ALPHA BETA DISCS

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1) Introduction

Following earlier work of Lynden-Bell & Pringle (1974) and Lightman (1974a, 1974b), Bath & Pringle (1981) have presented a simple method for studying the time-dependent evolution of viscous accretion discs. These models are axisymmetric, with the vertical structure reduced to integrated averages of local physical conditions. Published work examines models of dwarf nova eruptions driven by mass transfer bursts (Bath & Pringle 1981 - Paper I), eruptions produced by global viscous changes within the disc (Bath & Pringle 1982a - Paper II), and the time-dependent properties of giant discs in symbiotic binaries (Bath & Pringle 1982b - Paper III).

In the next section we use these models to examine the theoretical basis of the observed correlation between the eruption decay-time and binary-period of dwarf novae (Bailey 1975, Mattei and Klavetter 1982). The main section of the paper describes an improved method for treating the interaction between the mass-transfer stream and disc matter, and presents results for bursting mass-transfer dwarf-nova models.

2) Theoretical eruption decay-rates

Bailey (1975) first pointed to the existence of a correlation between the outburst decay-time and the binary period for dwarf novae. More recently Mattei and Klavetter (1982) have conducted an extensive analysis of AAVSO data and confirm that the decay-rate, t_d, defined as the slope of the most linear portion of the declining light curve, in day per magnitude, depends on the binary period, with the best fit straight line being $t_d = 11.1P + 0.11$ (see Fig.1).

We have examined the predicted decay-rate/binary-period relation with time-dependent α -disc evolution models and

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conclude that outbursts caused by sudden bursts in the flow rate of the mass-transfer stream obey the observed decay-rate/binary-period relation subject to the following conditions being satisfied: Firstly, the mass-transfer burst must decay on a faster timescale than the viscous evolution timescale of the disc. Secondly, to fit the observed decay times, the viscosity in the disc must have a value corresponding to $\alpha \simeq 1.5 \pm^{1.5}_{0.5}$. The observed relation is then simply a consequence of the increase in the time needed for matter to diffuse through larger discs in longer binary-period systems.

We construct models according to Paper I, with minor and unimportant changes to the opacity. The dominant parameters which determine the eruption decay-time in any system are, in order of decreasing importance, the size of viscosity (i.e. α), the maximum outer disc radius, R_{OUT} (assumed to be equal to the tidal radius, $R_{OUT} = 0.88 R_{RL}$, where R_{RL} is the average Roche lobe radius (Papaloizou and Pringle 1977, Paczynski 1977)), and the mass of the accreting white dwarf, which determines the inner disc radius, R_{IN}, and the gravitational potential. Note $R_{OUT} = R_N$ and $R_{IN} = R_1$ in the notation of paper I.

To determine the variation of R_{OUT} with period we specify the white dwarf mass, M_1. The condition that Kepler's law is satisfied, the secondary obeys a main-sequence mass-radius relation, and fills its Roche lobe then determines the size of the primaries Roche lobe, R_{RL}, and hence R_{OUT}. The circularization radius, R_K, of the mass transfer stream is obtained from the binary separation by interpolation from tables given in Lubow and Shu (1975). The values of R_{OUT} and R_K are given in Table 1 for periods in the range $0.05 < P(\text{days}) < 0.5$ and $1 M_\odot$ white dwarf primary.

Table 1

q	M_2 ($M_\odot$)	Period (days)	R_{OUT} (cm)	R_K (cm)
0.59	1.68	0.50	7.52×10^{10}	1.85×10^{10}
0.67	1.49	0.45	7.05×10^{10}	1.75×10^{10}
0.77	1.30	0.40	6.56×10^{10}	1.65×10^{10}
0.89	1.12	0.35	6.05×10^{10}	1.55×10^{10}
1.06	0.94	0.30	5.53×10^{10}	1.45×10^{10}
1.29	0.77	0.25	4.97×10^{10}	1.34×10^{10}
1.65	0.61	0.20	4.36×10^{10}	1.23×10^{10}
2.23	0.45	0.15	3.71×10^{10}	1.12×10^{10}
3.38	0.30	0.10	2.96×10^{10}	9.90×10^{9}
6.85	0.15	0.05	2.00×10^{10}	8.13×10^{9}

Theoretical visual light-curves of dwarf nova eruptions have been computed for the case of a mass transfer burst, with the same burst profile independent of binary period, and the decay time of the most linear portion of the light curve determined. We find that the light curve decay-time, t_d, is not sensitive to the form of the mass-transfer changes so long as the burst shuts off on a timescale shorter than the viscous disc diffusion-time. Variations of total burst mass, the maximum transfer rate in the burst, and the quiescent mass transfer rate have insignificant effect on t_d.

