Black Holes from Stars to Galaxies – Across the Range of Masses
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Humpy LNRF-velocity profiles in accretion discs orbiting rapidly rotating Kerr black holes: a possible relation to epicyclic oscillations

Zdeněk Stuchlík, Petr Slaný and Gabriel Török

Institute of Physics, Faculty of Philosophy and Science, Silesian University in Opava, Bezručovo nám. 13, CZ-746 01 Opava, Czech Republic

email: zdenek.stuchlik@fpf.slu.cz, petr.slany@fpf.slu.cz, terek@volny.cz

Abstract. Coordinate-independent definition of the characteristic, so-called "humpy" frequency related to the positive gradient of the orbital velocity in locally non-rotating frames around nearly extreme Kerr black holes is given and compared with the epicyclic and orbital frequencies for both Keplerian thin discs and limiting marginally stable thick discs.

Keywords. accretion disks – black hole physics – relativity – instabilities

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Gradient sign change of the orbital velocity $\mathcal{V}^{(\varphi)}$ related to the locally non-rotating frames (LNRF) in Kerr backgrounds (see, e.g., Bardeen, Press, Teukolsky 1972) has been found for accretion discs orbiting rapidly rotating Kerr black holes with spin $a > 0.9953$ for thin (Keplerian) discs (Aschenbach 2004) and $a > 0.99979$ for marginally stable thick discs characterized by the uniform distribution of the specific angular momentum, $\ell(r,\,\theta) = \text{const}$ (Stuchlík *et al.* 2005). Such a "humpy" orbital velocity profiles occur close to but above the marginally stable circular geodesic of the black hole spacetimes.

Non-monotonic behaviour of the orbital velocity in the marginally stable ($\ell = \text{const}$) tori is characterized by topology change of the von Zeipel/equivelocity surfaces, $\mathcal{R}(r,\,\theta) \equiv \ell/\mathcal{V}^{(\varphi)} = \text{const}$. In addition to the open surface crossing itself under the inner edge of the torus and existing for all values of the rotational parameter a, for $a > 0.99979$ the second self-crossing (marginally closed) surface together with toroidal surfaces occur. Toroidal von Zeipel surfaces exist under the newly developing cusp, being centered around the circle corresponding to the minimum of the equatorial LNRF velocity profile. The whole effect, elucidated by the toroidal von Zeipel surfaces, is located inside the ergosphere of a given Kerr spacetime.

The maximal positive rate of change of the orbital velocity in terms of the proper radial distance $\tilde{R}$,

$$\nu_{\text{crit}}^{\tilde{R}} = \left.\frac{\partial \mathcal{V}^{(\varphi)}}{\partial \tilde{R}}\right|_{\max}, \quad \mathrm{d}\tilde{R} = \sqrt{g_{rr}}\,\mathrm{d}r, \tag{1.1}$$

where r is the Boyer-Lindquist radial coordinate, introduces a locally defined critical frequency characterizing possible disc oscillations connected with the velocity hump. Comparing the "humpy frequency" related to distant observers,

$$\nu_{\text{h}} = \sqrt{-(g_{tt} + 2\omega g_{t\varphi} + \omega^2 g_{\varphi\varphi})}\,\nu_{\text{crit}}^{\tilde{R}}, \tag{1.2}$$

where $\omega = -g_{t\varphi}/g_{\varphi\varphi}$ is the angular velocity of the LNRF, with the radial and vertical epicyclic frequencies, ν_{r}, ν_{v} (their analytic expressions can be found, e.g., in Aliev &

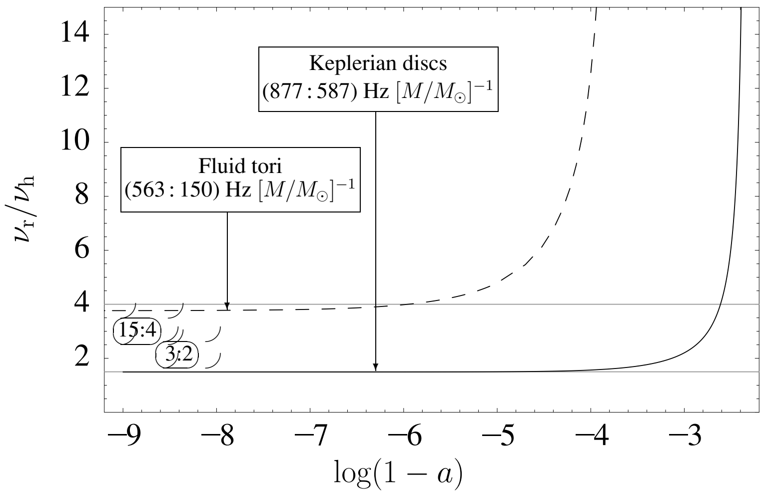

Figure 1. Spin dependence of the ratio of the radial epicyclic frequency and the "humpy frequency" related to distant observers for Keplerian discs and $\ell = \ell_\mathrm{ms}$ fluid tori. The ratio is given in the radius of definition of the humpy frequency. Both the ratios rapidly falls down to the asymptotic values of $3:2$ for Keplerian discs and $15:4$ for fluid tori. Then an exact 1/M scaling holds with frequencies depicted in the figure.

Galtsov 1981, Nowak & Lehr 1998), we can show that in the Keplerian discs orbiting extremely rapid Kerr holes ($1-a<10^{-4}$) the ratio of the epicyclic frequencies and the humpy frequency is nearly constant, i.e., almost independent of a, being $\sim 3:2$ for the radial epicyclic frequency and $\sim 11:2$ for the vertical epicyclic frequency. For black holes with $a \sim 0.996$, i.e., when the resonant phenomena with ratio $3:1$ between the vertical and radial epicyclic oscillations occur near the radius of the critical humpy frequency, there is ratio of the radial epicyclic and the humpy frequency $\sim 12:1$, i.e., the critical frequency is close to the low-frequency QPOs related to the high-frequency QPOs in such spacetimes. For $a > 0.996$ the resonant orbit $r_{4:1}$ (with the ratio $4:1$ between the vertical and radial epicyclic oscillations) occurs in the region of the hump. In the case of thick discs, the situation is more complex due to the dependence on the distribution of the specific angular momentum ℓ characterizing the disc rotation. For $\ell = \mathrm{const}$ tori and $1-a<10^{-6}$ the ratios of the orbital and epicyclic frequencies to the humpy frequency are again almost constant and independent of both a and ℓ. Moreover, the resonant orbit $r_{4:1}$ lays very close to the maximum rate of change of the velocity gradient and the ratio of the radial epicyclic and the humpy frequency is close to $4:1$ there. Spin dependence of the ratio $\nu_\mathrm{r} : \nu_\mathrm{h}$ for both Keplerian discs and fluid tori is illustrated in Fig. 1.

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