Limitations in Helicopter Design

By J. A. J. BENNETT, D.Sc., F.R.Ae.S.

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H. A. MARSH, A.F.C., A.F.R.Ae.S., IN THE CHAIR.

INTRODUCTION BY THE CHAIRMAN

Ladies and Gentlemen,

As most of you know this is the Fourth and last lecture of our present session, the other three having been given by Gp/Capt. LIPTROT, Mr. C. G. PULLIN and Mr. R. A. C. BRIE, covering an historical survey, design and operation of helicopters, etc., and to-day our lecturer is discussing "Limitations in Helicopter Design."

It gives me very great pleasure to introduce Dr. BENNETT to you, for I have known him personally for fifteen years or more, during the whole of which time he has been intimately associated with rotary wing and helicopter design.

For some years prior to the war, Dr. BENNETT was with Messrs. G. & J. Weir and the Cierva Autogiro Co.; and during the war was C.T.O. at the Airborne Forces Experimental Establishment and spent two years in the U.S.A. where he latterly was attached to Wright Field as project engineer on helicopter research. He is at present head of the Helicopter Branch of the Fairey Aviation Co. Ltd., and besides his many academical qualifications he is a F.R.Ae.S., a Founder Member and a Member of the Council of our Association, a Member of the Helicopter Committee of the Aeronautical Research Council and a Member and past Vice-President of the Helicopter Society of America.

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It can be readily seen that he is eminently suited to talk to us on the subject he has chosen.

On behalf of the Association it gives me great pleasure to welcome our guests this afternoon, who I feel sure will be well rewarded for coming along.

DR. J. A. J. BENNETT

MR. CHAIRMAN, Members of the Helicopter Association and Guests, I regret that I have chosen as the subject of my talk: "Limitations in Helicopter Design." I think that a more appropriate title for this afternoon would have been: "Limitations in Design for Living." However, we have already heard enough about that from the Ministry of Fuel and I shall endeavour to confine my further remarks to the subject of helicopters. I should like to say first of all how gratifying it is, to those of us who were concerned with rotary wing development before the War, to witness, in the immediate post-war period, a general acceptance of the helicopter as a practical aircraft, and a widespread enthusiasm for its unique characteristics. There is no longer any doubt about the future possibilities of the helicopter, thanks to the intensive work of IGOR SIKORSKY and his colleague MICHAEL GLUHAREFF, under the direction of whom the helicopter first became a fully-fledged flying machine.

In spite of this historic achievement and the subsequent production of military helicopters in the United States, I believe that we still have a long way to go in the investigation of the basic problems of rotary wing flight. Fortunately, due to the work of JUAN DE LA CIERVA and those who had the foresight to sponsor his experiments, there is available a background of rotary wing experience that helps in the appreciation of the inherent limitations which have impeded, and continue to impede, progress in helicopter development. I propose this afternoon to discuss briefly some of these limitations in the course of a general review of the present technical position.

The hinged rotor blade

Probably no other single feature contributed more towards the achievement of practical rotary wing flight than did the flapping hinge. Although it had been described in early helicopter patents as a means for suppressing vertical bending at the root of each blade, it became of primary importance for single rotor aircraft in balancing the dissymmetry of lift in forward flight (Fig. 1), which with rigidly-mounted blades caused a rolling couple of increasing magnitude as the forward speed increased. In the words of the main claim of the original Autogiro patent, the intention was that the

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rotating wing (Fig. 2) should adopt at every instant the position required for equilibrium between the centrifugal force produced by the speed of rotation and the lift due to the action of the wind on the wing. This intention was frustrated, however, by the inertia of the blade.

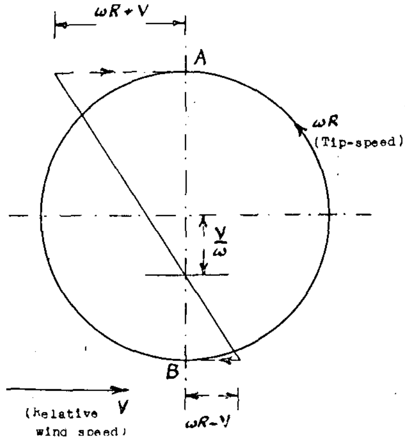

Fig. 1. Dissymmetry in forward flight.

Inertia imposes a natural oscillation (like that of a pendulum) on the motion of the blade about the flapping hinge, thereby preventing a condition of equilibrium between centrifugal force and lift. If the angle of the blade about the flapping hinge in any azimuth be denoted by x and the rotor

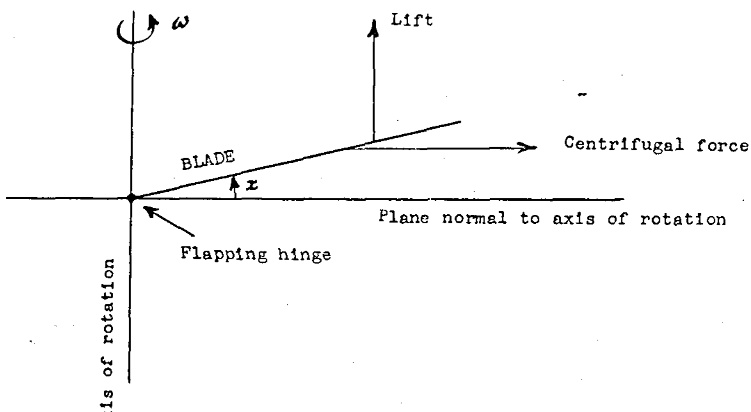

Fig. 2. Hinged rotor blade.

angular speed by w, the equation of motion of the free undamped oscillations is $\ddot{x} + w^2 x = 0$ which is identical with that of a simple pendulum of frequency w. Hence the natural frequency in flapping is equal to the speed of rotation.

Now the periodic disturbing force due to the dissymmetry of forward flight has the same frequency and is, therefore, in resonance with the natural flapping oscillation of the blade. It is when resonance occurs in any mechanical vibration that there is a phase lag between the disturbing force and the resulting displacement of exactly 90°. That is why, in a gyroplane, the flapping displacement of the blade is greatest after it has passed the nose of the machine although the position of maximum lift is on the advancing side of the rotor disc. Instead of the tip-path plane tilting laterally due to the dissymmetry of forward flight, it is tilted mainly longitudinally. Flapping, therefore, exchanges rolling of the aircraft for pitching. This phase lag between lift variation and flapping has been a basic limitation in rotary wing design, affecting adversely control response and, by inducing longitudinal dissymmetry in forward flight, affecting also stability, trim and rotor vibration.

