COLLAPSE DYNAMICS AND COLLAPSE MODELS

Richard B. Larson
Yale University Observatory

1. INTRODUCTION

Although we now possess a wealth of information about the properties of molecular clouds and the circumstances in which stars form, we still have little direct information about the crucial question of how the material in molecular clouds actually becomes condensed into stars. In this report we shall discuss briefly the current status of theoretical attempts to understand this problem, based on calculations of the dynamics of collapsing clouds and protostars.

Many collapse calculations have treated the collapse of individual protostars idealized as isolated objects, but a complete understanding of star formation requires also a consideration of how gravitational collapse proceeds on scales larger than that of individual protostars. The properties and evolution of protostars depend on the conditions under which they are formed, and their observable characteristics are particularly influenced by the properties of the material surrounding them. The common occurrence of young stars in close groups and multiple systems indicates that forming stars cannot in general be considered as isolated systems. Also, it is evident that any understanding of such long-standing problems as the initial stellar mass spectrum and the efficiency of star formation requires an understanding of the collapse problem in a larger context than that of individual protostars. Thus, while most collapse calculations refer to such special cases, it will be important to keep in mind their possible implications for the more general problems of how clouds collapse and fragment into stars.

2. THE COLLAPSE PROBLEM

In the evolution of a collapsing interstellar cloud or cloud fragment we may, at least conceptually, distinguish two phases: at first, the cloud is not gravitationally bound but is compressed by

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external forces; later, the self gravity of the cloud becomes dominant over other forces (i.e., its free-fall time becomes shorter than all other time scales) and the cloud collapses gravitationally. Here we shall not consider the stages leading to the formation of a gravitationally bound cloud; instead, we are interested in how the cloud develops subsequently and how the results depend on the conditions existing when gravity takes over and collapse begins. It is not clear that such a separation into two main phases of evolution is always possible, especially on the scale of individual protostars, but this is a question on which multi-dimensional collapse calculations such as those mentioned below can potentially shed some light.

The collapse of an interstellar cloud and the formation of stars in it involve many different processes, and many questions arise when we attempt to understand these various parts of the collapse problem. During the early stages of the collapse, the most important questions concern the process of fragmentation: How do condensations form in a collapsing cloud, and what determines their properties? Do the condensations, once formed, continue to fragment indefinitely or do they generally form only a single star or close multiple system? Do large scale gas flows and shock compression in collapsing clouds play an important role in producing the conditions required for fragmentation into protostars? Considering the later stages, we wish to understand the evolution of the individual collapsing fragments or protostars: How does the material in a protostar ultimately become condensed to stellar density? What is the role of accretion phenomena, such as accretion shocks and accretion discs? What determines whether a single star or a binary is formed? How much mass eventually goes into the star(s), and how much is dispersed? How do newly formed or forming stars react back on the collapsing cloud and influence its further evolution? Finally, and perhaps most importantly, how do all of these processes work to determine the masses and the mass spectrum of the stars finally formed?

Many calculations using a variety of different assumptions and methods have been made to study the collapse of gas clouds, and while definitive answers to the above questions are not yet in hand, the results of these calculations provide valuable insights into the way in which collapse proceeds. The case of spherical collapse without rotation or magnetic fields has been relatively well explored, and the effects of initial conditions, boundary conditions, and thermal and radiative properties of the material have been investigated in a number of studies. Also, the spherical case is the only one in which it has been possible to follow the collapse all the way to stellar densities and the formation of hydrostatic pre-main sequence stars. Results for collapse with axial symmetry are more limited, and calculations of this type have so far concentrated on studying the effect of rotation and the question of whether rings are formed. Finally, some preliminary results are available for the general 3-dimensional collapse problem, obtained with both a grid method and a finite-particle method; these results provide new information on the fragmentation problem, and support the general validity of certain results of the spherical and axisymmetric calculations. In the following sections we shall review the results of each of these types of calculation in turn.

3. SPHERICAL COLLAPSE

The basic features of spherical collapse have previously been discussed at some length by Larson (1973, 1974, 1975), and more recent studies by Ferraioli & Virgopia (1975), Kondo (1975), Appenzeller & Tscharnuter (1974, 1975), Westbrook & Tarter (1975), and Yorke & Krügel (1976) confirm the basic qualitative results discussed earlier, although with some quantitative differences which we shall consider later. In all cases the collapse is nonhomologous and the density distribution approaches the approximate form $\rho \propto r^{-2}$ if the collapse is nearly isothermal. Eventually the inner regions become opaque and a hot dense stellar core is formed, surrounded at first by an extended infalling envelope that still contains most of the mass, and the material in the envelope later falls into the core through an accretion shock at its surface.