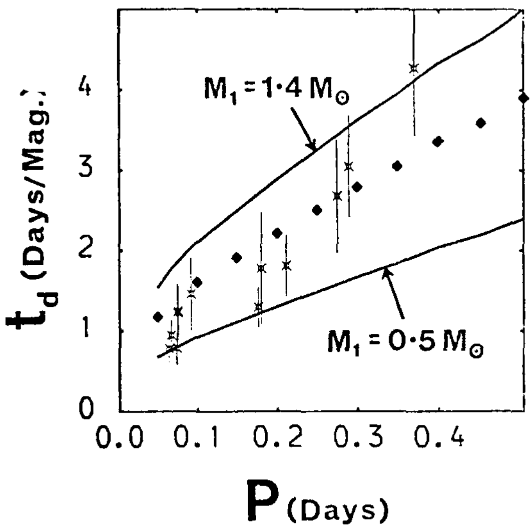

Fig 1. Decay time, t_d, versus binary period for observed systems (stars with error bars), and computed models with $\alpha = 1.5$ and $M_1 = 1M_\odot$ (solid diamonds). The model relation for $M_1 = 1.4\ M_\odot$ and $M_1 = 0.5\ M_\odot$ is also shown.

In Fig.1 the theoretical values of t_d are plotted (filled diamonds) together with the observed values of individual systems from Mattei and Klavetter (1982) (stars with error bars). For the assumed value of $1M_\odot$ for M_1, the best fit is obtained with a value of $\alpha = 1.5$. Also shown are the limits on t_d for $1.4\ M_\odot > M_1 > 0.5\ M_\odot$. The observed scatter can clearly be accounted for by variations in white dwarf mass from system to system. Equivalently, variations in α in the range $3.0 > \alpha > 0.5$ would account for the observed scatter, with a $1\ M_\odot$ white dwarf. It is evident from this study that the allowed variation of α is small, and that the decay-time/binary-period relation is a natural consequence of increasing disc size in longer period binaries.

Note furthermore that the decay-time is independent of the duration of the mass-transfer burst. Supermaximum outbursts can be obtained with long duration, larger mass flux bursts. These decay on the same timescale as normal bursts, and satisfy Bailey's relation, as is observed (Mattei and Klavetter 1982).

3) Mass input and stream/disc mixing

The method described in Bath and Pringle (1981) for computing disc evolution assumes instantaneous mixing of the mass-transfer stream with disc matter. Stream material is fed into the disc with the appropriate specific angular momentum, the change in disc radius computed, and the input mass mixed in an appropriate ratio in the two outer zones at the disc edge, R_{EDGE}, where angular momentum balance is achieved. Whether the disc edge remains at its maximum radius, R_{OUT}, or shrinks within this radius is determined by the balance between angular momentum transport within the disc (which tends to expand the disc towards R_{OUT} or keep it in contact with R_{OUT}), and the angular momentum of the mass transfer stream (which tends to shrink the disc towards R_K).

At R_{OUT} it is assumed that all excess angular momentum can be efficiently removed by tidal interaction with the companion. That is, the effective couple in the disc is zero at R_{OUT}, and $\partial(\Sigma\nu R^{\frac{1}{2}})/\partial R = 0$ at that point (note that the outer boundary condition is stated incorrectly in papers I and II. The computations in both papers use the correct boundary condition, ensuring mass conservation).

The instantaneous mixing hypothesis is employed largely for computational convenience. It conserves mass and angular momentum appropriately, but the extent of the region in which mixing takes place is arbitrarily determined by the zone size.

