

three chapters. It ranges from Greek philosophy to mathematical developments in the 20th century, allowing the author to display his scholarly erudition and wide cultural interests.

In 1959 Bourbaki stated in a private conversation that the chapter on structures really ought to have been a chapter on categories. This is even more true today. The chapter starts with a horrendous definition of certain types that will deter most readers from going further. It concerns a hierarchy of expressions built up from given sets with the help of the cartesian product and the power set operation. The latter is only presented as an object function and its variance is not discussed in general. Many species of structures are dealt with, not only primitive classes of algebras, but also classes of relational systems, the class of all topological spaces, etc. Unfortunately there is no general definition of what constitutes a map in a given species. (These deficiencies can be remedied, as has been shown by Z. Hedrlin, A. Pultr and V. Trnková, see "General topology and its relations to modern analysis and algebra II", Proceedings of the Second Prague Topological Symposium, 1966, pages 176-181). The highpoint of the chapter is the so-called universal mapping problem, which appears to be more general than the problem of finding the adjoint to a functor. It has a solution under certain conditions that resemble Freyd's solution set conditions. There is an interesting collection of important examples from many parts of mathematics. It would seem to be a worthwhile project to rewrite these 50 pages in the language of categories and functors.

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Éléments de mathématique, fascicule 17; Théorie des ensembles
Chapitres 1 et 2. by Nicolas Bourbaki. Hermann, Paris, 1966.
Troisième édition, 137 pages. 36 francs.

This is a revised and corrected edition of Bourbaki's treatment of the foundations of mathematics.

Chapter 1 deals with a description of formal mathematics. The reader learns to his surprise that the usual linear formulas dear to logicians are only abbreviations of certain non-linear assemblies made up from signs, blanks, and links. However, after the first few pages, he is allowed to forget this.

The propositional calculus uses "or" and "not" as fundamental notions. There are four simple axiom schemes, the rule of modus ponens, and two rules of substitution. One of the latter allows passage from one theory to another. One quickly arrives at the natural rules of inference employed in ordinary mathematical discourse: the deduction rule, *reductio ad absurdum*, argument by cases, the method of the auxiliary constant.

Quantifiers are defined in terms of Hilbert's ϵ - symbol, here called " τ ". Thus " $\tau_x A(x)$ " represents a privileged object b for which $A(b)$, if there are such objects, and " $\exists_x A(x)$ " is short for " $A(\tau_x A(x))$ ". If there is no object b for which $A(b)$, one might be tempted to say that " $\tau_x A(x)$ " represents no object; however, in Bourbaki's ontology it represents an object "about which one can say nothing" (see page 16). Unfortunately, having raised this reader's curiosity about this mysterious object, Bourbaki sticks to his guns and says nothing about it. There is one axiom for introducing the existential quantifier.

The theory of equality deals with a symbol " $=$ " and requires two axiom schemes, the usual substitution rule and a rule which says that if $\forall_x (A(x) \iff B(x))$ then $\tau_x A(x) = \tau_x B(x)$.

Chapter 2 is devoted to the theory of sets, which deals with the symbol " ϵ ". The axioms postulate extensionality, the existence of the set x, y , the existence of the set of all subsets of x , and the existence of some infinite set. In addition, there is a curious scheme of "selection and reunion". It asserts that if $\forall_y \exists_w \forall_x (R(x, y) \implies x \in w)$ then the set $\{x \mid \exists_y (y \in z \text{ and } R(x, y))\}$ exists for all z . The axiom of choice need not be postulated but can be deduced from properties of the symbol " τ ". Pairs, products, relations, functions, etc. are defined in the usual manner.

The book contains many exercises and a foldout displaying the axioms and axiom schemes.

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An introduction to the theory of numbers, by Ivan Niven and Herbert S. Zuckerman. John Wiley and Sons, New York, 1966. 2nd edition. 280 pages. \$7.95.

After looking at this lucid exposition, I was overcome by a feeling of nostalgia. How much nicer it would be to teach this than the series of unmotivated definitions that goes under the name of algebra!

We have here a leisurely introduction to divisibility and congruences, which any freshman can read, followed already on page 69 by Quadratic Reciprocity. Then there is a chapter on special number theoretic functions (which graduate student still knows the Moebius inversion formula?), followed by chapters on Farey fractions, simple continued fractions, $\pi(x)$ (including Bertrand's postulate), algebraic numbers, and the partition function. The last chapter deals with the density of sequences of integers and culminates in the Schnirelmann and $\alpha\beta$ theorems. There are many carefully arranged exercises