

Local characterizations of geometries

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One of the best ways to understand the nature of finite simple groups is through geometries associated with them. This approach for classical and exceptional groups of Lie type has been quite successful and has led to the development of the concept of buildings and polar spaces. The latter have been characterized by simple systems of axioms with a combinatorial-geometric flavour. It has been observed recently that geometries similar to buildings can be associated with finite sporadic simple groups. However, most of the known characterizations of such geometries for sporadic groups require additional assumptions of a group-theoretic nature. One aim of this thesis is to present characterizations of geometries for the sporadic groups J_2 , Suz , McL , Co_3 , Fi_{22} , Fi_{23} , Fi_{24} and He , which are in the same spirit as the characterizations of buildings and polar spaces mentioned above, in particular without any assumption on the automorphism groups of the geometries. A by-product of these results for J_2 , Suz and He is a proof that certain presentations for those groups are faithful.

Most of this work may be viewed as a contribution to the theory of graphs with prescribed neighbourhood. The result on graphs of (+)-points of $GF(3)$ -orthogonal spaces, which is also used for characterizations of geometries for Fi_{23} and Fi_{24} , may be considered (along with the results for the groups Fi_{22} , Fi_{23} and Fi_{24}) as a generalization of a well known theorem on locally cotriangular graphs.

Several of the geometries to be characterized are extensions of polar spaces. Hyperovals of polar spaces are natural generalizations of hyperovals of projective planes of even order and play an important role in investigations of extensions of polar spaces. A computer-aided enumeration of hyperovals was performed. As a by-product new extended generalized quadrangles were found as hyperovals of the polar spaces $Q_5^+(4)$ and $H_5(4)$.

The main part of this thesis consists of several research papers, given as References below. We refer the reader of this abstract to them for further discussion and literature.

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