The phase angle of 90° is unaffected by damping of the flapping motion so long as the ratio between the forced and natural frequencies $\frac{W}{W_n}$ is unity. This can be seen by reference to standard textbooks on the subject of vibration, where the amplitude of forced vibration and phase angle between force and displacement are shown (Fig. 3) as functions of

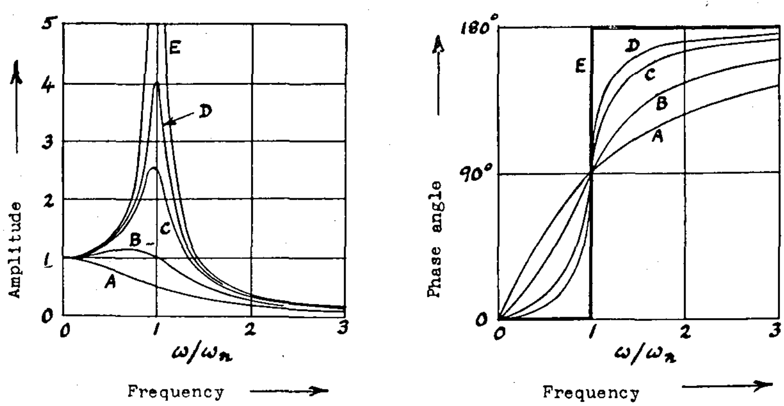

Fig. 3. Amplitude and phase angle of forced vibration

the frequency ratio for different values of damping. The equation of the damped free flapping motion is of the form:

$$I\ddot{x} + c\dot{x} + kx = 0$$

where the three terms represent respectively inertia, damping and restoring moments. The blade is, therefore, not merely in equilibrium between lift and centrifugal force but between these three moments. If the natural frequency were altered, for example, by locating the flapping hinge outboard from the axis of rotation or by inclining the flapping hinge in the plane of rotation or by restraining the motion elastically by means of a spring, the phase angle between the periodic disturbing force and flapping displacement would no longer be 90°, and it would be dependent also on damping.

Flapping, Hunting and Feathering

These three terms define the angular oscillation of the blade about three mutually perpendicular axes. Flapping is the angular oscillation of the blade about a pivot (the " flapping hinge ") which allows the zenithal angle of the blade to be varied; hunting is the angular oscillation of the blade about a pivot (the "drag hinge") which allows the blade to be displaced angularly in azimuth with respect to the rotor head; and feathering is the angular oscillation of the blade about a pivot (the "feathering hinge") which allows the blade angle to be varied.

We have seen how the flapping phase angle causes the tip-path plane to tilt longitudinally whenever there is lateral dissymmetry of lift, and of course to tilt laterally for any longitudinal dissymmetry of lift. The blade, therefore, rises and falls cyclically, thereby causing a variation in incidence

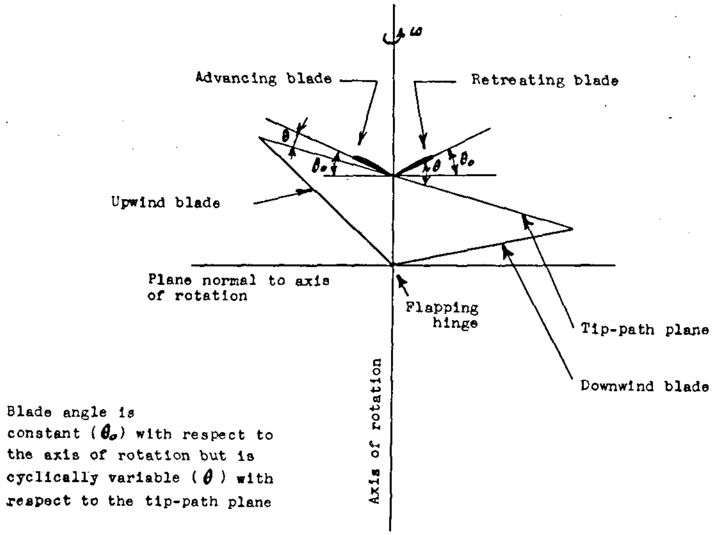

Fig. 4. Cyclic feathering with respect to the tip-path plane due to flapping.

with respect to the tip-path plane, i.e., cyclic feathering (Fig. 4). As flapping is merely a method of providing cyclic feathering of the blades with respect to the tip-path plane, feathering hinges can be used to do the same thing, in which case the tip-path plane need not be allowed to tilt but may remain at right angles to the axis of rotation.

The tilting of the tip-path plane causes a further dissymmetry (Fig. 5), viz. a cyclic force tending to oscillate the blades in azimuth. This force arises from the inertia of the blade and is known in rigid dynamics as the Coriolis force, because it was discovered by CORIOLIS more than a century ago as the component force normal to the path of a given mass moving in a rotating plane. Hence the necessity for a drag hinge, which relieves the blade root from the periodic bending moments which tend to promote blade failure by fatigue. However, the introduction of the drag hinge, giving each blade an additional degree of freedom of oscillation, has been responsible for rotor vibration troubles probably more than any other step in rotary wing development. The low natural frequency of one of the possible modes in which the blade may oscillate about the drag hinge is not very far removed from the frequency of the Coriolis forces, i.e., the angular

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speed of the rotor, with the result that the hunting oscillation is unstable unless the freedom of movement of the blade about the drag hinge is is restricted by damping or by a spring restraint to increase the natural frequency of the hunting oscillation. Hunting instability manifests itself as a violent rocking of the aircraft during starting or stopping of the rotor on the ground under the additional restraint and damping of the undercarriage, and on take-off or landing with a forward run, and there have fortunately been very few occasions of such instability during flight. With

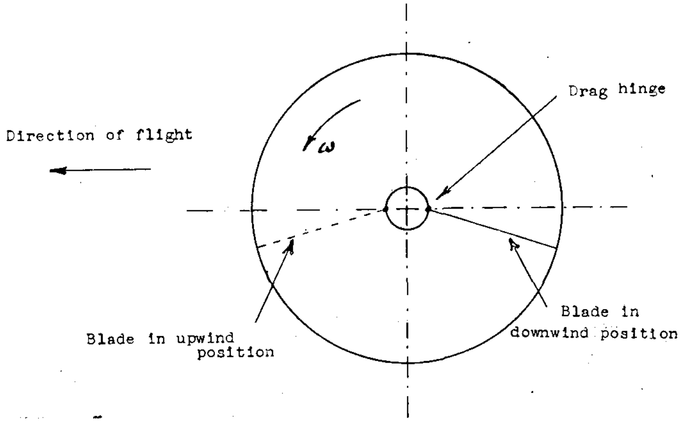

Fig. 5. "Hunting" oscillation of blade.

insufficient damping, the oscillations can increase in amplitude sufficiently to cause severe damage to the aircraft, and not the least perturbing feature of the instability is that it is self-excited. Hence the drag hinge, originally conceived as an assurance against blade fatigue, has become a potential instrument of self-destruction and, although it may be said to have successfully overcome one of the limitations of the flapping hinge, with which it forms a universal joint, it has achieved this result only at the expense of another limitation equally as critical as the first.

