P. J. Lorimer, A Study of T_2-Groups, McGill University (Supervisor: H. W. E. Schwerdtfeger).

In the past few years, the literature has contained a number of group-theoretic characterisations of the groups PLF(2, F), of Moebius transformations over a finite field F. The concept of a T_2-group, defined by Professor H. W. E. Schwerdtfeger, gives a characterisation which is simpler than those previously given.

A group G is called a T_2-group if it contains a subgroup H and

(i) $a \notin H$, $bab^{-1} \notin H$, and $a^2 \neq 1 \Rightarrow \exists$ unique $h \in H \ni hah^{-1} = bab^{-1}$.

(ii) $a \notin H$, $bab^{-1} \notin H$, and $a^2 = 1 \Rightarrow \exists$ exactly two elements $h_1, h_2 \in H \ni h_1ah_1^{-1} = h_2ah_2^{-1} = bab^{-1}$.

A T_2-group G is called an S_2-group if G - H contains an involution, i.e. condition (ii) is not empty.

The following are S_2-groups:

(1) $G = (0, 1)^{\alpha}$ where (0, 1) is the group with two elements, and α is any cardinal number. H is any subgroup of G which is isomorphic to (0, 1).

(2) H is any Abelian group with exactly one involution and G is obtained from H by adjoining an element t which obeys the laws, $t^2 = 1$; $tht^{-1} = h^{-1}$ for all $h \in H$.

(3) G = PLF(2, F), where F is a field of characteristic $\neq 2$, and H is the subgroup of all similarities $z \to \frac{az+b}{d}$.

The following theorems are proved:

THEOREM 1. If G is an S_2-group, and either
(i) The subgroup H is normal in G, or
(ii) The centre of G is non-trivial,
then G and H are the group and subgroup of either (1) or (2).

THEOREM 2. If G is a finite S_2-group and either
(i) The subgroup H is not normal in G, or
(ii) The centre of G is trivial,
then G and H are the group and subgroup of (3).

Thus all finite S_2-groups are known; they are given by (1), (2), or (3), and Theorem 2 gives a characterisation of the groups PLF(2, F), when F is finite and the characteristic of F is not equal to 2.