The nonhomologous collapse found in the spherical case occurs also in the two- and three-dimensional collapse calculations (see below), and has many important implications for the later evolution of collapsing clouds and protostars. Many properties of protostars, for example, depend strongly on the existence of a remnant infalling envelope, and on how much its collapse is retarded relative to that of the core. There are two reasons for a nonuniform collapse: (1) any initial density inhomogeneities in a collapsing cloud tend to be amplified because the densest regions collapse fastest, and (2) the collapse of the outer parts of the cloud is retarded relative to the collapse of the core by pressure gradients; even if not present initially, such pressure gradients are soon established by the inward propagation of a rarefaction wave from the boundary of the cloud. The importance of both of these effects depends on how closely the initial conditions satisfy the Jeans criterion. Density irregularities on scales smaller than the Jeans length will tend to be smoothed out by acoustic waves before the cloud can collapse very far, but larger scale inhomogeneities can survive and grow if the cloud size exceeds the Jeans length. On the other hand, the propagation of rarefaction waves is most important if the cloud size is comparable to the Jeans length, i.e. the sound travel time is comparable to the free fall time. In either case, the collapse is nonuniform, the dominant scale of the density inhomogeneities being comparable to the Jeans length. As will be discussed, however, quantitative results can be rather sensitive to the initial conditions, so it is important for quantitative purposes to understand in some detail just how collapsing clouds and protostars are formed and what their initial conditions are.

The predicted nonhomologous collapse of spherical clouds and protostars has important implications for the dynamics of collapse in general. If some parts of a collapsing cloud become much more condensed than the rest, the dynamics of the remaining diffuse material is increasingly dominated by the effects of these dense condensations. In particular, local self-gravitational forces in the residual gas become unimportant in comparison with the tidal forces produced by the mass concentrations, and therefore further fragmentation is inhibited and the collapse of the remaining material takes on the character of an accretion process, with the dense condensations acting as the centers of accretion. This qualitative picture is supported by the results of 3-dimensional collapse calculations (below). Since the time scale for the collapse and internal evolution of the condensations is relatively short, stars can begin to form in them and react back on the remaining gas in a variety of ways; for example, radiation pressure, ionization, or stellar winds may blow away the remaining gas and prevent it from condensing into stars. Thus the masses finally attained by the forming stars and the efficiency of star formation may depend strongly on the extent to which the collapse is nonhomologous, leaving behind much diffuse gas after stars have begun to form.

4. AXISYMMETRIC COLLAPSE

(a) Nonrotating

A number of calculations of the isothermal collapse of nonrotating prolate and oblate configurations have been made by Larson (1972a). The results showed the same kind of nonhomologous collapse with increasing central concentration as had previously been found in the spherical case. It was also found that if the initial conditions approximately satisfy the Jeans criterion, pressure gradients remain important and tend to prevent the growth of large deviations from spherical symmetry during an isothermal collapse. These results suggest that if the Jeans criterion is nearly satisfied, fragmentation will not occur and a single central object will form, just as in the spherical case. The only configuration which showed any tendency toward fragmentation was a cold cylinder of gas which was much longer than the Jeans length and hence was already unstable to fragmentation initially; in this case, two centers of condensation formed. While not conclusive, these results suggest that fragmentation is not likely to occur during nonrotating isothermal collapse unless the initial configuration is already unstable to fragmentation; otherwise, the collapse proceeds roughly spherically and only a single central star is formed, very much as in the spherical case.

(b) Rotating

There has been greater interest in studying the collapse of rotating axisymmetric clouds, and such calculations have been made by Larson (1972a), Tscharnuter (1975), Fricke et al. (1976), Black & Bodenheimer (1976), and Nakazawa et al. (1976). All calculations find once again a strongly nonhomologous collapse with the development of a small region of high density near the center, even when centrifugal force and gravity are initially nearly in balance. The high density near the center is attained largely as a result of infall of matter along the axis of rotation, where infall is not impeded by centrifugal force. However, the various calculations disagree in the detailed form of the high density region near the center: the results of Larson, Black & Bodenheimer, and most of those of Nakazawa et al. show the formation of a ring structure near the center, while Nakazawa et al. find that no ring is formed in a case with higher initial temperature, and Tscharnuter and Fricke et al., using a different numerical method, find no tendency toward ring formation in any circumstances.

It is clear that numerical errors must be present in some (or all) of these calculations, but the results also suggest that the formation of a ring may be sensitive to the initial conditions and to the detailed distribution of angular momentum in a collapsing cloud. The grid methods of Larson, Black & Bodenheimer, and Nakazawa et al. contain an inherent numerical viscosity which may lead to inaccuracies in conservation of angular momentum; on the other hand, it is not clear that the Legendre expansion method of Tscharnuter and Fricke et al. is adequate to represent the formation of rings (Nakazawa et al. 1976). In any case, all investigations have assumed inviscid flow with strict conservation of angular momentum for each fluid element, but in reality it is likely that torques due to magnetic forces, turbulent viscosity, or deviations from perfect axial symmetry would alter the results importantly. Until these various numerical and physical problems are better understood, it seems reasonable to conclude that there probably exist circumstances in which rings are formed and other circumstances, perhaps not very different, in which rings are not formed.