The Rigidly-mounted Blade.

Owing to the foregoing difficulties with hinged blades, investigations are currently in progress into alternative methods of balancing the dissymmetry of lift in forward flight. In single rotor aircraft, the feathering hinge can effectively replace the flapping hinge for maintaining lateral trim, in which case the absence of Coriolis forces in steady flight renders the drag hinge unnecessary. With twin laterally-opposed rotors, lateral trim is, of course, no longer dependent on cyclic feathering, and the blades may be rigidly-mounted as in the orthodox propeller. That is not the end of flapping and feathering, however, or of hunting, because, even in the absence of a flapping hinge, the blade partially flaps owing to its own longitudinal flexibility. On the other hand, even in the absence of a feathering hinge, the blade partially feathers on account of periodic twist within the blade itself. The pitching moment of the aerofoil may be zero, the blades may be designed to be straight for a given condition of flight, e.g., hovering, but the bending deflection at the blade tip in other conditions of flight combined with variation in torque causes internal feathering and there is also a pitching moment tending to diminish the blade angle owing to the leading edge of the aerofoil being at a greater coning angle than the trailing edge. This pitching moment obviously increases with blade angle and chord.

The rigidly-mounted blade is subject to periodic stress arising from:

(a) periodic variation of lift in forward flight;

(b) gyroscopic moments; and

(c) cyclic flap control (if blade flaps are provided instead of feathering hinges) including the longitudinal control necessary for trim at low airspeeds.

If rigidly-mounted blades could be built with sufficient structural strength to withstand the combination of periodic bending and the steady centrifugal force load, they would appear to have a number of advantages over hinged blades:

(a) smoothness in operation due to the suppression of the unstable hunting oscillations;

(b) light control forces owing to the blade loads being taken through the aircraft structure and substantially isolated therefore from the controls;

(c) improved dynamic stability by the location of the rotor axes at a considerable distance from the centre of gravity of the aircraft (15% of the rotor radius has been achieved); and

(d) elimination of the sluggishness inherent in the control response of hinged blades.

The Two-bladed Rotor.

Few factors affect the cost of helicopter manufacture more than the present limitation in the number of blades. The simplification of hub and controls that results from a reduction in the number of blades from three to two would make the two-bladed rotor an attractive arrangement were it not for the associated rotor vibration. This vibration is due fundamentally to the variation of the lift, drag and pitching moments of each blade element in proportion to the square of the speed which is itself periodic. These moments are therefore not only cyclic but bicyclic and may be balanced for all practical purposes in a three-bladed rotor but not in a two-bladed one.

Considering a blade element at distance r from the rotor axis, the velocity is

$$w\ r + V \sin A$$

where w is the rotational speed, V the translational speed and A the blade azimuth angle. The lift L_1 of one element is therefore proportional to this quantity squared and may be written

$$L_1 = a + b \sin A - c \cos 2A$$

i.e., the algebraic sum of a constant term, a cyclic term and a bicyclic term.

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In a three-bladed rotor, the blades being displaced in azimuth by 120°, the lift of similar blade elements can be expressed as

$$L_2 = a + b \sin (A + 120°) - c \cos 2 (A + 120°)$$

and

$$L_3 = a + b \sin (A + 240°) - c \cos 2 (A + 240°)$$

The sum of L_1, L_2, and L_3 is a constant quantity, viz., 3a, all the periodic terms disappearing.

For a two-bladed rotor, the corresponding expressions are

$$L_1 = a + b \sin A - c \cos 2A$$

and

$$L_2 = a + b \sin (A + 180°) - c \cos 2 (A + 180°)$$

The sum of L_1 and L_2 is

$$2a - 2c \cos 2A$$

which is no longer a constant quantity but contains a term of twice rotor frequency.

Similar expressions are obtained for the variation of lift, drag and pitching moments. Obviously the bicyclic variation of these moments cannot be eliminated by cyclic pitch variation but would require a bicyclic pitch variation. There are, therefore, in a two-bladed rotor, inherent forced vibrations of twice rotor frequency which result from the additive vibration from each blade. The bicyclic vibrations from the several blades of a rotor having more than two blades are in counterphase, and rotors having three blades or more are not subject to this limitation.

It should be mentioned here that, although no means have yet been developed for eliminating these vibrations at the source, they have been rendered comparatively innocuous through the employment of vibration isolation methods. A flight survey has shown that it is possible to minimise the transmission of vibration to the fuselage from a two-bladed rotor sufficiently for practical purposes in small helicopters designed specially to overcome this limitation.

Another method that has been tried is the counter-phasing of two rotors, i.e., counterbalancing the vibration from a two-bladed rotor by an equal and opposing vibration from another similar rotor. As this procedure necessitates the use of four blades, it can scarcely be considered a solution of the two-bladed rotor problem. Except for the counter-rotation of twin rotors to give torque balance, which makes counter-phasing of vibration more difficult, the vibration of twin rotors would be equivalent to that of a single rotor with double the number of blades.

The Single-Rotor Torque Problem

Torque balance is one of the most troublesome limitations of the single main-rotor helicopter and has led to the use of twin contra-rotating rotors, even in small helicopters, in spite of their mechanical complexity. It is a fundamental axiom in dynamics, as expressed by NEWTON in his third law of motion, that to every action there is an equal and opposite reaction. If the blades are driven by the application of torque at the rotor hub, the fuselage will spin in contra-rotation unless prevented by an external couple such as that arising from the thrust of an auxiliary tail rotor. The torque of the main rotor, being the ratio of the power applied at the hub to the angular speed, can be reduced only by applying some of the available power elsewhere or by increasing the angular speed. The latter alternative is limited by the blade-tip speed which must remain well below the speed of sound to avoid high profile drag losses and by the disc loading which requires a relatively large rotor diameter if a high induced drag loss is to be avoided.