In practice the mass transfer stream will penetrate the disc on a "quasi-elliptical" orbit. How deeply the stream penetrates before all the stream material has been sheared off by shocks driven by disc matter in circular orbits (as discussed for the case of low viscosity discs by Lubow & Shu 1975) is unknown. However there is evidence (e.g. Stover 1981) that extensive penetration of the stream occurs in some systems. We have therefore developed an alternative approach to the mass input problem which is fully conservative, and includes mass and angular momentum sources implicitly in the conservation equations for disc structure.

If the mass flux fed from the stream into the disc at radius R and time t is $\partial(\dot{m}(R,t))/\partial R)$ then, in the thin disc approximation, conservation of mass gives,

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$$\frac{\partial \Sigma}{\partial t} = \frac{-1}{R} \frac{\partial (R\Sigma v_R)}{\partial R} + \frac{1}{2\pi R} \frac{\partial \dot{m}}{\partial R} \qquad (1)$$

where Σ is the surface density and V_R the radial velocity. If the stream has specific angular momentum which would place it in an orbit at radius R_K, then conservation of angular momentum implies,

$$\frac{\partial (\Sigma R^2 \Omega)}{\partial t} = \frac{-1}{R} \frac{\partial (R^3 v_R \Sigma\Omega)}{\partial R} + \frac{1}{R} \frac{\partial}{\partial R}\left(R^3 \nu\Sigma \frac{\partial \Omega}{\partial R}\right) + \frac{R_K^2 \Omega_K}{2\pi R} \frac{\partial \dot{m}}{\partial R} \qquad (2)$$

where ν is the effective kinematic viscosity. Combining (1) and (2), eliminating V_R, and assuming Keplerian angular velocities, leads to an evolution equation for the surface density as a function of time of the form,

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R} \frac{\partial}{\partial R}\left(R^{\frac{1}{2}} \frac{\partial (\nu \Sigma R^{\frac{1}{2}})}{\partial R}\right) + \frac{1}{2\pi R} \frac{\partial \dot{m}}{\partial R} + \frac{1}{\pi R} \frac{\partial}{\partial R}\left(R\left(1 - \frac{R_K^{\frac{1}{2}}}{R^{\frac{1}{2}}}\right) \frac{\partial \dot{m}}{\partial R}\right) \qquad (3)$$

This is the revised evolution equation including stream material as a source implicitly. The first term is the basic diffusion term describing matter and angular momentum redistribution due to viscous stress, and is the fundamental expression controlling disc evolution (see e.g. Pringle 1981). The second term is the stream source term from equation (1). The third term describes the influence on the evolution of Σ of the angular momentum of the mass transfer stream. The first, diffusion term drives material out (and in) into a disc by viscous stress, whilst the third term describes the tendency of new input material with angular momentum appropriate to an orbit at R_K to squeeze the disc towards an annulus at radius R_K.

To solve equation (3) the rate at which matter is stripped off the stream and redirected into circular orbits, $\partial \dot{m}/\partial R$, must be specified. This is not a well defined problem. In order to make progress, and in particular to examine the effect on disc evolution of deep disc penetration as opposed to instantaneous mixing, we have adopted the following parametrization. We assume that the stream is stripped at a rate which is a fraction β times the circular mass flux in the disc at that radius, i.e.

$$\frac{\partial \dot{m}}{\partial R} = \beta\ \Sigma \left(\frac{GM_1}{R} \right)^{\frac{1}{2}} \qquad (4)$$

If $\beta = 1$ the momentum imparted to stream material by "collision" of disc matter moving in the azimuthal direction is that of the disc material itself (though note that the form of equation (3) ensures that angular momentum is conserved). It is clear that $\beta \simeq 1$ is an upper limit, and in general $\beta < 1$.