If a ring forms, it seems inescapable that the end result will be the fragmentation of the ring into (at least) a binary system of condensations orbiting around each other. This result is suggested by stability considerations, and has been found in preliminary 3-dimensional calculations by Black & Wilson (1976) of the evolution of such rings. In view of the very general susceptibility of rapidly rotating systems to non-axisymmetric instabilities, it even seems possible or likely that a rapidly rotating cloud will fragment directly into a binary system even before a well-defined ring is formed. This result is in fact a frequent outcome of the crude 3-dimensional collapse calculations described below. If it can be assumed that each of the resulting condensations forms a single star, this would provide an attractive explanation of the observed preponderance of binary stars (Abt & Levy 1976).

However, it remains to be understood how the two orbiting condensations resulting from such a fragmentation process would evolve subsequently. Do they in fact form single stars, or does further fragmentation take place? As members of a binary system, they are now subject to external forces of a non-axisymmetric nature, and the idealization of axisymmetric collapse is no longer justifiable. Tidal torques will tend to transfer angular momentum from spin to orbital form within the system, thereby possibly enabling the condensations to collapse into single stars without further fragmentation. Also, if the condensations begin with small mass and accrete most of their mass from a common surrounding envelope, the accretion process may tend to select mass elements with small angular momentum relative to the two accreting centers, so that they end up with little spin angular momentum and form single stars. It will be important to try to answer these questions with 3-dimensional collapse calculations.

In summary, the axisymmetric collapse calculations suggest that in the absence of rotation, fragmentation does not generally occur unless the initial configuration is already unstable to fragmentation. In the presence of rotation, fragmentation into a binary or small multiple system seems likely to be the most common result. In neither case, however, do the calculations suggest the occurrence of an extensive hierarchial fragmentation process of the sort that has often been assumed in discussions of star formation. Instead, the results suggest that successive fragmentation occurs only to a very limited extent, generally not proceeding beyond the formation of binary systems, and that the properties of the fragments are largely determined by the initial conditions existing at the time when gravitational collapse begins.

(c) Magnetic

Although no calculations of collapse with magnetic fields have yet been made, Mouschovias (1976a,b) has calculated sequences of equilibrium models of self-gravitating clouds partially supported by magnetic fields, which provide some indication of how collapse might proceed with magnetic fields. These models are moderately flattened and strongly centrally condensed, the degree of central condensation increasing as self-gravity becomes more important until, at a critical point, equilibrium is no longer possible and collapse must occur. This is qualitatively the same behavior as is found without magnetic fields, and this suggests that the collapse of a magnetic cloud will take place in qualitatively the same nonhomologous way as the collapse of a nonmagnetic cloud, with ever increasing central concentration. Mouschovias has also emphasized the probable role of magnetic fields in retarding the collapse of the outer layers of the cloud even after the inner part has decoupled from the magnetic field; this would enhance the separation into a dense core and a diffuse envelope that is already implied by the results for non-magnetic clouds. Magnetic fields may thus play an important role in reducing the efficiency of star formation.

5. THREE-DIMENSIONAL COLLAPSE

(a) Grid Methods

We mention first some calculations which, although not strictly 3-dimensional, probably display many features of a realistic 3-dimensional collapse. These calculations have been made by Theys (1973) and Quirk (1973) using the "beam scheme" to calculate the dynamics of self-gravitating thin discs of gas. Theys, in studying possible models for ring galaxies, calculated the collapse of rapidly rotating gas discs and found that rings form in some cases but not in others, depending on the assumed initial density distribution. In all cases, such rings were found to be very unstable, and rapidly fragmented into a number of dense lumps. Quirk calculated the collapse of initially nearly uniform rapidly rotating discs with different temperatures, and found rapid fragmentation into systems of orbiting condensations or knots, the sizes of the fragments being approximately consistent with the initial Jeans length. The fragments tended to become more centrally condensed with time, but showed no tendency toward further fragmentation. A considerable part of the initial mass remained as diffuse gas swirling around the condensations, some being slowly accreted by them and some being dispersed. Many of these features are expected to occur in a realistic 3-dimensional collapse; however, besides being limited to a thin disc, these calculations contain an inherent numerical viscosity which, although it may qualitatively mimic real effects, is difficult to justify quantitatively.

Preliminary calculations with a 3-dimensional grid have been made by Black & Wilson (1976) to study the evolution of self-gravitating rings such as those found in several calculations of the collapse of axisymmetric rotating clouds. The results show that the ring begins to fragment into two major condensations orbiting around each other, thus supporting the conjecture that the collapse of a rotating cloud will ultimately result in a binary system.

