Proceedings of the Edinburgh Mathematical Society (1985) **28**, 419–425 ©

BOOK REVIEWS

STRĂTILĂ, S. *Modular theory in operator algebras* (Editura Academiei, Bucharest, and Abacus Press, Tunbridge Wells, 1981), 492 pp. £33.

Takesaki's monograph *Tomita's theory of modular Hilbert algebras and its applications* (Lecture Notes in Mathematics 128) appeared in 1970, and opened a new chapter in the study of von Neumann algebras; by the mid 1970s a vast new body of results had appeared. Their principal author was Alain Connes, but important contributions were made also by Arveson, Haagerup, Takesaki, and others.

Before the advent of the Tomita–Takesaki modular theory, von Neumann algebras, and in particular factors, of type III were rather poorly understood. In his thesis Connes defined several invariants for type III factors and, using ideas of Arveson, developed various powerful tools for analysing their structure. One of the most striking results of the theory is that a type III von Neumann algebra can be expressed as a crossed product of a type II algebra by an action of $\mathbb{R}$ or $\mathbb{Z}$. The effect of this is to reduce the study of type III von Neumann algebras to that of type II algebras and their automorphisms.

The book under review, in some ways a sequel to Strătilă and Zsidó's *Lectures on von Neumann algebras* (Abacus Press, 1979), presents a unified account of this theory together with much useful background material. It is by far the most comprehensive exposition of these topics to have appeared so far as a book. As such it will prove extremely valuable both to specialists and those attempting to learn the subject.

There are six substantial chapters, devoted successively to normal weights, conditional expectations and operator-valued weights, automorphism groups, crossed products, continuous decompositions, and discrete decompositions; there is also a short appendix on unbounded operators. The presentation is clear and to the point, except perhaps in the chapter on crossed products, where the approach, using Kac algebras, is rather more general than the context requires. A more straightforward approach (cf. A. van Daele, *Continuous crossed products and type III von Neumann algebras*, LMS Lecture Notes 31) would be less daunting for the novice. I find also that the layout is in many places unduly cramped, and this makes some of the more intricate proofs rather tiresome to read through.

These minor reservations notwithstanding, this volume is a valuable addition to the literature on operator algebras. Presenting as it does some of the most acclaimed work in functional analysis of the past fifteen years, it is an essential acquisition for any university library, and I am sure that nobody working in operator algebras would wish to be without it.

SIMON WASSERMANN

ITÔ, K. *Introduction to probability theory* (Cambridge University Press, 1984), 211 pp. £18.50 cloth, £6.95 paper.

This book is a translation by the author of the first four chapters of a more extended Japanese volume entitled "Probability Theory". The book is intended to "explain basic probabilistic concepts rigorously as well as intuitively" and I presume it is intended for students attending their first serious course on analytic probability theory. The only essential prerequisite for the material is a basic knowledge of point set topology.

The distinctive features of this text include (i) a determined effort to develop the formal machinery in a way that corresponds to our intuitive thinking; (ii) the early introduction of the topological aspects of measure theory; and (iii) some traditionally difficult and frequently omitted theorems presented in a clean accessible manner. In this latter aspect the work is excellent. One could certainly recommend this book to a student and particularly Chapters 2 and 4 if he wanted to see nice proofs of any of the following: the theorems of Glivenko, Lévy and Bochner relating characteristic functions and convergence in distribution, the characterisation of Borel measures on Polish spaces by their values on compact sets, and generally for theorems concerning the pointwise and fluctuation behaviour of sums of independent (but not identically distributed) random variables.

However, one is left with the feeling that the features (i) and (ii) mentioned above make this book unsuitable as a general textbook—at least for British undergraduates. The approach is very interesting but over-formal for a first introduction. There is little looking ahead or verbal explanation—and little appeal to the intuition—rather it is an attempt to do intuitive mathematics formally. Undoubtedly this would be good for those who already have the intuition but would cause difficulties for a genuine beginner.

Chapter 1 is concerned with finite trials (finite sample spaces) and develops the idea of probability, random variable, law or distribution and real-valued random variable. It also introduces the idea of direct composition of two finite trials and tree composition (the former is just the product sample space and measure). Because these are introduced before the somewhat simpler ideas of conditional probability and independence (for finite event spaces) one can't help suspecting that students would become confused, particularly on pages 18–21 where one "deduces" appropriate trials to model multiple coin tossing games and multiple selection without replacement before one has even introduced the notion of independence as it applies to the various coin tosses or selections.

The second chapter is a nice description of basic measure theory. It starts with Carathéodory's construction of Lebesgue integral, and proceeds to consider standard measure spaces and measures on Polish spaces (but in common with most others ignores Doob's truly simple proof of Lusin's theorem). The chapter finishes with a discussion of probability measures on $\mathbb{R}$, proving that they are a Polish space with respect to convergence in distribution and giving the usual characterisations of this convergence in terms of characteristic functions. Again, to bolster the motivation of the novice reader some mention of sums of independent random variables would have been a good idea when convolutions of probability measures were treated.

The third chapter is on random variables (rather than measurable functions). Here a certain problem arises. In this book a probability measure μ uniquely determines a σ-algebra $\mathscr{D}(\mu)$ on which it is typically complete. If X is a map which is measurable from $\mathscr{D}(\mu)$ to say the Borel sets on $\mathbb{R}$ then the law μ^X is defined not as a Borel measure but on every set for which $X^{-1}(A)$ is μ-measurable. Thus μ^X might not be completely determined by its values on Borel sets. To overcome this the author follows Kolmogorov by considering random variables as being defined on a restricted class of measurable spaces (perfect, separable). This is fine; however, when combined with the enthusiasm for completions it makes the exposition slightly complicated. Conditional expectation is also treated in a non-standard way.

The final chapter discusses sums of random variables giving Lindeberg's central limit theorem and a law of the iterated logarithm. In this "harder" analysis the book is at its best and that is very good.

This book can easily be recommended to anyone preparing a course on (analytic) probability and to the able student looking for additional insights. It also has some good do-able problems.

It is a great pity for us that Professor Itô has so far only translated the first four chapters of his book; it would have been very interesting to see what the master had to say about stochastic processes.

T. J. LYONS

ApSimon, H. *Mathematical byways in Ayling, Beeling and Ceiling* (Recreations in Mathematics No. 1, Oxford University Press, 1984), 97 pp. £5.95.

This little collection of eleven recreational problems has been accumulated by the author over a period of about thirty years. They involve only elementary algebra, geometry and trigonometry and could be enjoyed by anyone with a mathematical bent over the age of sixteen.