We have explored the effects of different values of β with numerical models. Low values of β allow deep stream penetration, high values of β ($\beta = 1$) approach instantaneous mixing. To obtain numerical solutions equation (3) is transformed to variables $X = 2R^{\frac{1}{2}}$ and $S = X\Sigma$ (see paper I).

$$\frac{\partial S}{\partial t} = \frac{12}{X^2} \frac{\partial^2 (\nu S)}{\partial X^2} + \frac{4(GM_1)^{\frac{1}{2}}}{\pi X^2} \frac{\partial}{\partial X}\left(\left(1 - \frac{X_K}{X}\right) \beta S\right) + \frac{4(GM_1)^{\frac{1}{2}}}{\pi X^3} \beta S \qquad (5)$$

Equation (5) is solved with a centred finite difference scheme with appropriate boundary conditions which ensure conservation of mass within the grid, allow mass and angular momentum loss at R_{IN} and angular momentum loss at R_{OUT}. The vertical structure is solved as in Paper I, with values of $\alpha = 1.5$, $R_{OUT} = 5 \times 10^{10}$ cm, $R_K = 1.5 \times 10^{10}$ cm, $R_{IN} = 5 \times 10^8$ cm, and $M_1 = 1\ M_\odot$.

In Figs 2 and 3 the change in structure of the disc following a burst of mass transfer is shown for values of $\beta = 1.0$ (a), $\beta = 0.1$ (b), and $\beta = 0.01$ (c). In Fig 2 the radius of the disc, R_{EDGE}, together with the point at which complete mixing of the stream into the disc is reached, R_{STRIP} is shown as a function of time. Material is being progressively stripped off the stream between R_{EDGE} and R_{STRIP} according to the prescription of equation (4). In Fig 3 the change in surface density as a function of radius during the rise in the mass transfer burst is shown. The surface densities increase with time as matter is injected into the disc from the enhanced flux in the stream. In Fig 3 each curve is separated by 2×10^4 sec evolution time, starting at 1×10^5 sec with a steady-state disc (curve A) and finishing at 2×10^5 sec (curve B) when the burst reaches its maximum rate of 10^{19} g s^{-1}. The form of the mass transfer burst is given in Fig 4.