Hence the problem is resolved into finding a method of absorbing some of the available power usefully elsewhere. The powered glider with rotating wings (Fig. 6) conceived by JUAN DE LA CIERVA, was one solution. Here the whole of the available power was applied to a forward propeller which ensured the necessary translational speed to keep the rotor revolving

Fig. 6.

at zero torque. A large portion of the rotor blade, acting as a windmill (Fig. 7) absorbed sufficient energy from the air to propel the remainder of the blade (Fig. 8) which, acting in the helicopter state, expended energy on the air. The limitation of the Autogiro as an aircraft of practical utility is that it is fundamentally a glider and can ascend only on tow. Whenever the towing force of the propeller ceases, it must descend. This fact becomes most apparent at take-off and on landing in confined areas, when manœuvres to utilise momentarily the kinetic energy of the rotor are necessary to enable the Autogiro to take-off and land in still air without a forward run. The inertia take-off is only a palliative, accentuating the necessity of applying power directly to the rotor.

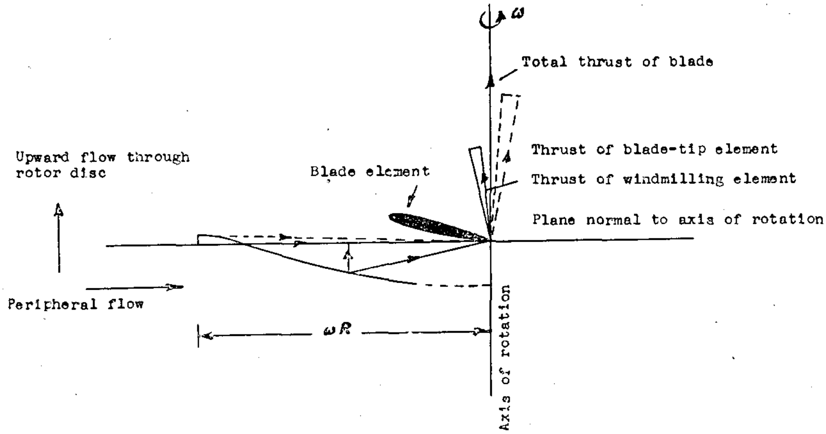


Fig. 7. The mechanism of autorotation.

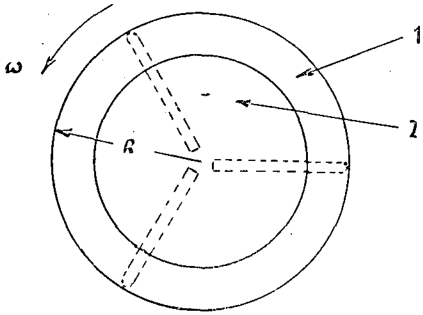
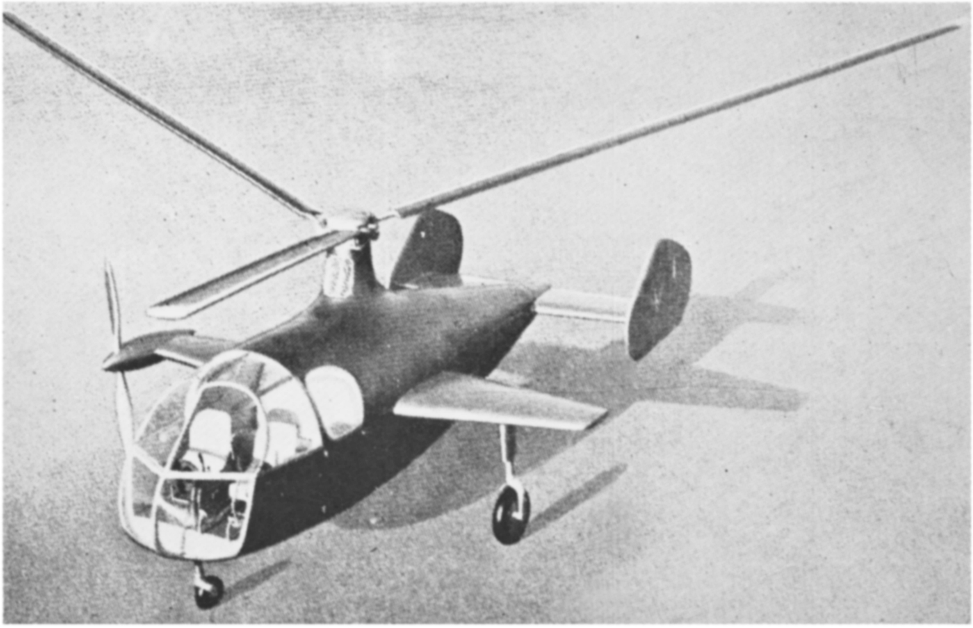


Fig. 8. Torque balance of blade elements in autorotation.

1. *Portion of rotor disc where energy is expended on the air.*

2. *Portion of rotor disc where energy is absorbed from the air.*

Fig. 9. Gyrodyne.



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The gyrodyne (Fig. 9) is an alternative arrangement in which this limitation of the Autogiro may be overcome without sacrificing the advantages of low pitch. The greater part of the available power is utilised for rotor thrust in vertical flight, the remainder being supplied to the propeller which is located outboard for balancing the rotor torque reaction. It is true that in this arrangement power is wasted in vertical flight but the forward thrust of the propeller contributes to propulsion of the aircraft. Provided the disc loading and the available power loading are sufficiently low, the gyrodyne is capable of slow vertical take-off and landing which are the unique advantages of the helicopter.

Similar in principle to the gyrodyne method of balancing torque is the auxiliary tail rotor arrangement developed by SIKORSKY. In one respect this arrangement overcomes the main limitation of the gyrodyne, viz., the large proportion of power applied to the non-lifting propeller, which is useful only in forward flight. To minimise this power loss, the non-lifting propeller must be located as far as possible from the rotor axis and it is then more convenient to support it at the tail than on a long lateral outrigger. There it is known as a rotor rather than a propeller because its thrust is normal to the line of flight and it is subject to similar dissymmetry in forward flight as the main rotor. In fact, the machine is really a twin rotor helicopter. A limitation of this arrangement is the long tail-rotor transmission with the necessary supporting structure and a relatively high disc loading which results in a high induced power loss in vertical flight.

