ANALYSIS OF LUNAR OCCULTATION DATA

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Abstract. The technique of model fitting has been applied to the analysis of data obtained from photoelectric measurements of lunar occultations. A model occultation curve is generated and fitted, by least squares, to the observed light curve, and by this method the values of the model parameters are determined, together with their formal errors. This technique is contrasted with Scheuer's deconvolution procedure by applying both methods to the same observed data.

1. Introduction

In his pioneering work on lunar occultations Whitford (1939) showed that the process of model fitting could be used to extract useful data from an occultation curve. He demonstrated that a model curve based on monochromatic Fresnel diffraction by a straight edge, suitably modified to include the effects of detecting a finite range of optical wavelengths, would fit the observed data from point-source stars extremely well. He later reported that he had measured the diameter of two stars by this technique (Whitford, 1946) but has not published the data from which the diameters were obtained.

We have extended Whitford's data reduction procedure primarily by making use of high-speed computing facilities not available at the time of his measurements, and by including some additional factors in the generation of the model curves. We have also written a computer program which realizes the deconvolution procedure proposed by Scheuer (1962), and are thus in a position to compare these two techniques for the analysis of lunar occultation measurements.

2. The Model Curve

The basic procedure of model fitting is based on the assumption that a mathematical model of an observed process can be constructed, whose details are controlled by a physically meaningful set of variable parameters, and that these parameters can be adjusted to obtain a good fit between the model and the observed data. Williams (1939) accepted the suggestion by Eddington (1909) that the occultation phenomenon could be described by Fresnel diffraction at a straight edge, and showed how the diffraction pattern is modified by stars with apparent angular diameters larger than about 0.001 sec of arc. Whitford (1939) showed that the finite range of detected wavelengths also modifies the pattern and must be included in the model. A detailed description of the model fitting process we have developed has been published elsewhere (Nather and McCants, 1970) and will only be summarized here. Our process of model curve generation begins with the spectral type of the star under observation. From Allen (1963) we obtain the effective color temperature of the star and substitute

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this into the black body radiation formula to obtain the spectral distribution of light from the star. This distribution is modified by the sensitivity of our detector (and filter, if one is used) and results in a curve describing the proportion each wavelength contributes to the total diffraction pattern. This distribution curve is then divided into 100 Å segments and a separate monochromatic diffraction pattern is produced for each segment, weighted by the contribution of the segment. The sum of these monochromatic patterns is the model curve which would be obtained from a point source observed by a point detector.

The modification of the occultation pattern due to the finite size of the telescope aperture is usually negligible compared with the bandwidth effect. A telescope 1 meter in diameter has an effect equivalent to an optical bandpass of 50 Å. The two effects are similar but not identical, so each must be modelled separately. A change in wavelength causes a change in the scale of the diffraction pattern, while the effect of a finite aperture is to smear the pattern in the spatial dimension, leaving its scale unchanged. Arguments of symmetry can be invoked to show that the angular diameter of the telescope, as seen from the limb of the Moon, has the same effect on the diffraction pattern as would a star showing the same angular size, and can be modelled in the same way.

3. Fitting the Observations

Most occultation curves obtained in practice are those of a single, unresolved point source of light. A model curve generated to fit such an observation requires four parameters:

(1) The intensity of the background light (mostly scattered moonlight).

(2) The intensity of the star under observation.

(3) The effective velocity of the lunar limb perpendicular to itself.

(4) The time at which the intensity of the star drops to 25% of its free-field value, called the time of geometric occultation.

In our fitting procedure the values of these four parameters are adjusted simultaneously to obtain the best fit to the observed data (in the sense of least squares). The output of the procedure is the adjusted value for each parameter, the formal error for each, and a matrix indicating the degree of correlation between the various parameters. For a point source the two parameters of interest are the time of geometric occultation, which can be obtained to an accuracy of about 1 msec from a good trace, and the effective limb velocity. This latter value is compared with the velocity computed from the Jet Propulsion Laboratory Lunar Ephemeris on the assumption that the limb is level at the point of occultation; any difference can be interpreted as due to a slope of the limb. The ability of this procedure to fit a point source occultation is shown in Figure 1.