(b) Finite-Particle (N-Body) Method

Finally, we mention some preliminary results obtained with a "finite-particle" method in which a fluid is simulated by a system of representative particles or fluid elements of finite mass whose motions are followed individually, as in N-body calculations of stellar dynamics. In addition to a modified inverse-square gravitational force, the particle interactions include a pressure force between neighboring particles, supplemented by an "artificial viscosity" term simulating the effect of shock waves; these terms are analogous to those appearing in standard Lagrangian hydrodynamic codes. The form adopted for the pressure term assumes that the gas remains isothermal, but the temperature is an adjustable parameter. To avoid the necessity of calculating the very rapid internal dynamics of the tightly bound condensations that form, the particles have been merged when they come very close together.

The first applications of this method have been made with systems of 100 particles which are initially scattered randomly in a sphere and given a rigid rotation; the main parameters which have been varied are the temperature and the initial angular velocity. The collapse of this system of particles or "cloud" is found to depend on a modified Jeans criterion similar to that of Larson (1972a) for axisymmetric collapse, in which the kinetic energies of thermal motion and rotation contribute approximately additively to counterbalancing gravity. The outcome of the collapse depends strongly on how closely the initial condition satisfies this modfied Jeans criterion, and we briefly summarize here the different types of results found.

If the initial condition nearly satisfies the Jeans criterion, i.e. if gravity is almost balanced by pressure plus centrifugal force, the cloud remains very extended and roughly spherical in overall structure and develops a single central condensation, surrounded by a diffuse envelope containing most of the mass. This is qualitatively the same result as was previously found in the spherical and axisymmetric calculations, except that here no tendency for ring formation is evident; however, it is not clear that the small number of particles involved could adequately represent a ring structure. The central condensation rapidly forms a single object containing many particles, and this central object continues to accrete material from an extended and moderately flattened "accretion envelope" around it. The innermost part of the accretion envelope is more strongly flattened and may be described as an accretion disc. Depending on the initial angular velocity, at least 20 percent and perhaps as much as half of the initial mass is eventually accreted on the central object, the remainder being gradually dispersed.

In these results, an important role is apparently played by the "turbulent viscosity" produced by the interactions between the individual particles; this viscosity acts to transfer angular momentum outward in the accretion envelope, allowing the inner part to fall onto the central object. In this respect the model resembles the floccule theory of star formation proposed by McCrea (1960). While a number of mechanisms probably operate to redistribute angular momentum in a collapsing cloud, including magnetic and gravitational torques as well as viscosity due to random motions, these effects are not yet quantitatively well understood, so it is not clear to what extent the finite-particle viscosity is quantitatively realistic; thus the results must be considered as only illustrative rather than definitive. Nevertheless, they do indicate the likely importance of viscous forces in collapsing clouds, and they support the suggestion from spherical and axisymmetric calculation that fragmentation is not likely to occur if the initial conditions closely satisfy the Jeans criterion.

As the initial ratio of pressure plus centrifugal force to gravity is reduced, a point is soon reached where the cloud fragments into a binary system. There are two effects which favor fragmentation: (1) a reduction in the initial Jeans length, and (2) an overall contraction of the cloud which raises its mean density, further reducing the Jeans length in comparison with the size of the cloud. The exact mode of fragmentation into a binary seems to depend on the initial angular velocity: in a case in which the initial centrifugal force was small enough to allow a significant overall contraction, the cloud collapsed to a relatively compact bar-like configuration which then separated into two lumps, whereas in a case with higher angular momentum the cloud developed a more diffuse and elongated bar-like structure with initially only one major condensation, and the remaining bar material then formed a "spiral arm" structure in which a second condensation later appeared. In either case, each of the two condensations collapses without further fragmentation to form a single condensed object, and the two objects thus formed continue to accrete matter from the remainder of the cloud. If they are well separated, each is surrounded by its own accretion envelope, but if they are close together their accretion zones overlap and they share a common accretion envelope. In most cases, there appears to be a tendency for the accretion process to approximately equalize the masses of the two accreting objects, and together they may eventually accrete as much as half of the initial mass of the cloud.

As the temperature of the cloud is further reduced, fragmentation into a larger and larger number of distinct condensations is observed. The number of condensations formed is approximately inversely proportional to the initial Jeans mass, but the amount of fragmentation taking place and the fraction of the cloud mass going into dense condensations are enhanced by an initial overall contraction of the cloud which becomes more important as the initial angular velocity is decreased. With faster initial rotation, on the other hand, a larger fraction of the mass remains in diffuse form and is eventually dispersed. In systems which form many condensed objects or "stars", there is a strong tendency toward the formation of subgroups consisting of binary or small multiple systems, often with a hierarchial structure reminiscent of that observed for multiple stars. Binary systems generally appear to form as described above, and a hierarchial triple system forms if the temperature and Jeans mass are small enough that one of the condensations in a binary system that is beginning to form fragments further into a closer binary pair.