Torque reaction may be avoided entirely by driving each blade at or near the tip instead of at the hub. There are a number of alternative

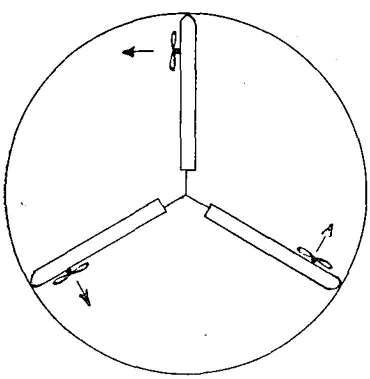

Fig. 10. *Blade-mounted propellers.*

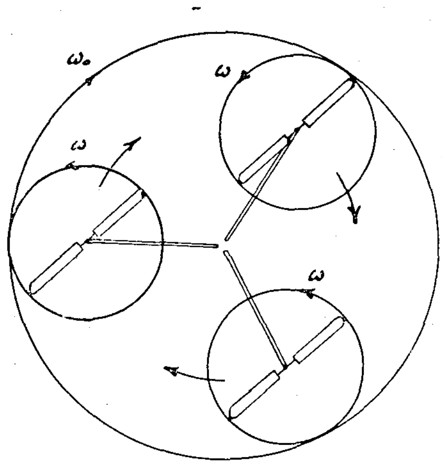

Fig. 11. *Epicyclic rotors.*

arrangements which could conceivably be used to produce thrust at the blade tip. Propellers (Fig. 10) have in the past been mounted on the rotor blades for this purpose and recently blade-tip jets have enabled vertical flight to be attained by an aircraft that is designed to fly normally as an Autogiro. The high propeller losses, gyroscopic couples and power transmission problems are limitations of blade-mounted propellers and the principal defect of blade-mounted jets at present is concerned with fuel consumption. If and when the fuel combustion problem of small jet units developing relatively low thrust is solved, jet propulsion of helicopter rotors may become a practical proposition. Instead of blade-mounted propellers, one could visualise blade-tip rotors (Fig. 11) which would be powered similarly to the propellers and would rotate epicyclically about the main hub axis, the necessary propulsive thrust being obtained either from the additive torque of the epicyclic rotors or by inclining their tip-path planes in the direction of rotation of the main rotor. A further method is to replace each blade by a system of power-driven paddle blades rotating about a radial axis, the necessary thrust for rotation about the main axis being produced by cyclic pitch adjustment of the paddle blades as in the cyclogyro.

For given values of tip speed, power loading and disc loading, torque increases as the cube of the linear dimensions. Hence the torque problem becomes rapidly worse with increase in size and, unless torqueless rotors can be developed to a practical stage, the use of a single rotor is confined to relatively small machines, large helicopters requiring multiple rotors.

Limiting Power Loading

The minimum power P that would be absorbed by a rotor supporting a gross weight W, if the profile drag were zero, would be that required to give a uniform induced velocity v, where

$$P = Wv.$$

The extreme limit of power loading $\frac{W}{P}$ is equal therefore to the reciprocal of the induced velocity. According to the momentum theory of airscrews, the induced velocity v in ft./sec. at sea level is related to the disc loading w in lbs./sq.ft. by the equation

$$v = 14.5\sqrt{w}$$

Hence the power loading has, as an extreme limit, the value 1/v or, in lbs./h.p.

$$\frac{W}{P} = \frac{38}{\sqrt{w}}$$

A rotor of disc loading 2.3, for example, would not be expected ever to support more than 25 lbs. per horse power in hovering flight away from the " ground cushion ".

In practice, the distribution of lift is far from uniform over the blade length and, as a result, the effective disc loading is greater than the nominal value. In other words, in current helicopters, sustentation is caused by the downward acceleration of air by the rotor blades over an annulus (Fig. 12) of the disc and not over the whole disc. This effect causes the induced

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velocity to increase by a factor which appears at present to be about $\frac{2}{\sqrt{3}}$ The induced power loading is therefore, reduced by the reciprocal factor to about $\frac{33}{\sqrt{w}}$.

Fortunately, although the effect of lift distribution is to increase the induced velocity and, therefore, to adversely affect the induced power loading, the induced velocity is decreased at take-off or landing because the

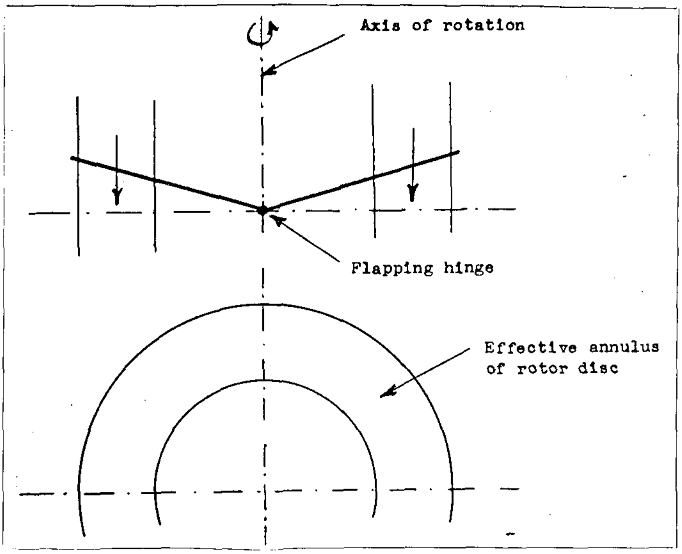

Fig. 12. *Effect of non-uniform flow distribution.*

air is compressed between the rotor and the ground. This cushioning effect of the ground may well enable a helicopter which is overloaded and cannot hover in still air away from the ground, to take-off and land vertically within the " ground cushion ".