A double star with an angular separation of a few milliseconds of arc can be resolved by the occultation technique. In this case two additional parameters are included to account for the intensity of the second star and its occultation at a slightly different time. Derived parameters include the time of occultation of each component,

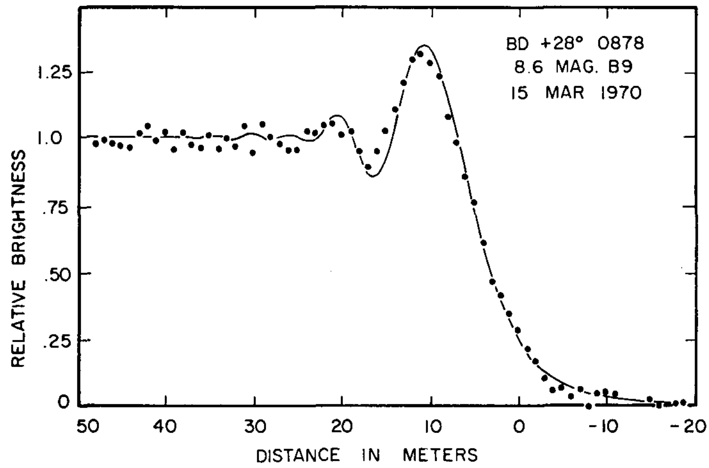

Fig. 1. An occultation measurement of a faint point-source star. The solid line is the fitted model curve, from which the values of the four model parameters are derived.

from which the separation of the pair in the direction of lunar motion can be obtained, and the relative intensity of each star in the color of light in which the observation was made. The application of this analytical method to double star measurements has already been reported (Nather and Evans, 1970).

A star of sensible angular diameter is modelled by dividing its disk into a series of strips parallel to the lunar limb and replacing each strip with a point source of equivalent brightness. The model curve is the sum of these point source patterns. Any brightness distribution across the disk can be approximated easily, and non-symmetrical distributions can be modelled should the need arise. The formal error of the diameter determination is obtained along with its correlation with the other parameters. The diameter of the red giant λ Aqr has been determined by this procedure (Nather *et al.*, 1970).

4. Deconvolution

The procedure of deconvolution, proposed by Scheuer (1962) for the reduction of occultation observations of radio sources, and reduced to practice by Von Hoerner (1964), is attractive because it does not require any assumptions about the brightness distribution across the source; indeed, the basic output of the process is this brightness distribution. We have developed a computer program, following Von Hoerner, making those changes required by the reduction in effective wavelength by $\sim 10^6$, and have applied the process to both artificial and observed occultation traces. Two difficulties became immediately apparent:

(1) The effective bandwidths used in optical observations are so much wider than those normal in the radio region that the monochromatic approximation used by Scheuer in his derivation may not be acceptable. The observed curve is no longer a simple convolution of the brightness distribution with the Fresnel diffraction pattern,

but is the integral of this convolution over the bandwidth involved. Only if the smoothing beamwidth is chosen wide enough to dominate the bandwidth effect can the results be considered valid. If this precaution is not observed the brightness distribution obtained will show a spurious width, due to the bandwidth effect, and might be falsely interpreted as being due to a measurable stellar diameter.

(2) The deconvolution procedure requires that the effective velocity of the lunar limb be specified as an input parameter, which is equivalent to the assumption that the slope of the limb at the point of occultation is known. While this assumption may be acceptable in the radio region, where the scale of the phenomenon is 1000 times larger, our observations indicate clearly that this assumption can introduce serious error into optical measurements.

It seems likely that the first difficulty can be overcome by an extension of present techniques. If a suitable restoring function can be derived from the bandwidth-modified curve of a point source, rather than from its monochromatic equivalent, it should be capable of completely valid restoration for the bandwidth chosen. Such a procedure might obviate the need for the artificial smoothing Gaussian which Scheuer introduced to keep the mathematics tractable.

The second difficulty seems to be more fundamental, and affords a clear contrast between the model fitting and deconvolution procedures. With the former we can derive a value for the lunar velocity, and hence allow for the effects of a lunar slope, but we must assume a brightness distribution across the source. In the latter we derive the brightness distribution, but we must assume we know the value of the lunar slope. At our present state of knowledge it appears safer to assume a star appears optically as a point or a disk than to assume we know the slope of the lunar surface over a region perhaps 50 m in extent. Just the opposite assumption may be preferable in the radio region, and perhaps in the infra-red as well.

It should be noted that this difference between the two procedures obtains only when diffraction effects are dominant; i.e., for single and double point sources, and for stars with apparent angular diameters less than about 0.010 to 0.015 sec of arc. For stars of angular diameter greater than this the diameter and velocity parameters in the model fitting process become so highly correlated that they cannot be determined separately, and the velocity parameter must be assumed for either procedure. We must look further, then, if we are to compare the two techniques as applied to large stars.