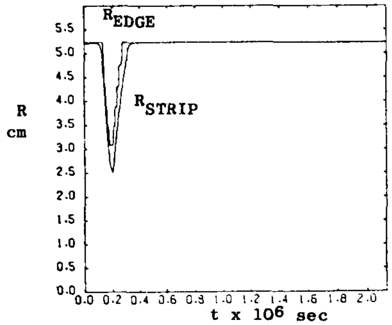

Fig 2a $\beta = 1.0$

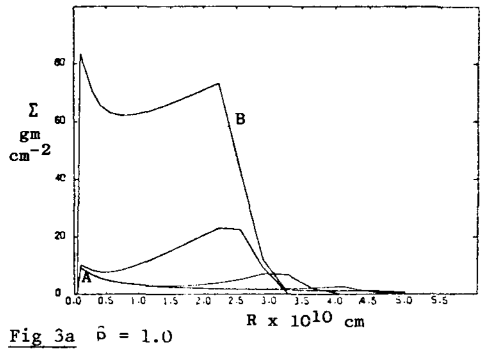

Fig 3a $\beta = 1.0$

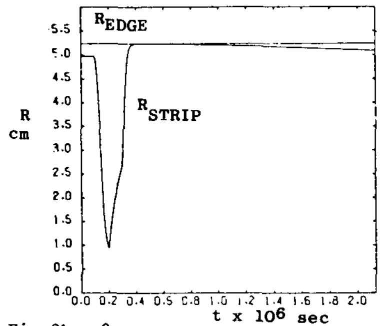

Fig 2b $\beta = 0.1$

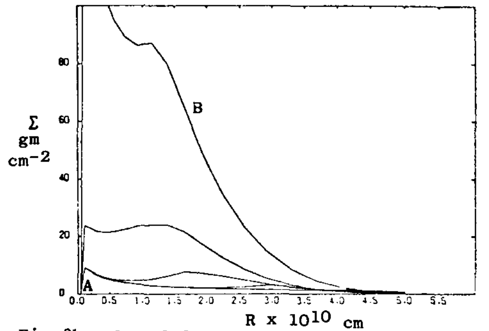

Fig 3b $\beta = 0.1$

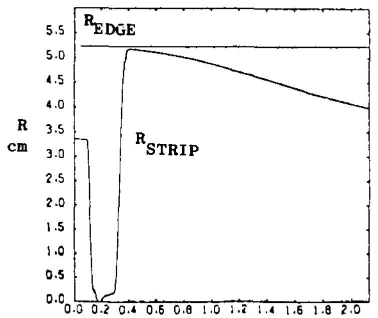

Fig 2c $\beta = 0.01$

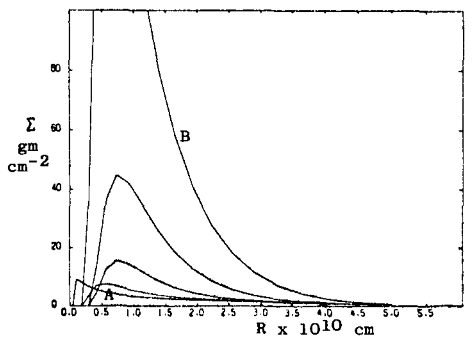

Fig 3c $\beta = 0.01$

Fig 2. Variation of disc edge, R_{EDGE}, and stream penetration radius R_{STRIP}, with time.

Fig 3. Evolution of surface density during the rise in the mass transfer rate. Curves labelled A correspond to steady state conditions at $t = 1 \times 10^5$ sec and B to the state at $t = 2 \times 10^5$ sec when the maximum mass transfer rate is achieved. Each curve is separated by $T = 2 \times 10^4$ sec.

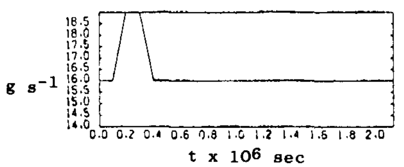

Fig 4. Mass transfer variation with time.

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At a value of $\beta = 1.0$ the stream is stripped in the outermost region of the disc (Fig 2a).The behaviour is close to the instantaneous mixing of Paper I. During the mass transfer burst the disc edge shrinks due to deposition of low angular momentum material in the outer disc rim, and then expands back to R_{OUT} at 5×10^{10} cm. The form of the surface density distribution (Fig 3a) shows how the stream builds up the outer edge of the disc with new material which subsequently diffuses inward.

At a value of $\beta = 0.1$ the stream penetrates deep into the disc in the burst (Fig 2b), down to a radius of 1.0×10^{10} cm (R_K). The disc edge no longer shrinks, but, as can be seen in Fig 3b, R_{EDGE} remains at R_{OUT} and material is stripped progressively off the stream as it penetrates all the way through the disc down to R_{STRIP}.

More extreme disc penetration occurs at the lowest value of $\beta = 0.01$. Even in the steady-state before outburst the stream penetrates well into the disc (as must be occurring in RU Peg, see Stover 1981). During the burst most of the material is deposited close to R_K, and stripping extends all the way down to the minimum distance of approach to the white dwarf by the stream. Because this material has angular momentum which tries to force it into an orbit at radius R_K, and viscosity has not yet had time to operate effectively, the surface density profile peaks at a point close to R_K. As expected, with low values of β most of the stream material goes into an orbiting annulus near R_K. This persists until viscosity acts to spread the new burst of matter out into a disc.

Although the value of β has a major influence on the surface density evolution of the disc, the effect on the form of the eruption light curve is much less significant. The only difference between the light curves with $\beta = 0.01$ compared to $\beta = 1.0$ is enhanced ultraviolet flux at maximum light (Fig 5). This enhanced ultraviolet emission is due to the increased fraction of matter deposited directly in high temperature inner disc regions. After the transient effects associated with the period of enhanced mass flux are over, the subsequent viscous evolution is closely similar and independent of β. The value of β affects the rise and maximum light behaviour, but the decay is unaffected. Thus the decay-time/period relation discussed in Section 2 is insensitive to the processes discussed in this Section. One way in which an estimate of β in individual systems may eventually be obtained is from the changes in the continuum spectral distribution during the rise and at maximum light. In Fig 6 the continuum distribution during the rise (at the same time intervals as Fig 3) is shown. Major differences between the three cases are apparent, and in particular during the rise, and at