By definition, the thrust and torque coefficients C_T and C_Q respectively, based on disc area and tip-speed, are related by the equation

$$\frac{W}{E.H.P.} = \frac{C_T}{C_Q} \frac{550}{wR}$$

which can be written

$$\frac{W}{E.H.P.} = \frac{26.8}{\sqrt{w}} \frac{C_T^{\frac{3}{2}}}{C_Q}$$

The ratio of the actual limiting power loading $\frac{W}{E.H.P.}$ to the extreme limit of power loading $\frac{W}{P}$ (for zero profile drag and uniform induced velocity distribution) is

$$\frac{P}{E.H.P.} = \frac{26.8}{38} \frac{C_T^{\frac{3}{2}}}{C_Q}$$

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This ratio of limiting induced power to actual power is sometimes referred to as the "figure of merit" or "efficiency" of the rotor and is roughly about two-thirds in hovering flight for the optimum value of $\dfrac{C_T^{\frac{3}{2}}}{C_Q}$

Hence the limiting power loading, taking profile drag into account, is given approximately as

$$\frac{W}{E.H.P.} = \frac{25}{\sqrt{w}}$$

For example, a rotor of disc loading 2.3 should support between 16 and 17 lbs. per horse-power in hovering flight, at sea level conditions, away from the ground cushion.

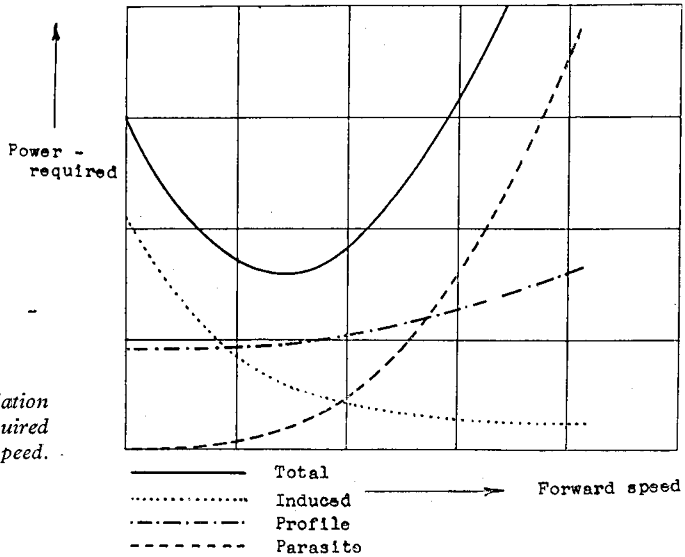

Fig. 13. *Variation of power-required with forward speed.*

When the power loading for vertical flight is exceeded, direct take-off is possible either with the assistance of the ground cushion or by making an inertia take-off as in the jump-start Autogiro. The helicopter may also make a tangential take-off like an aeroplane. The limiting power loading for a take-off with forward speed is nearly double that at zero forward speed (Fig. 13) because, at the forward speed (usually about 50 m.p.h.) where the power required is a minimum, the induced power is only a fraction of its value for hovering flight whereas the profile drag and the parasite drag change the power required relatively little at low airspeeds.

Pitch Limitations

Variation in rotor pitch is required

(a) to vary the rotor power independently of angular speed ;
(b) to compensate for the change in angle of attack of the blades due to the variation in axial flow through the rotor disc with forward speed; and
(c) to compensate for the change in density of the air with altitude.

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The necessity for control simplification has resulted in the development of devices for governing rotor pitch or engine speed, thereby eliminating a separate pitch control or a separate throttle control respectively. In the one case there is an automatic pitch reduction in the event of power failure and, if this occurs near the ground, the sudden reduction in pitch and consequent loss of lift before steady autorotation is established may cause the helicopter to land at an excessive sinking speed. In the other case, the pilot retains control of pitch and may utilise the kinetic energy of the rotor for momentary hovering prior to a power-off landing, but there is a risk of over-control in flight. When this occurs, an increase in pitch causes the aircraft to sink rather than climb, the rotor blades slowing down and eventually stalling.

The immediate reduction in pitch on power failure is an essential requirement for those helicopters which operate at a pitch beyond that at which the blades will autorotate. A simple calculation shows that the kinetic energy of the rotor is sufficient only for a very few seconds of power-off flight when the limiting autorotative pitch is exceeded. For a rotor of total blade weight equal to six per cent. of the gross weight (W lbs.) of the aircraft, the kinetic energy in lbs. ft. is roughly $\frac{W}{3000}$ times the square of the tip speed in ft./sec. If the E.H.P. at the operating speed is six per cent. of the gross weight, energy is absorbed by the blades at the rate of 33 W lbs.ft./sec. Therefore, the whole of the kinetic energy of the rotor is expended by the blades every *t* secs. where

$$t = \frac{1}{10}\left(\frac{wR}{100}\right)^2$$

i.e., two and a half seconds if the tip speed is 500 ft./sec. In other words, the inertia of the rotor will not prevent the blades from rapidly decelerating at high pitch. Hence the necessity for automatic pitch reduction in the event of power failure.

The risk in operating helicopters beyond the limiting autorotative pitch can be lessened by the provision of twin power plants, but it is not essential for helicopters to take this risk at all. The power can be absorbed equally well at low pitch by a rotor of low blade loading or high tip speed, except at very high altitudes.

A further high-pitch limitation is one associated with the forward inclination of the rotor for propulsion (Fig. 14). The rotor disc then makes a negative angle of incidence with respect to the flight path, thereby causing the axial flow through the rotor to increase with forward speed and change the blade angle of attack unequally from root to tip. Least affected by a change in axial flow is the tip portion of the blade. Consequently, when the main collective pitch of the blades is increased to compensate for the increased axial flow, the blade angle at the tip becomes excessive and may approach the stall cyclically at high transational speeds where the angle of attack on the retreating blade is already high due to blade flapping or cyclic feathering. This periodic variation in lift distribution at maximum speed not only impairs the propulsive efficiency of the forwardly inclined rotor but limits the operation of the helicopter to the inherent roughness at the higher airspeeds.

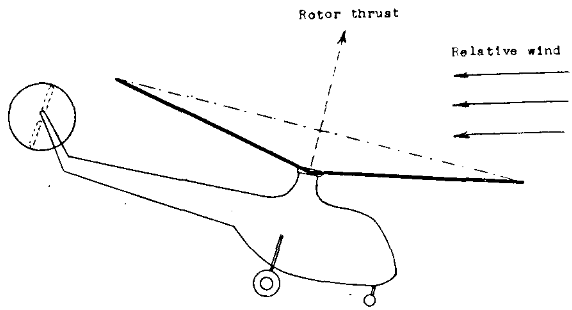

Fig. 14.
Propulsive rotor.