Another phenomenon observed in these results is that secondary condensations may sometimes form in the accretion envelopes or discs around more massive accreting objects; this can occur because of the relatively high density and high velocity of the accreted gas around a massive object, which can result in strong shock compression. This process may lead to the formation of small companion stars or large planets like Jupiter around some stars. However, the formation of secondary condensations is also inhibited by the tidal force field of the accreting object, and this process does not seem to account for a majority of the "stars" formed in these calculations; the bulk of the fragmentation appears to occur as a one-shot process during the initial collapse of the cloud.

In summary, these crude 3-dimensional collapse calculations suggest that fragmentation is largely determined by the Jeans mass at the time when collapse begins. The condensations in a collapsing cloud are formed approximately satisfying the Jeans criterion, and depending in detail on how closely they satisfy it, they may form either single stars, binaries, or small multiple systems; however, extensive hierarchial fragmentation is not found. Provided that viscosity or other mechanisms for redistributing angular momentum are present, individual collapsing condensations or protostars evolve much as in the spherical case, forming a dense accreting core surrounded by an extended accretion envelope, which has a flattened disc-like structure in the innermost region near the accreting core. The principal difference from the spherical case is that the inflow of matter into the core may not occur in free fall but at a rate determined by the viscosity of an accretion envelope or disc. The formation of an accretion disc offers an attractive possibility for understanding the origin of planetary systems, and accretion disc models for the formation of the solar system have been studied by Cameron (1976).

We note that important implications for star formation theory follow if extensive hierarchial fragmentation does not occur in collapsing clouds, as has generally been assumed. If fragmentation is determined by the conditions existing when collapse begins, then it is important to understand the dynamical and thermal evolution of clouds prior to the onset of collapse; in particular, it is necessary to understand the processes of cloud compression and cooling which lead to the high densities and low temperatures which are observed in molecular clouds and which are necessary for the collapse of protostars in the normal stellar mass range.

6. EVOLUTION OF PROTOSTELLAR CORES

The only existing calculations of the later stages of protostellar evolution have all assumed spherical symmetry, so we now return to consider in more detail the spherical collapse models discussed in section 3. As we have seen, these idealized spherical models are probably qualitatively correct, but may need quantitative revision to take into account various effects such as rotation which they neglect.

According to the spherical models, protostars during their later stages of evolution consist of two very distinct regions which can for many purposes be treated separately. The accreting core is essentially a hydrostatic star, and its properties and evolution depend on those of the infalling envelope primarily through the rate of infall of matter onto its surface; the detailed structure of the envelope is not important except insofar as it determines the inflow rate. On the other hand, the envelope is affected by the core only through the core's mass and luminosity, and perhaps during later stages by other effects such as stellar winds. In this and the following section we consider in turn the properties of protostellar cores and envelopes.

(a) Low Mass Protostars

It is convenient to distinguish low mass protostars with masses less than a few solar masses from more massive protostars, since the evolution of the core is qualitatively different in the two cases. In the low mass case, radiative energy transfer remains unimportant in the core during the entire accretion process, and until an outer convection zone appears during the final stages of core accretion, each mass element in the core retains the entropy which it had immediately after passing through the accretion shock. Thus the entropy distribution and the structure of the core are determined by the thermal properties of the surface layers just inside the accretion shock, which depend in turn on the properties of the infalling matter just outside the shock.

If the protostellar envelope is extremely dense and compact and falls rapidly onto the core, the infalling material may be so opaque that no significant radiative energy losses can occur and the surface layers of the core end up with a very high specific entropy; as a result, the core of 1 $M_\odot$ protostar attains a large radius of $\sim$ 100 $R_\odot$ and a correspondingly high luminosity of $\sim$ 500 $L_\odot$ at the end of the accretion phase. An example of this type of evolution is provided by the models of Narita et al. (1970).

If, on the other hand, the early stages of the collapse are very nonhomologous and leave a much more diffuse and extended envelope surrounding the core, as the calculations described above lead us to expect, the envelope eventually becomes sufficiently optically thin at infrared wavelengths to allow radiation to escape from the surface of the core during the accretion process. In this case the kinetic energy of the infalling matter is largely radiated away at the surface of the core and the surface temperature is determined by the kinetic energy inflow rate, which depends in turn on the rate of mass accretion; a higher accretion rate implies a higher surface temperature and hence a higher entropy and a larger radius for the core. If the collapse is strongly nonhomologous and the time scale for envelope accretion is as long as the initial free fall time, as in the models of Larson (1972b) and the 3-dimensional calculations described above, the entropy inside the accretion shock is relatively low and the core ends up with a radius of only a few $R_\odot$ and a luminosity of a few $L_\odot$ when the accretion ceases to be important. These results have been numerically confirmed within a factor of two by Appenzeller &

Tscharnuter (1975), who obtained a core radius about twice as large as Larson, possibly partly because of the different opacities assumed.