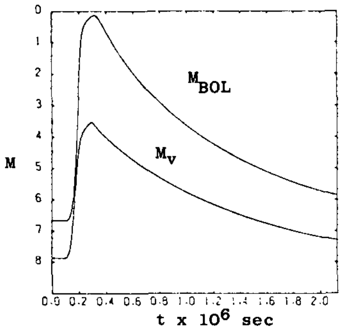

Fig 5a β = 1.0

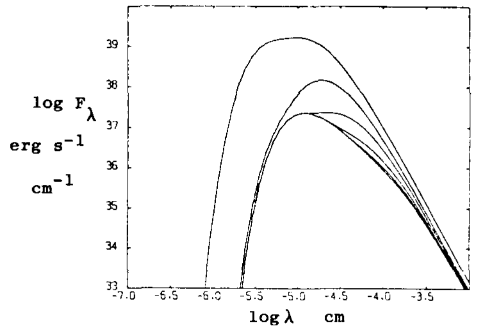

Fig 6a β = 1.0

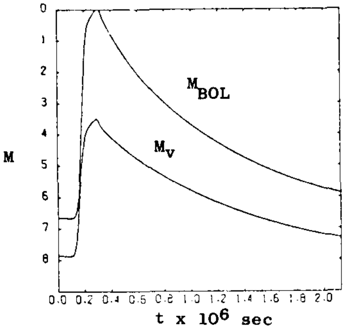

Fig 5b β = 0.1

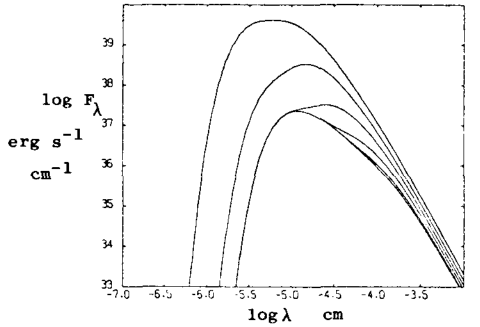

Fig 6b β = 0.1

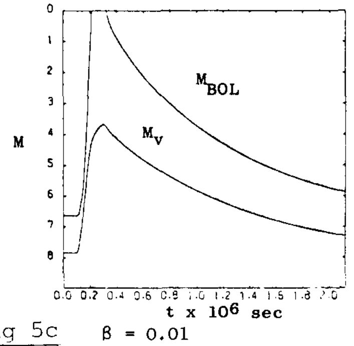

Fig 5c β = 0.01

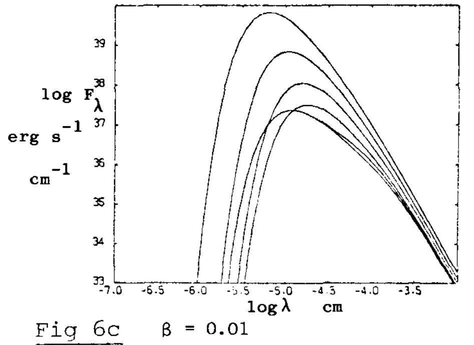

Fig 6c β = 0.01

Fig 5. Bolometric and visual light curves.

Fig 6. Spectral evolution during the rise in the mass transfer rate for the same times as Fig 3.

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maximum light, the slope is not a steady-state $\lambda^{-2.33}$ power law. During these phases, the effects of mass input and time-dependent evolution introduce significant deviations from the predictions of steady models.

Hassall et al. (1982) find that on the rise to outburst the UV flux in VW Hydri lags at least one day behind the optical, with the continuum spectral distribution rising initially only in the visual, as we find for models with $0.1 < \beta < 1.0$. Comparing their results with models in Fig.6 we find that $\beta > 0.1$. There is no need at this stage to invoke optically thick winds in dwarf novae to account for deviations of the continuum from steady state predictions as Klare et. al (1982) have claimed.

An upper limit on β can be set from the observed variation of the "hot-spot" luminosity during the outburst rise. It is well established that during the rise the hump luminosity does not change in proportion to the overall increase in luminosity from the disc. Indeed it is often claimed that this indicates no change in the mass flux in the stream, and that the outbursts must be produced by disk instabilities. However this conclusion is based on the naive assumption that all the available stream kinetic energy is released within a distance of order the height of the disc, H at the disc edge. If stream penetration occurs this will not be the case. In Fig.7 the light curve of VW Hyi in outburst (Warner 1974) is shown together with the hump luminosity. The delay of the rise in the hump behind the overall outburst is evident.