Limiting Translational Speed

Although the compressibility of the air no longer appears to be an insuperable barrier to the speed of fixed wing aircraft, it is still a serious limitation in regard to the horizontal speed attainable by helicopters. Even if boundary layer control and jet propulsion can be applied successfully to rotor blades, the fluctuation in relative air speed and therefore in lift coefficient at the blade tip is so great at high forward speeds that bending and torsional deflections of the blade and their effect on vibration and airworthiness of the aircraft would appear to place a definite limit on

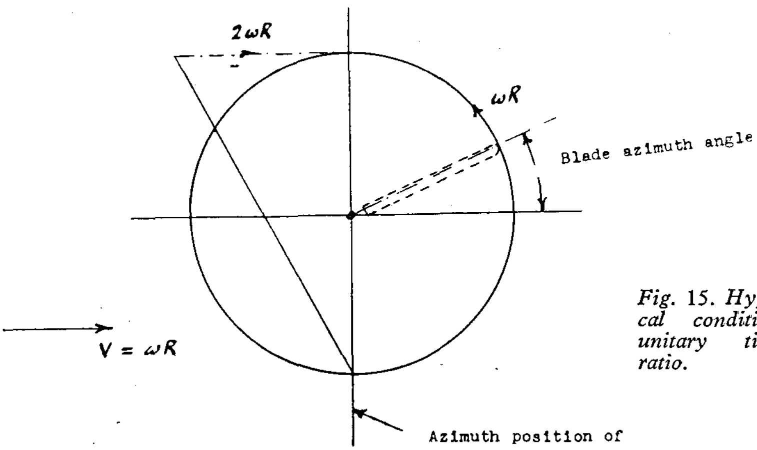

Fig. 15. *Hypothetical condition of unitary tip-speed ratio.*

forward speed. This limit is governed by the maximum relative air speed permissible at the tip of the advancing blade and it is not anticipated that this will exceed about 3/4 of the speed of sound.

With regard to tip-speed ratio, i.e., the ratio of forward speed to the steady peripheral speed at the blade tip, this ratio was as high as 2/3 in early Autogiros and it has been suggested that it could ultimately be increased to unity (Fig. 15) in which case the tip of the retreating blade would have zero air speed and the remainder of the retreating blade would have negative air speed. The lift of each blade would then have to fluctuate from zero in each of the two lateral azimuths to its maximum

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value in the fore and aft azimuths. It is thought that vibration will limit the tip-speed ratio of helicopters to about 2/3 for a long time to come. This corresponds to a limiting forward speed of 0.3 of the speed of sound if the relative air speed at the tip of the advancing blade is limited to 3/4 of the speed of sound.

It is unlikely, therefore, that helicopters will be developed to fly eventually at much over $\frac{1}{4}$ of the speed of sound, which, at sea level, corresponds to less than 200 m.p.h.

Limitations in Size

Strange as it may appear at the present time when large helicopters are being contemplated as a matter of course, it was only a few years ago that the proposal to develop helicopters for shipboard use met with considerable opposition because of the contention that the limiting size of helicopter would be one carrying a maximum useful load of slightly over half a ton. The argument behind this assertion was that the rotor weight would increase as the cube of the rotor diameter and the gross weight only as the square. Consequently, the rotor weight would become such a large proportion of the gross weight that the useful load would eventually tend to zero.

Rotary wing experience had shown that it was possible to keep the ratio of rotor weight to gross weight practically constant, independent of size. In a rotor blade consisting essentially of a main spar to carry the centrifugal tension and an outer shell to give aerodynamic shape and stiffness, all the dimensions did not require to be increased in the same proportion as the size increased. The thickness of the skin could remain substantially constant and it was easy to show that for constant tensile stress in the spar and for constant blade-tip speed, the area of the spar needed only be proportional to the rotor diameter. These broad arguments suggested that the blade weight would be a constant proportion of the gross weight.

While the English-speaking nations were debating whether the cube law or square law applied to rotor size, with the destiny of the helicopter at stake, Germany (or as Dr. Sissingh is present, shall I say: half of Germany) had already decided that the law was a linear one and that the ratio of blade weight to rotor weight was therefore inversely proportional to the diameter, an increase in size resulting in an increased proportion of useful load. Here then are three different laws that give widely different results. Which of them is correct?

The answer is that they are all correct, but they apply to separate ranges of rotor size. The three factors that govern the ranges are:

(a) Angular speed (b) Blade-tip speed (c) Coning angle

The ratio of rotor weight to gross weight is inversely proportional to the product of these three quantities. The linear law applies when the angular speed and coning angle are kept constant and the blade-tip speed is increased with rotor size. When the limit of blade-tip speed has been reached (at present just over half the speed of sound) the square law begins to apply. In this case, the angular speed varies inversely, and the coning angle directly, with rotor diameter, the blade-tip speed remaining constant.

When the coning angle and therefore the downwash gradient become excessive (apparently about one-sixth of a radian—or $9\frac{1}{2}°$) rotor vibration limits the range of size to which the square law applies. Beyond this limit, with coning angle and blade-tip speed constant, the rotor weight increases as the cube of the rotor diameter for a given value of disc loading. An increase in disc loading with size does not affect this question very much because, as we have seen, the limiting power loading decreases with increased disc loading.

For a total blade weight equal to six per cent. of the gross weight and a coning angle in hovering flight, of one-tenth of a radian (about $5\frac{3}{4}°$) the limiting rotor diameter, beyond which the blade-tip speed cannot be increased, is about 50 feet. Beyond this diameter, the coning angle must increase and reaches the present limit of one-sixth of a radian at a diameter of 84 feet. Until rotor vibration at larger coning angles can be kept within practical limits, rotor diameters in excess of 84 feet can be used only at the expense of percentage useful load, unless a corresponding saving can be effected in the percentage weight of other items.

Certain items of the rotating system other than the blades themselves will also be affected by the cube law. A useful load equal to one-quarter of the gross weight has already been achieved and, if the cube law applies to one-eighth of the present gross weight, the variation in useful load for rotors above 84 feet diameter is proportional to

$$\frac{3}{8}\left(\frac{d}{84}\right)^2 - \frac{1}{8}\left(\frac{d}{84}\right)^3$$

It can be shown that this quantity is a maximum when d = 2 x 84 and becomes zero when d = 3 x 84. Hence the cube law for blade weight variation gives a limiting useful load per rotor of about 3 tons at a rotor diameter of 168 feet. Beyond this size the useful load would decrease and tend to zero at a diameter of about 252 feet.

I think you will agree that the cube law does not forecast too limited a future for the helicopter.

