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CORRESPONDENCE

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SIR,

Reflection coefficient at a dielectric interface

I have been involved in recent correspondence and discussions with glaciologists who are assessing the possible internal structure of polar ice sheets through a study of the strength of reflecting horizons observed during pulsed radio-echo sounding. Requests have been made for a formulation of the reflection coefficient at an interface separating dielectrics whose permittivities and loss tangents are marginally different. In preparation for an earlier paper with Dr Gordon Robin, I undertook such an analysis, but published only the result of the derivation (Paren and Robin, 1975). In view of current interest in internal layering in ice sheets, and for clarification of the validity of my formulae for reflection coefficients, I wish to present the analysis I originally developed.

The power reflection coefficient $|R|$ between two media of complex intrinsic admittance $\Upsilon_1^\star$ and $\Upsilon_2^\star$ is

$$|R| = |(\Upsilon_1^\star - \Upsilon_2^\star)/(\Upsilon_1^\star + \Upsilon_2^\star)|^2, \tag{1}$$

where normal incidence is assumed at a single smooth boundary. This is a well-known result, and is derived, for example, in chapter 10 of Bleaney and Bleaney (1957).

In a situation where the media are of very similar characteristics, I set

$$\Upsilon_1^\star = \Upsilon^\star + \Delta\Upsilon^\star, \qquad \Upsilon_2^\star = \Upsilon^\star.$$

Equation (1) becomes, to first-order approximation,

$$|R| = |\Delta\Upsilon^\star/2\Upsilon^\star|^2 = |\tfrac{1}{2}\Delta[\ln \Upsilon^\star]|^2. \tag{2}$$

The intrinsic admittance $\Upsilon^\star$ of a medium of complex relative permittivity $\epsilon^\star$ normalized to the free-space admittance is given (cf. Bleaney and Bleaney, 1957, equation 10.29) by

$$\Upsilon^\star = \sqrt{\epsilon^\star}.$$

The complex permittivity may be expressed either in terms of polar coordinates

$$\epsilon^\star = r \exp(-\mathrm{j}\delta),$$

or by its real component, the relative permittivity ϵ', where

$$\epsilon^\star = \epsilon'(1 - \mathrm{j} \tan \delta)$$

and $\epsilon' = r \cos \delta$, $\tan \delta$ being the loss tangent of the dielectric medium. Thus

$$\begin{aligned} \ln \Upsilon^\star &= \tfrac{1}{2} \ln \epsilon^\star = \tfrac{1}{2}[\ln r - \mathrm{j}\delta] \\ &= \tfrac{1}{2}[\ln \epsilon' - \ln \cos \delta - \mathrm{j}\delta]. \end{aligned}$$

I now consider the change in admittance caused by variations of $\Delta\epsilon'$ in ϵ' and $\Delta\delta$ in δ.

$$\begin{aligned} \Delta\Upsilon^\star/\Upsilon^\star &= \frac{\mathrm{d}(\ln \Upsilon^\star)}{\mathrm{d}\epsilon'}\Delta\epsilon' + \frac{\mathrm{d}(\ln \Upsilon^\star)}{\mathrm{d}\delta}\Delta\delta \\ &= \tfrac{1}{2}(\Delta\epsilon'/\epsilon' + [\tan \delta - \mathrm{j}]\,\Delta\delta). \end{aligned} \tag{3}$$

The full expression for the power reflection coefficient at a single boundary is calculated from Equations (2) and (3)

$$|R| = \tfrac{1}{16}([\Delta\epsilon'/\epsilon' + \tan \delta\, \Delta\delta]^2 + [\Delta\delta]^2). \tag{4}$$

The equation may be simplified for changes in one variable alone.

For permittivity changes:

$$|R| = (\tfrac{1}{4}\Delta\epsilon'/\epsilon')^2. \tag{5}$$

For loss tangent changes:

$$|R| = \tfrac{1}{16}(1+\tan^2\delta)\,\Delta\delta^2;$$

this expression is equivalent to

$$|R| = (\tfrac{1}{4}\cos\delta\,\Delta(\tan\delta))^2,$$

which for small phase angles approximates to

$$|R| = (\tfrac{1}{4}\Delta(\tan\delta))^2. \tag{6}$$

Equations (5) and (6) are the formulae for the power reflection coefficients given by Paren and Robin (1975).

Reflections from within polar ice sheets observed during pulsed radio echo-sounding are due to discontinuities in permittivity or loss tangent or a simultaneous change in both. Formulae for the returned pulse strength have been conventionally derived for a discontinuity in permittivity, and contain factors which multiply the single-boundary reflection coefficient (Equation (5)) to take the pulse length and layering statistics into account. These formulae may be used where a discontinuity only occurs in the loss by substituting $\Delta\tan\delta$ for $\Delta\epsilon'/\epsilon'$. In the general case, Equation (4) should be used as the basis for calculations of echo strength in a layered medium.

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REFERENCES

Bleaney, B. I., *and* Bleaney, B. 1957. *Electricity and magnetism.* Oxford, Clarendon Press.

Paren, J. G., *and* Robin, G. de Q. 1975. Internal reflections in polar ice sheets. *Journal of Glaciology*, Vol. 14, No. 71, p. 251–59.