In Fig.8 the changes in the disc, spot and stream luminosity are shown for a $\beta = 0.1$ disc. The spot luminosity, L_{spot} is computed as that fraction of the stream-disc impact energy liberated within a region of thickness H of the disc edge (i.e. that flux which could produce an asymmetric radiation pattern with the properties of the so-called "hot-spot"). The stream luminosity, L_{stream}, is the stream kinetic energy liberated deeper within the disc (which may be radiated anisotropically but not with the expected distribution in binary phase of the "hot-spot").

It is evident that the spot luminosity does not rise simultaneously with the outburst rise, but is delayed. This is simply due to the fact that until the surface density at the outer edge has grown by viscous transport of material deposited interior to the disc edge, the stream continues to penetrate, and the spot flux is unaffected by the outburst. A simultaneous increase of L_{disc} and L_{spot} occurs only if $\beta = 1.0$, i.e. with instantaneous mixing. We conclude that there is no observational support for the view that mass transfer variations cannot be driving the outbursts. With $\beta = 0.1$ all the major observed features presently known to be associated with the disc component of dwarf nova outbursts are realised. Further evidence in support of this interpretation is given in a subsequent paper in this volume.

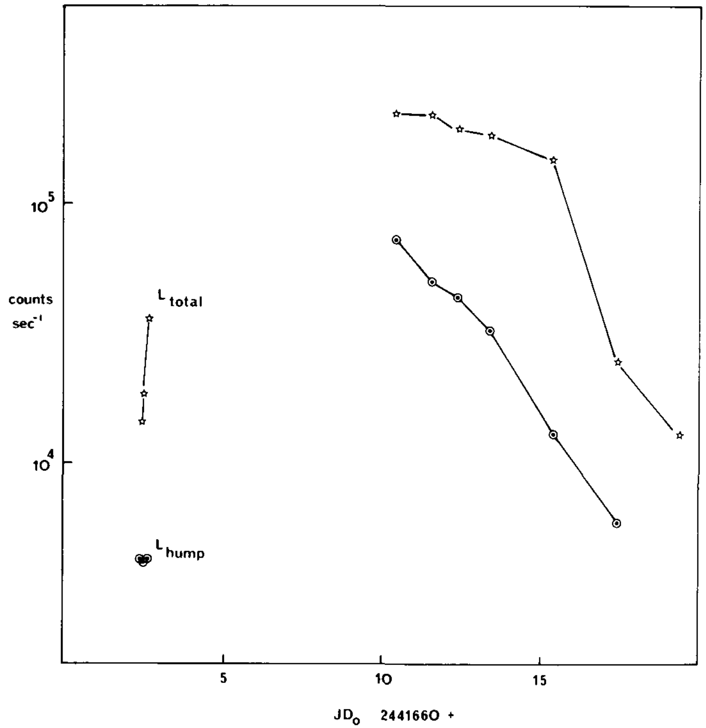

Fig.7. Observed total flux variation and hump luminosity through outburst in VW Hydri.

