# A Generalization of Certain Properties of Laguerre Polynomials.

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Let

$$\Pi_n(z) = e^z \left(\frac{d}{dz}\right)^n \{e^{-z} A_n(z)\}$$

where

$$A_n(z) = a_0 z^n + \binom{n}{1} a_1 z^{n-1} + \binom{n}{2} a_2 z^{n-2} + \ldots . + a_n,$$

the $a$'s being constants. Then the following relations hold:

$$\text{(i)} \quad \sum_{s=0}^{\nu} (-1)^s \binom{\nu}{s} \frac{n!}{(n-s)!} \Pi_{n-s}(z) = (-1)^n \sum_{s=0}^{n-\nu} (-1)^s \binom{n-\nu}{s} \frac{n!}{(n-s)!} A_{n-s}(z),$$

and

$$\text{(ii)} \quad \sum_{s=0}^{\nu} (-1)^s \binom{\nu}{s} \frac{n!}{(n-s)!} \Pi'_{n-s}(z) = -n \sum_{s=0}^{\nu-1} (-1)^s \binom{\nu-1}{s} \frac{(n-1)!}{(n-s-1)!} \Pi_{n-s-1}(z),$$

the accent denoting differentiation with respect to $z$. For, it follows from the definition of $\Pi_n(z)$ that

$$\Pi_{n-s}(z) = \sum_{\nu=0}^{n-s} (-1)^\nu \binom{n-s}{\nu} \frac{(n-s)!}{\nu!} A_\nu(z).$$

Substituting this in the left-hand side of (i) and summing with respect to $s$, we obtain the right-hand side by Vandermonde's Theorem. Equation (ii) is obtained similarly on using the relation

$$A'_\nu(z) = \nu A_{\nu-1}(z).$$

It may be noted that (i) is of a reciprocal nature, being also valid when the $\Pi$'s and $A$'s are interchanged, as is evident on changing $\nu$ into $(n-\nu)$ on both sides.

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