J. Austral. Math. Soc. (*Series A*) **26** (1978), 383–384

ON THE FIXED POINTS OF SYLOW SUBGROUPS OF TRANSITIVE PERMUTATION GROUPS: CORRIGENDUM

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(Received 30 August 1977)

Abstract

The proof of Theorem 5 in a paper with the same title is incorrect. In this note weaker versions of that theorem are proved.

Subject classification (*Amer. Math. Soc.* (*MOS*) 1970): 20 B 05.

In Herzog and Praeger (1976) we stated Theorem 5 which is incorrect for $p > 2$. Theorem 1 and Corollaries 2–4 are unaffected, as well as Lemmas 2.1 and 2.2. It follows from Praeger (1978b) and Theorem 1 that Corollary 7 is true.

Using the results of Praeger (1978b) we shall prove the following weaker version of Theorem 5.

THEOREM 5′. *Let G be a transitive permutation group on a set Ω of n points, and let P be a Sylow p-subgroup of G for some prime p dividing $|G|$. Suppose that P has t long orbits and f fixed points in Ω, and suppose that $f = tp-1$. If P has an orbit of length p, then $t = 1$, $n = 2p-1$ and $G \supseteq A_n$.*

PROOF. By Praeger (1978a) it follows that all long orbits of P have the same length, namely p. Hence $f = tp-1 = \frac{1}{2}(n-1)$, and by Praeger (1978b) $t = 1$, $n = 2p-1$ and $G \supseteq A_n$.

Finally we shall show that Theorem 5 holds for $p = 2$ and $f > 0$.

THEOREM 5″. *Let G be a transitive permutation group on a set Ω of n points, and let S be a nontrivial Sylow 2-subgroup of G. Suppose that S has t long orbits and f fixed points in Ω, and suppose that $f = 2t - i_2(n) > 0$. Then $t = f = i_2(n) = 1$ and G is 2-transitive. If the long S-orbit has length 2, then $n = 3$ and $G \cong S_3$.*

PROOF. If $n \leqslant 3$, then Theorem 5″ clearly holds. Assume, by induction, that the result is true for transitive groups of degree less than n. By Wielandt (1964) 3.7, $|N(S):S|$ is divisible by $f = 2t - i_2(n)$. Since $|N(S):S|$ is odd, f is odd, and hence $i_2(n) = 1$.

Let $\Sigma = \{B_1, \ldots, B_r\}$ be a set of blocks of imprimitivity for G in Ω. Since $f > 0$ and since S fixes setwise any block containing a point of $\mathrm{fix}_\Omega S$, it follows that $\mathrm{fix}_\Sigma S$ is non-empty. Let $B \in \mathrm{fix}_\Sigma S$ and set $f_B = |\mathrm{fix}_B S|$, $f_\Sigma = |\mathrm{fix}_\Sigma S|$. Denote by t_B and t_Σ the number of long S-orbits in B and Σ, respectively. Suppose first that S acts nontrivially on B. Then by Herzog and Praeger (1976) Theorem 1, $f_B = 2t_B - d$ for some $d \geqslant 1$. Hence by Herzog and Praeger (1976), Lemma 1.2,

$$2t-1 = f = f_\Sigma f_B = 2f_\Sigma t_B - f_\Sigma d \leqslant 2t - f_\Sigma d$$

as $f_\Sigma t_B$ is the number of long S-orbits in $U\{B \mid B \in \mathrm{fix}_\Sigma S\}$. Therefore $f_\Sigma = d = 1$ and $t_B = f_\Sigma t_B = t$, from which we conclude that $|\Sigma| = 1$. On the other hand, if S acts trivially on B, then by Herzog and Praeger (1976), Lemma 1.2 and Theorem 1,

$$f = |B| f_\Sigma, \quad t = |B| t_\Sigma \quad \text{and} \quad f_\Sigma = 2t_\Sigma - d$$

for some $d \geqslant 1$. Hence $2t-1 = f = 2t - |B| d$ and so $|B| = 1$. Thus G is primitive on Ω.

Let $\alpha \in \mathrm{fix}_\Omega S$ and let $\Gamma_1, \ldots, \Gamma_r$, $r \geqslant 1$, be the orbits of G_α on $\Omega - \{\alpha\}$. By Wielandt (1964), 18.4, S acts nontrivially on each Γ_i. Let S have f_i fixed points and t_i long orbits in Γ_i for $1 \leqslant i \leqslant r$. Then by Herzog and Praeger (1976), Theorem 1, $f_i = 2t_i - d_i$ for some $d_i \geqslant 1$, $1 \leqslant i \leqslant r$, and so

$$2t-1 = f = 1 + \sum f_i = 1 + \sum (2t_i - d_i) = 2t + 1 - \sum d_i,$$

that is, $\sum d_i = 2$. If $r > 1$, then $r = 2$ and $d_1 = d_2 = 1$. By induction G_α is 2-transitive on Γ_1 and Γ_2, a contradiction to Wielandt (1964), 17.7. Hence $r = 1$, that is G is 2-transitive. If $f > 1$, then by Wielandt (1964), 3.7 applied to G and to G_α, $|N(S):S|$ is divisible by the even integer $f(f-1)$, a contradiction. Hence $f = 1$ and so $t = 1$. Finally, if S has an orbit of length 2, then $n = 3$ and $G \cong S_3$.

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https://doi.org/10.1017/S1446788700011903 Published online by Cambridge University Press