5. Antares Revisited

We have compared the two techniques by applying each to the occultation curve of Antares obtained by Evans *et al.* (1954) on 13 April 1952 at the Radcliffe Observatory. This curve is one of the series analyzed by Taylor (1966), who used the deconvolution process to derive a brightness distribution across the star, and is the least noisy of the series. The observed data are shown in Figure 2, together with the fitted model curve for a uniformly illuminated disk of diameter 0.044 sec of arc. The fit appears to be acceptable everywhere but in the region of the 'toe', where the star shows brightness

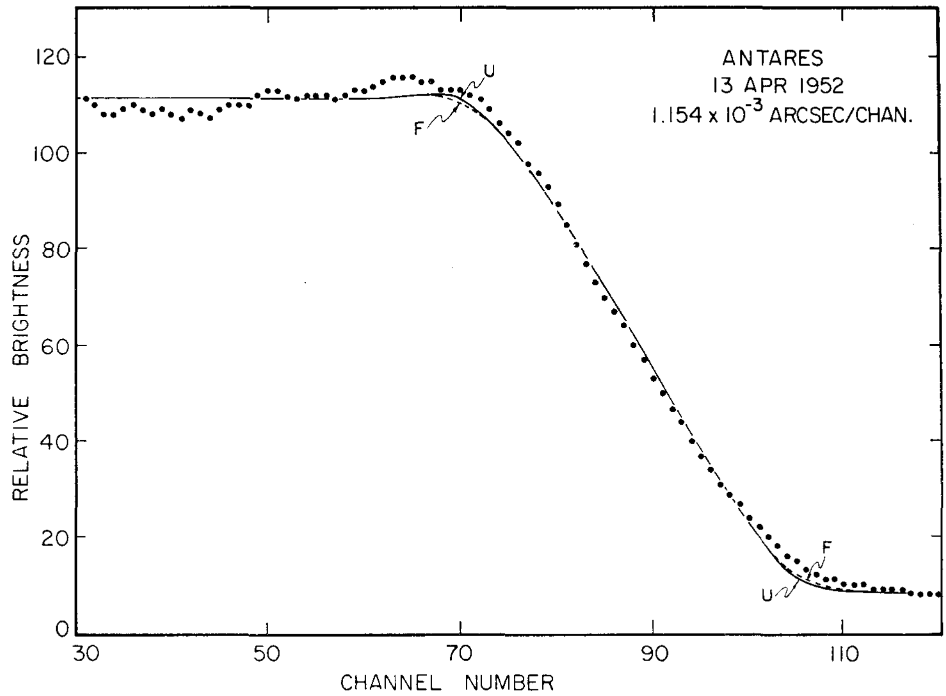

Fig. 2. Antares reappearance observed in 1952. The solid line (U) shows the fitted model curve assuming the star appears as a uniformly illuminated disk, while the dashed line (F) shows the curve obtained assuming a cosine law of darkening and a fully darkened disk.

not exhibited by the model curve. This cannot be an instrumental lag because the curve represents the reappearance of the star from behind the Moon, and therefore brightens too early.

The assumption of a cosine law of darkening and a fully darkened disk yields a diameter of 0".050 but is not an appreciably better fit. The difference between this curve, indicated in Figure 2, and that generated from the uniform disk model is extremely small; it seems unlikely that limb darkening can be determined from occultation data even for the largest stars.

Figure 3 shows this same trace deconvolved, using a smoothing beam width of 0".005. The distribution expected from a uniform disk 0".044 in diameter is shown as a solid line. For both analyses the lunar limb velocity of 0.4267 m/msec, computed from the Jet Propulsion Laboratory Lunar Ephemeris and assuming a level limb, has been used; the uncertainty in its value, if modified by a lunar slope, is the dominant one in the diameter evaluation.

This brightness distribution shows quite clearly the deviation in the 'toe' region, but offers no clue as to its cause. It is perhaps significant that another observation of this same event (Cousins and Guelke, 1953) fails to show this same asymmetry, suggesting it may be an effect caused by the lunar limb.