Dynamic Stability

At one time it was thought that the dynamic instability of single main-rotor helicopters about their rolling and pitching axes in the hovering condition would be a major limitation and that it would be necessary to devise means for overcoming this defect before practical helicopter flight could be achieved. Experience with existing helicopters has shown that dynamic instability at zero forward speed may be relatively unimportant.

The rolling or pitching motion of the helicopter, following a small disturbance, possibly the result of a gust, is statically stable if the initial tendency of the helicopter is to return to its equilibrium condition. This is ensured by the high position of the rotor above the centre of gravity of the aircraft.

Dynamic stability is concerned with the subsequent motion (Fig. 16), which may be either non-periodic (in which case it is stable) or oscillating, in which case it is stable if successive oscillations are of decreasing amplitude but unstable if they are divergent. Both stable and unstable oscillations are characterised by:

(a) the period of time required for one complete oscillation ; and

(b) the damping or amplification factor which determines the rate at which the amplitude of successive oscillations decreases or increases respectively.

It is found that in certain present-day helicopters where the period of

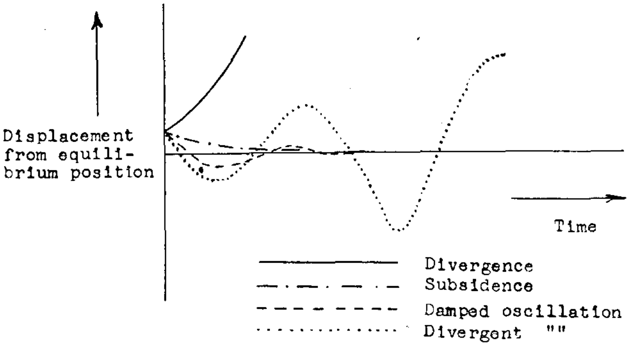

Fig. 16. Motion of aircraft following a small disturbance.

oscillation is about 12 secs. and the time required to double the amplitude is 18 secs., the unstable motion is not at all critical or difficult to control.

The equations of motion of the aircraft (Fig. 17) when expressed in terms of angular displacement and linear velocity as independent variables, result in an equation of the form:

$$An^3 + Bn^2 + Cn + D = 0,$$

where n is the frequency of the oscillation and C = 0 if the variation in inclination of the tip-path plane with respect of the axis of rotation is small compared with the amplitude of oscillation of the helicopter.

The condition for stability is that the roots of the frequency equation should be real and negative, or imaginary with their real parts negative, and ROUTH has shown in his textbook "Advanced Rigid Dynamics" of 1892 that this condition is satisfied provided the coefficients A, B, C and D are positive and, in addition,

$$BC > AD$$

So long as C =0, therefore, the aircraft must be unstable.

In other words, the tip-path plane must be prevented from oscillating with the same amplitude as the body of the aircraft. It can be shown that if the oscillation of the tip-path plane is reduced in amplitude by a times the amplitude of oscillation of the body, C is no longer zero and the condition for dynamic stability can be satisfied. If l is the distance of the rotor hub above the centre of gravity and k is the radius of gyration of the aircraft about the rolling or pitching axis, a must satisfy the inequality

$$a > \frac{I}{I + \frac{l^2}{k^2}}$$

For example, if 1/k = 3/4, a must be about 2/3, i.e., more than 2/3 of the oscillation of the tip-path plane must be suppressed if complete dynamic stability is required.

This result applies to the motion about the pitching and rolling axes of single main-rotor helicopters but is applicable also to the pitching oscil-

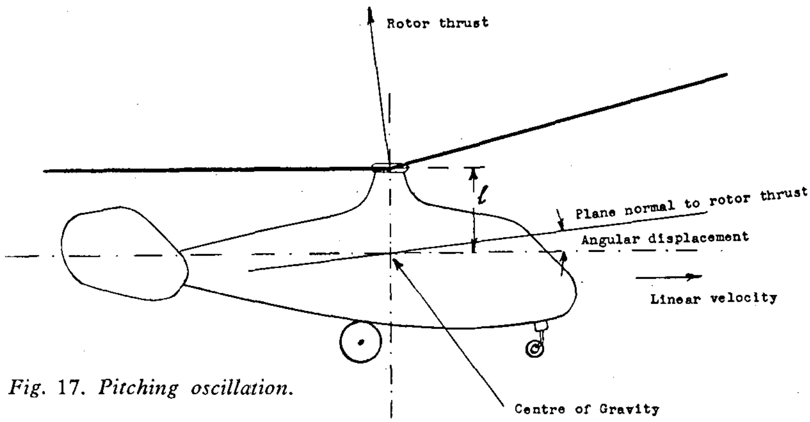

Fig. 17. *Pitching oscillation.*

lation of laterally-disposed twin rotors and to the rolling oscillation of twin tandem rotors. In multi-rotor helicopters any angular oscillation of the aircraft that causes the axial flow through one or more rotors to vary is heavily damped and dynamic stability can be provided without difficulty.

The Technical Outlook

It may be a very long time before we have perfectly stable, jet-driven, hingeless, two-bladed, large-sized, single-rotor helicopters that will operate smoothly and safely in every condition of flight ranging from zero translational speed to a quarter of the speed of sound. It may be many years before the helicopter is as fully developed as the aeroplane or as easy to handle as a car, but at least it has emerged from the laboratory stage. In spite of its present limitations, it is already established as a vehicle of practical utility—not a competitor of the aeroplane or other forms of transport but with unique uses of its own. The aeroplane will always be more suitable for long journeys by air, just as rail transportation is preferred to long journeys by road. The helicopter, like the car, will have its own restricted uses—I hope much more so, because although the helicopter can be brought safely to a standstill in flight, an air traffic jam over the cities is unpleasant even to contemplate.

Of the multifarious versions of the helicopter now under development throughout the world, the single main-rotor helicopter with a rotary tail unit —a configuration that must ever be attributed to IGOR SIKORSKY—appears to be the best compromise for low-powered machines of relatively low cruising speed. So long as vibration continues to be an important limitation in translational flight, an orthodox propeller may be preferred to the forwardly-inclined rotor for the propulsion of helicopters of high cruising speed and low power loading.

The rapid increase in rotor torque with size restricts the use of a single main-rotor to relatively small helicopters, unless jet assistance for vertical flight becomes practicable, in which case large single-rotor helicopters could operate in forward flight at reduced torque as a gyrodyne or at zero torque as a gyroplane. For larger machines, twin rotors, mounted in tandem, and possibly intermeshed, would seem to be the most straightforward arrangement at the present stage of development.

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