The final properties of the stellar core are related in a simple way to the time scale for infall of the envelope. During the final stages of the accretion process, the internal contraction luminosity of the core becomes comparable to the luminosity produced by accretion. Since the kinetic energy of the infalling gas is comparable to the thermal kinetic energy of the material in the core, this implies that the Kelvin-Helmholtz contraction time of the core must be comparable to the time required to accrete an amount of mass equal to the core mass. Even if the core first forms with a contraction time shorter than the accretion time, it would contract rapidly while still accreting matter until the two time scales become comparable. Thus if the accretion time is long, the final core must have a long Kelvin-Helmholtz contraction time, i.e. it must have a small radius and luminosity; conversely, an object of large radius and luminosity can be formed only by a correspondingly rapid accretion process.

The final properties of the core thus depend on the collapse time of the envelope, which depends on both the initial density and the degree to which the collapse is nonhomologous and leaves an extended envelope around the core. A higher initial density implies a shorter collapse time and a larger final radius; for example, Larson (1969a) found that an increase by a factor of 10^3 in the initial density increases the final radius by about a factor of 3. Much larger final radii were obtained in the calculations of Westbrook & Tarter (1975), in which pressure retardation of the outer layers of the collapsing cloud is apparently less important than in the models of Larson or Appenzeller & Tscharnuter, with the result that the bulk of the envelope falls onto the core relatively soon after the core has formed. The final core radius for a 1 $M_\odot$ model is about 90 $R_\odot$ in these calculations. While a difference in this direction from the results of Larson and of Appenzeller & Tscharnuter is expected because the lower initial temperature assumed by Westbrook & Tarter (3 °K instead of 10 °K) makes pressure retardation of the envelope collapse less important, several features of the Westbrook & Tarter calculations are crude and the quantitative results are difficult to interpret. In any case, the 3-dimensional collapse results described above suggest that the assumptions of Larson and of Appenzeller & Tscharnuter are more realistic. In addition, the lengthening of the accretion time scale by the effects of rotation and the formation of an accretion disc would tend to produce a smaller final core radius.

(b) Massive Protostars

If the mass of a protostar exceeds a few solar masses, radiative transfer begins to become important in the core before the infall of the envelope is completed, and the core approaches radiative equilibrium as a conventional radiative pre-main sequence object. The occurrence of radiative energy losses from the core on a time scale shorter than the accretion time means that the Kelvin-Helmholtz contraction time becomes shorter than the accretion time and the intrinsic core luminosity begins to exceed the infall luminosity. If the core mass exceeds approximately 3 - 5 $M_\odot$, the core can contract all the way to the main sequence and begin nuclear burning as a main sequence star while continuing to accrete matter and grow in mass. Once the internal luminosity of the core becomes dominant, its structure and evolution are no longer determined by the properties of the accretion shock and surface layers, and are altered only by the addition of mass.

The approach to radiative equilibrium is accompanied by a rapid jump in core luminosity, which may be very important for the evolution of the envelope and the observed properties of the protostar. In the 5 $M_\odot$ and 10 $M_\odot$ models of Larson (1972b) the effect is relatively modest, but in the 60 $M_\odot$ model of Appenzeller & Tscharnuter (1974) the outer layers of the core are heated so strongly and rapidly that hydrostatic equilibrium is destroyed and the outermost layers of the core and the entire infalling envelope are blown off when the core attains a mass of about 18 $M_\odot$. It is clear from this result that only a fraction of the material in a massive protostar actually becomes incorporated in a star, but more calculations of different cases will be necessary to provide a better understanding of this phenomenon and its possible observational implications. It seems possible, for example, that the predicted rapid flareup in luminosity may be related to the "FU Orionis phenomenon", but no quantitative agreement between models and observations can yet be claimed.

If an accreting protostellar core attains a mass in the upper main sequence mass range, probably having already become a main sequence star, its very high luminosity almost certainly begins to affect the dynamics of the infalling envelope. There are a number of ways in which the radiation from a massive protostellar core can cause the envelope to be blown off, and these probably set a limit of less than 100 $M_\odot$ to the mass with which a star can form (Larson & Starrfield 1971). Larson & Starrfield estimated that the most important limiting effect would be the ionization of the envelope when the core becomes an O star in the mass range 30 - 60 $M_\odot$. However, in a more detailed analysis of the effect of radiation pressure, Kahn (1974) concluded that radiation pressure would be more important and would limit the mass that can be attained by accretion to about 40 $M_\odot$, depending on the properties of the dust in protostellar envelopes. The models of Westbrook & Tarter (1975) for massive protostars show that an increasing fraction of the mass is blown off by radiation pressure as the total mass is increased; for example, a 50 $M_\odot$ model loses 35 $M_\odot$ in this way, leaving a core of only 15 $M_\odot$. In a detailed numerical study of the dynamics of protostellar envelopes, Yorke & Krügel (1976) found that the most important effect limiting the core mass is radiation pressure acting on the outer part of the envelope, and in an example with a total mass of 150 $M_\odot$, a star of only 35 $M_\odot$ was formed and the remainder of the material was dispersed by radiation pressure.

