

## CORRESPONDENCE.

## SICKNESS NOTATION.

*To the Editors of the Journal of the Institute of Actuaries.*

DEAR SIRS,—There are one or two points connected with Sickness Notation to which attention may, perhaps, usefully be drawn.

In some important sickness investigations there is an undesirable ambiguity as to the meaning of the words “rate of sickness.” Also, the symbol used is sometimes  $s_x$  and sometimes  $z_x$ . And in these cases, sometimes  $s_x = z_x$  and sometimes  $s_x$  does not  $= z_x$ .

Here are some examples :

In the Experience of Registered Friendly Societies (Blue Book 303 of 1896, William Sutton) the “rate of sickness” is called  $s_x$ . On examining the mode by which the Exposed to Risk were computed one finds that the deaths between age  $x$  and age  $x+1$  were regarded as at risk of sickness for the whole of the year of age  $x$ . Thus, this  $s_x$  denotes the rate of sickness as commonly understood.

In Mr. G. F. Hardy’s “Messenger Prize Essay on Friendly Societies,” the symbol  $z_x$  is used to denote the rate of sickness when, in computing the Exposed to Risk of Sickness, the deaths occurring between age  $x$  and age  $x+1$  are regarded as at risk of sickness for the whole of the year of age  $x$ . Thus, Mr. Sutton’s  $s_x$  and Mr. Hardy’s  $z_x$  seem to mean the same thing.

In Mr. Watson’s admirable volume upon the “Manchester Unity Experience 1893–1897,” the “rate of sickness” is called  $s_x$ . On examining the mode by which the Exposed to Risk of Sickness were computed, one finds that “the number dying in each year were assumed to be at risk until the middle of the year.” That is to say, these deaths were regarded as at risk of sickness for one-half of the year, not for the whole of the year. Thus, the Manchester Unity “sickness rate,”  $s_x$ , is the central sickness rate, or the force of sickness in the middle of the year of age  $x$  to  $x+1$ . Thus, the Manchester Unity  $s_x$  is not the same thing as Mr. Sutton’s  $s_x$ , nor is it the same thing as Mr. Hardy’s  $z_x$ . But, as pointed out, Mr. Sutton’s  $s_x =$  Mr. Hardy’s  $z_x$ .

Again, in the Institute of Actuaries’ Text Book, Part II., by Mr. George King, page 375, the “rate of sickness” is called  $z_x$ . And it is there defined in terms that agree with Mr. Hardy’s definition of  $z_x$ . Thus, Mr. King’s  $z_x =$  Mr. Hardy’s  $z_x$ . The symbol  $s$  is used by Mr. King for the value of a sickness benefit.

Other examples could be quoted that show an existing confusion between the symbols  $s_x$  and  $z_x$ , and between the “rate of sickness” and the “central sickness rate.” I am not suggesting there is any confusion in the minds of the authors of the books here quoted. The confusion arises when a reader finds the same symbol used to denote different things. And unless the reader is well on his guard he may be making mistakes.

I venture to suggest that it is desirable for an authoritative pronouncement to be made by the Institute as to the meaning of

the two symbols  $s_x$  and  $z_x$ , including a statement as to the mode of computing the Exposed to Risk of Sickness. For the mode of computing these exposures has an appreciable effect upon the "rate of sickness" obtained.

Another point that emphasizes the desirability to attach an exact meaning to items of sickness notation, is that in comparing the actual and the expected time of sickness and cost of sickness in any Friendly Society that may be under observation, the expected sickness being based upon this or that standard Sickness Experience, it is necessary to compute the Exposed to Risk of Sickness in the Friendly Society under observation by the same mode as that by which the Exposed to Risk of Sickness in the Standard Experience were computed. For instance, in testing a Friendly Society's experience by the  $s_x$  of the Manchester Unity the Exposed to Risk of Sickness in the Friendly Society should give only half a year's exposure to the deaths. But in testing the same Friendly Society's sickness by the  $s_x$  of Mr. Sutton in Blue Book 303 of 1896, the Exposed to Risk of Sickness in the Friendly Society under observation must be recomputed so as to give the full year's exposure to sickness in respect of each death.

Then again, when constructing commutation columns, the present absence of standardization of sickness notation is a quite possible source of error. For, in addition to the confusion of notation herein mentioned there is the further point that commutation columns based upon a rate of sickness have to be prepared differently from commutation columns based upon a central sickness rate.

These matters seem to need an authoritative statement by the Institute.

Yours truly,

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