and Manchester (1988). Curves corresponding to the inner and outer conal zones of the  $M$  stars,  $4.3^\circ P^{-1/2}$  and  $5.9^\circ P^{-1/2}$ , are indicated.

related with the sense of rotation of the linear position angle. Transitions from positive (LH) to negative (RH) are found to accompany negative (clockwise) rotations of the position angle, and *vice versa*.

In the general framework of models in which the radio power is curvature radiation emitted by charge bunches constrained to follow field lines, the linear polarization direction is intrinsic to the emission mechanism, and is, furthermore, a purely geo-

metric property independent of the polarity of the magnetic field or of the sign of the charges. The correlation we find then suggests that the antisymmetric circular polarization is also a purely geometric property of the emission process. Curvature radiation will have significant net circular polarization if there are gradients in the emissivity over angular scales comparable with the emission cone of a single charge (*i.e.*  $\gamma \simeq 1$ , where  $\gamma$  is the Lorentz

factor of a charge bunch). The observation of significant circular polarization therefore implies that  $\gamma < 20$  for the core emission. Furthermore, no net circular polarization is produced if the emissivity is circularly symmetric about the magnetic dipole axis. The sign of the correlation we have observed is consistent with an emission region more extended in longitude than in latitude.

## Geometry of conal emission

In an accompanying paper in this volume (Rankin 1992), equations were derived for the radius of the conal emission beam  $\rho$  in terms of  $\alpha$ ,  $\beta$  (which can be calculated from  $\alpha$  and the sweep rate of the linear polarization angle), and the outside, half-power width of the conal component pair  $\Delta\Psi$ ; and these relations were then applied to those pulsars with five-component (M) profiles.

The result of this analysis was that five-component (M) pulsars have 1-GHz inner and outer conal beam radii as follows

$$\rho_{\text{inner}} = 4.3^\circ P^{-1/2} \quad \rho_{\text{outer}} = 5.9^\circ P^{-1/2},$$

corresponding to emission heights of the inner and outer conal emission zones

$$h_{\text{inner}} \simeq 120 \text{ km, or } \simeq 12 \text{ stellar radii}$$

$$h_{\text{outer}} \simeq 210 \text{ km, or } \simeq 21 \text{ stellar radii,}$$

where we have assumed, for the sake of calculation, that the inner and outer conal zones are emitted along the same peripheral field lines at different heights. Clearly, other relative configurations of the two emission zones are also possible.

In any case, let us here compare these results for M stars with other pulsars with conal emission, looking first at those pulsars with core-single ( $S_t$ ) and triple (T) profiles. Exactly the same procedure can be followed: first compute the orientation of the magnetic axis  $\alpha$  by evaluating the width of the core component and then calculate  $\rho$ , the radius of the conal emission beam using the relations in the accompanying paper (Rankin 1992).

The results for  $S_t$  (open triangles) and T (solid triangles) pulsars are given in figure 8. Notice the overall  $P^{-1/2}$  trend in the curve, with significant departures. Note, however, that the  $S_t$  stars generally lie close to the lower curve; it is the T stars which have the least orderly behavior.

Finally, a few points representing  $S_d$ , D, and cT are also plotted on the diagram. None of these pulsars have core components, and so the technique described above is not possible. Instead,  $\alpha$  values (as well as the other observational data) have been taken from the analysis of Lyne and Manchester

(1988) as it is just for such cone-dominated profiles that their analysis apparently best motivated. Note that the conal single and double pulsars have  $\rho$  values which generally lie between the two solid curves, and that the conal triple values fall essentially on the outer curve as expected.

## Conclusions

We reemphasize that pulsars exhibit two modes or mechanisms of emission. Core emission is found in a majority (60–70%) of mostly younger pulsars, whereas conal emission predominates in the profiles only of a minority of older pulsars.

The width of core components have a regular dependence on rotation period and on the magnetic inclination angle  $\alpha$ . The magnitude of these widths suggests that the core radiation is emitted close to the neutron-star surface throughout the entire polar-cap region. Furthermore, its circular polarization properties seem compatible only with a radiating population of low  $\gamma$  particles in an asymmetric emission region—that is, one which is longitudinally extended.

The conal emission, by contrast, seems to be emitted at heights of  $\sim 100$ – $200$  km. This value stems from consideration of 1-GHz profiles; the lower frequency emission will then come from larger heights and *vice versa*. We find no reason to doubt the longstanding association of conal emission with relatively high  $\gamma$  particles, and this is further supported by its linear polarization, which closely follows the single-vector (Radhakrishnan and Cooke 1969) model.

The classification of pulsar profiles appears to give useful information about the geometry of the emission region. The orderly sequence of profile types as well as their generally orderly evolution with radio frequency suggests that the pulsar magnetic field is usually highly dipolar. Similarly, there appears to be little observational evidence for “patchy” or incomplete cones of emission as suggested by Lyne and Manchester (1988).

Finally, the five-component M pulsars exhibit the highest degree of profile complexity and appear to express the full morphological potentialities of the pulsar emission process. The inner conal zone has geometrical properties very similar to the type of conal emission exhibited by core-single ( $S_t$ ) pulsars at high frequency, and the the outer conal zone seems to have much in common with the conal radiation pattern seen in conal triple (cT), conal single ( $S_d$ ) and double (D) pulsars. Triple (T) pulsars are apparently less regular in their behavior and warrant further detailed study.