From all of these studies it is clear that it is difficult to form massive stars because a variety of effects, including radiation pressure and perhaps also stellar winds, will tend to blow off the protostellar envelope before all of it has been accreted on the core. These effects could plausibly account for the steep decline in the stellar mass function at large masses and for the apparent absence of stars with masses exceeding 100 $M_\odot$; indeed, there may even be some difficulty in understanding how the most massive observed stars can form. A very dense protostellar envelope is required to overcome the effects of radiation pressure or stellar winds and allow accretion to continue; thus it may be that the most massive stars form only in regions of unusually high density. Nonspherical accretion may also play a role; if a protostellar envelope has large density inhomogeneities or develops a flattened disc-like structure, the effects of radiation pressure, etc., may more easily be overcome.

7. PROTOSTELLAR ENVELOPES AND OBSERVED PROPERTIES

During most of its evolution, the protostellar core is completely obscured by the optically thick infalling envelope, and the observed properties of the system depend strongly on the structure of the envelope. The radial density variation in a spherical accretion flow has the form $\rho \propto r^{-3/2}$, and the envelope density distributions in spherical collapse models approach this form during the later stages of the collapse. Shu (1976) has shown that the development of an accretion flow with this type of density distribution can be approximately described by a similarity solution (which is of a qualitatively different nature from the similarity solution proposed for the earlier isothermal stages of collapse by Larson (1969a) and Penston (1969).) Until the protostellar envelope is almost completely accreted or dispersed, the luminosity generated in the accretion shock or in the core is absorbed by dust grains in the envelope and converted to infrared radiation, so that the emitted spectrum of the protostar depends on the transfer of infrared radiation through the extended optically thick envelope.

Approximate calculations of the spectrum of radiation emitted from protostellar envelopes with power-law density distributions have been made by Larson (1969b) and Rowan-Robinson (1975), and a more detailed numerical solution of the transfer problem has been made by Bertout (1976). In these studies the dust opacity was simply assumed to vary with a power of the wavelength, so only the gross features of the spectrum can be represented. The predicted spectrum resembles a blackbody spectrum, except that there is more emission at long wavelengths. As the protostar evolves and the envelope density decreases, the "photosphere" or surface of optical depth unity moves inward to higher temperatures, and the wavelength of peak emission decreases. The corresponding evolutionary tracks in an infrared HR diagram (Larson 1972b) fall in the same region as a number of infrared sources which are believed to be young objects or protostars.

Qualitative but not quantitative agreement is found between the predicted spectra and observed infrared spectra, which are generally broader than a blackbody spectrum and also show absorption features near 3 and 10 microns due to water and silicates. A more detailed treatment of the dust opacity is evidently required to reproduce such features, and Finn & Simon (1976) find that the 10 micron feature can be closely reproduced by a model with silicate grains in a protostellar envelope having a power-law density distribution. Another important factor is the geometrical structure of the envelope and the distribution of dust in it. For example, Cohen (1973) has obtained better fits to the observed infrared spectra by using more elaborate double shell models. Such a "double cocoon" structure is in fact predicted by the detailed envelope models of Yorke & Krügel (1976), which have an outer shell of ice grains at a temperature of $<$ 200 °K and an inner shell of refractory grains at a temperature of $\sim$ 1000 °K. Deviations from spherical symmetry of protostellar envelopes, such as a clumpy or flattened structure, are probably also important in producing regions with a range of temperatures and hence in making the spectrum broader than a blackbody.

When the protostellar envelope is no longer completely optically thick, a double-peaked spectrum is predicted showing contributions from both the envelope and the core, which by now is essentially a conventional hydrostatic pre-main sequence star. The predicted spectra qualitatively resemble those of a number of T Tauri stars, such as T Tau, R Mon, and R CrA, which show in varying degrees separate infrared emission peaks probably attributable to circumstellar dust. If accretion is still important, radiation from the hot layers inside the accretion shock may make an important contribution to the spectrum at short wavelengths (Walker 1972, Ulrich 1976), where enhanced emission is often observed.