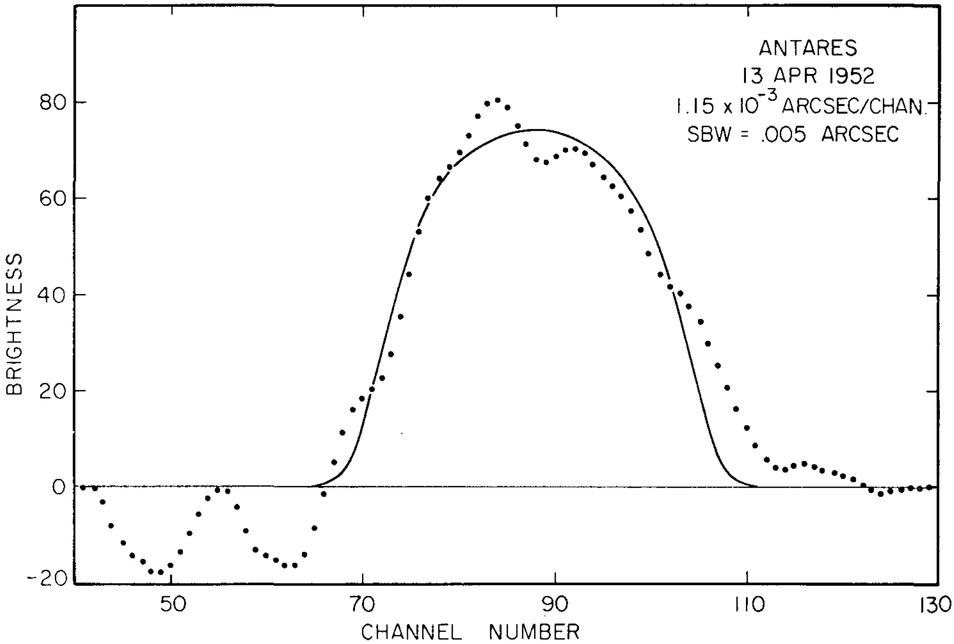

Fig. 3. The brightness distribution across the disk of Antares obtained by deconvolution. The solid line shows the distribution expected from a uniformly illuminated disk 0.044 arcsec in diameter.

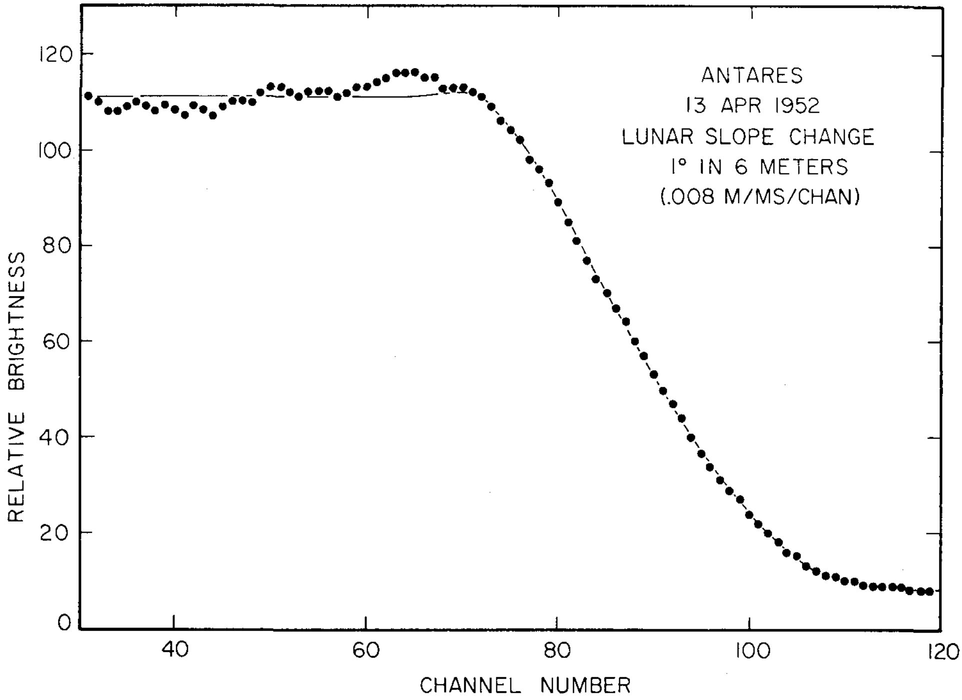

Fig. 4. Antares reappearance. The solid line shows the model curve based on a changing effective lunar velocity or an equivalent changing lunar slope.

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If the brightness distribution across the star is indeed that found by Taylor (1966), consisting of a bright core and an extended, somewhat dimmer envelope, it is not obvious in Figure 3. Our attempts to fit such a model (two concentric disks of differing diameters and surface brightnesses) to this trace failed to converge because of the noted asymmetry, and indicates that no stable solution of this type exists for this single observation.

An analysis of the time from 'first appearance' of Antares to the 50% point compared to the time from the 50% point to completely out from behind the Moon led to the conclusion that the 50% time was relatively too late. This could be caused by a lunar hill, or, equivalently, a changing lunar slope. A constantly changing lunar slope can be modeled by a constantly changing effective velocity of the lunar limb. Allowing the effective velocity to change by 0.008 m/msec every channel produces the very good fit to the data shown in Figure 4. This is equivalent to a change in lunar slope of 1° every 6 m or to a lunar hill 7 m maximum height, 140 m long, and 350 m radius of curvature.

6. Conclusions

A modernized version of Whitford's model fitting procedure has been automated and applied successfully to over 100 occultation events, most of them representing single point sources but with enough double stars and resolvable diameter measurements to keep interest alive. An attempt to transplant Scheuer's elegant deconvolution technique into the optical region has met with difficulties not yet resolved. The best available trace of Antares can be satisfactorily fitted with the help of a small lunar hill at the point of occultation.

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