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POSITIVE SCALAR CURVATURE AND CALLIAS-TYPE INDEX THEOREMS FOR PROPER ACTIONS

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This thesis by publication is a study of the equivariant index theory of Dirac operators and Callias-type operators in two distinct settings, namely on cocompact and noncocompact manifolds with a Lie group action.

The first two chapters are a short resumé of Dirac operators and index theory and form a common introduction to the papers in the appendices.

The first appendix [2] is joint work with my supervisors, Elder Professor Mathai Varghese and Dr Hang Wang. For G an almost-connected Lie group acting properly and cocompactly on a manifold M, we study G-index theory of G-invariant Dirac operators. By establishing Poincaré duality for equivariant K-theory and K-homology, we are able to extend the scope of our results to include all elements of equivariant analytic K-homology, which we also show is isomorphic to equivariant geometric K-homology. Our results are applied to prove: a rigidity result for almost-complex manifolds, generalising a vanishing theorem of Hattori; an analogue of Petrie's conjecture; and Lichnerowicz-type obstructions to G-invariant Riemannian metrics on M.

The second appendix [1] studies the much more general situation where the quotient M/G is noncompact and G is an arbitrary Lie group. We define *G-Callias-type operators* and show that they are $C^*(G)$-Fredholm by adapting analysis by Kasparov to new Hilbert $C^*(G)$-module analogues of Sobolev spaces. Questions of adjointability, regularity and essential self-adjointness are addressed in detail. The estimates on G-Callias-type operators are based on the work of Bunke in the nonequivariant context. We construct explicit admissible endomorphisms for G-Callias-type operators from the K-theory of the Higson G-corona of M, a highly nontrivial group. The index

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theory developed here is applied to prove a general obstruction theorem for G-invariant metrics of positive scalar curvature in the noncocompact setting.

References

[1] H. Guo, 'Index of equivariant Callias-type operators and invariant metrics of positive scalar curvature'. Preprint, 2018, arXiv:1803.05558.
[2] H. Guo, V. Mathai and H. Wang, 'Positive scalar curvature and Poincaré duality for proper actions'. *J. Noncommut. Geom.*, to appear. Preprint, 2016, arXiv:1609.01404.

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