At present it is controversial whether any features in the optical spectra of T Tauri stars can be explained by the effects of infall from a remnant protostellar envelope. The conventional interpretation of T Tauri emission line profiles is in terms of outflow of matter, and it has been argued that this outflow is important for the final dispersal of protostellar envelopes (Strom et al. 1975). Only a few stars, the so-called YY Ori stars (Walker 1972), have been found in which the line profiles sometimes seem to indicate infall rather than outflow. However, Ulrich (1976) has challenged the conventional interpretations, and has shown by a detailed study of the kinematics and radiative transfer in an infall model that some spectral features usually attributed to outflow can also be explained as radiation from the accretion shocks in an infall model. Lynden-Bell and Pringle (1974) have also proposed an accretion disc model to explain some of the properties of T Tauri stars. As we have seen, the formation of accretion discs seems to be an almost inescapable result of the collapse process, and provides an attractive framework for understanding the formation of planetary systems. However, none of the available collapse calculations have incorporated the effects of stellar winds, largely for want of any quantitative understanding of this phenomenon, and it is entirely possible that such winds could eventually dominate the dynamics and reverse the infall in protostellar envelopes, perhaps even before the core becomes visible, in which case infall would never be observed.

Many interesting phenomena related to star formation appear to be associated with the final dispersal of protostellar envelopes. For example, Strom et al. (1975) have suggested that winds from obscured T Tauri stars first clear out holes in the surrounding cloud material, allowing these stars to illuminate or excite patches of nebulosity which are then observed as Herbig-Haro objects. Herbig (1970), Field (1974), and Silk (1976) have suggested that the dissipation of protostellar envelopes or "solar nebulae" may provide an important source of dust grains for the interstellar medium. The possible separation of gas and dust in protostellar envelopes has been studied by Edmunds & Wickramasinghe (1974), and the formation and dynamics of dust shells or "cocoons" have been studied by Davidson (1970), Burke & Silk (1976), Yorke & Krügel (1976), and Cochran & Ostriker (1976). These dust shells are driven outward by radiation pressure, and this phenomenon may play a dominant role in the final evolution of protostellar envelopes and in determining the upper mass limit for star formation. It is perhaps also in these cocoons that the conditions required for OH and H_2O maser emission are produced (de Jong 1973).

8. SUMMARY

Because of the complexity of the dynamics of collapsing clouds, the existing crude and/or idealized calculations cannot be expected to provide detailed or definitive predictions of their evolution, but should be regarded as providing illustrative examples of how collapse might proceed in various idealized cases. Nevertheless, a number of qualitative features are predicted with considerable generality in many of these calculations. In all cases, if a cloud collapses at all it does so in a nonuniform fashion, developing a centrally condensed or lumpy structure. During the earliest stages of 3-dimensional collapse the cloud often shows a bar- or spiral-shaped structure, in which condensations later form. An individual collapsing region or cloud fragment may form a single dense core, but the available results suggest that a more common outcome is the formation of a binary system of two orbiting centers of condensation. Once a dense core has formed, it grows in mass by accretion from the surrounding envelope, finally becoming a conventional hydrostatic star when all of the surrounding material has been accreted or dispersed.

All calculations indicate the importance of accretion processes of some form for star formation. In cases where an accreting core is relatively isolated, it seems almost inescapable that a flattened, rotating accretion envelope or accretion disc will form around it. It is in this type of situation that many problems related to star formation must be treated; for example, the "angular momentum problem" becomes the problem of understanding how angular momentum is transported in accretion envelopes or discs by viscous or other torques. Various studies of the dynamics of discs in other contexts may thus turn out to be relevant to star formation as well. Also, if the solar system may be viewed as a remnant of the solar accretion disc, it may ultimately yield valuable information about the way in which the sun formed.

The calculations indicate that the size of the fragments that form in a collapsing cloud is given approximately by the Jeans criterion, as expected classically. Because of the very non-homologous collapse of the fragments, little tendency for further fragmentation beyond the formation of binary or triple systems is observed. However, fragmentation can be enhanced by a significant overall collapse of the cloud; because collapse velocities are generally supersonic, this means that shock compression can be important for fragmentation into small masses. Also, because of the sensitivity of collapse results to the assumed temperature, it is clear that any cooling occurring during the collapse will be important for fragmentation; this effect is not present in most of the calculations, which have assumed isothermal collapse. The importance of understanding the dynamical and thermal evolution of clouds prior to the onset of collapse is also evident.

On the basis of rather general time-scale arguments, it may be anticipated that newly formed stars of low mass first become visible as pre-main sequence stars on the lower part of their Hayashi tracks, while massive stars do not become visible until they are already on or near the main sequence. During the very earliest stages of stellar evolution, the observable properties of stars are predicted to be dominated by the remnant protostellar envelopes around them, which absorb or scatter much of the stellar luminosity and reradiate it at infrared wavelengths. These predictions seem to be consistent in a general way with the observed properties of newly formed stars, but because of the many uncertainties and complexities of detail, more quantitative predictions and comparisons with observations can probably not yet be considered significant.

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