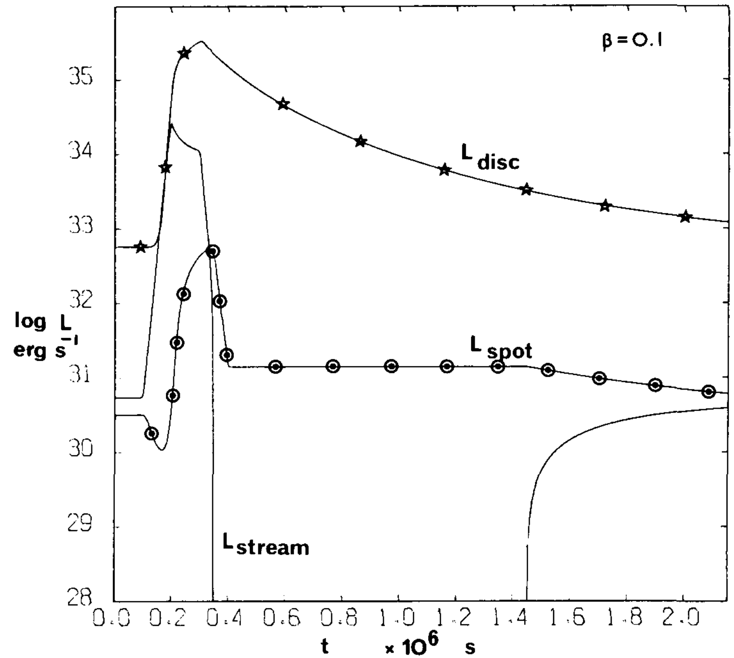

Fig.8. Changes in disc, spot, and stream luminosity in a β = 0.1 disc.

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DISCUSSION FOLLOWING G. BATH'S TALK

LAMB: It seems to me that the distinction you are making between the luminosity of the spot and the luminosity of the whole stream is a little shaky.

BATH: I agree. Let me just clarify, this is just an approach to study a problem which at the moment is being pushed under the carpet and upon which an awful lot of arguments depend. All I am saying is that this is the first attempt to study it and when I do, it turns out that many of the arguments that have been made in the past are not valid, they are only valid if $\beta=1$, which is an extreme case.

ROBINSON: Even if you accept that there is an observational distinction between L_{stream} and L_{spot} it looks to me like L_{spot} does achieve a new bright equilibrium value long before the eruption has decayed completely so in this model, the spot should reappear at some point in the eruption.

BATH: I am going to talk about details of the anisotropic pattern of this in my second talk.

SCHATZMAN: My question concerns the turbulent viscosity which you assumed in order to find values of α and β close to 1, do you know of any work which gives more justification than just the parametrization of the scales?

BATH: No. I know that people have done work, but I don't know of any work that has been accepted.

RITTER: You have shown how the stream penetrates into the disk if you release a blob of matter at L_1. What about disk penetration in the quiescent state?

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BATH: These were evolved in a steady state, constant $\dot{M}$. No stream penetration with $\beta=1$ in the steady state, slight penetration with $\beta=0.1$ and extensive penetration with $\beta = 0.01$.

RITTER: So this means that if you make observations of dwarf novae in the minimum state, then the instantaneous mixing assumption is not too bad.

BATH: Right. In most cases, except I am worried about Stover's observations of U Peg.

RITTER: I have another question. When you have shown the radiation coming out, you gave the argument that if you have a very low β, you have in the bolometric flux a huge peak in the radiation, but there you have obviously assumed that all the matter which is streaming in, is circularized at the corresponding orbit, because if you have $\beta=0.01$ then the stream behaves as a more or less non dissipative.

BATH: It goes around ten times in fact, before it gets completely stripped, with $\beta=0.01$.

LIVIO: Concerning the Bailey relation, it seemd to me that if the last observational point should be taken seriously that the theoretical graphs that you have obtained, always did not have the same slope as the slope of the observations.

BATH: I don't think that this is important. I think it is amazing that with an "α viscosity" here, which is not real physics, we get as good a correlation as we have. Also, α may be changing as a function of time. I would like to mention here that it was John Whelan who in the conference in Cambridge in 1975 suggested that maybe mass transfer bursts generate disk instabilities.

MATTEI: The points on left side of the graph of the Bailey relation are all for SU UMa stars and the ones after the gap of course are for U Gem and Z Cam type stars and even though we can fit a line like the one in the figure, there appears to be a better fit it we fit one line through Z Cam and U Gem stars and another one through SU UMa stars. Is there any way of explaining the different values of α, in order to have two relations?

BATH: If there is a distinction, I don't understand what it is at the moment. I would like to make the comment that it is very difficult to fit this relationship with disk instability models of a simple type. This is because if you vary the viscosity globally over the disk, the decay of the light curve is determined initially by the diffusion time through the disk, but at later stages of the outburst it is determined by the rate of change of viscosity itself, and that has to scale with binary period if it is going to fit the Bailey relationship and I don't see any reason why that should be the case.

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