ENERGY TRANSFER BY CIRCULATION IN W URSAE MAJORIS SYSTEMS

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1. INTRODUCTION

After Lucy (1968) introduced the contact-binary model with a common convective envelope, it was envisaged by Hazlehurst & Meyer-Hofmeister (1973) that a sideways flow of convective elements would carry energy from the more luminous star, the primary, to the less luminous star, the secondary, as a result of horizontal pressure variations. Webbink (1977) extended this picture by noting that the interaction between vertical entropy gradients and large-scale smooth circulation currents in the common envelope would provide the necessary redistribution of flux. That is, energy is absorbed by the flow during its vertical motion in the primary and is released during its vertical motion in the secondary. Webbink (1977) mentioned two mechanisms by which a large-scale circulation could be generated: (1) the non-spherically symmetric force field due to rotation and tides which will drive an analogue of classical Eddington-Sweet circulation and (2) differential heating of the base of the common envelope. Although these mechanisms are conceptually different, they are not in practice easy to disentangle, and will certainly both be operating in contact binaries.

2. EDDINGTON-SWEET CIRCULATION

Any non-spherically symmetric perturbation to a star will give rise to departures from radiative equilibrium and result in circulation currents. If the combined influence of rotation and tides is to cause a flow rising in one component and sinking in the other, then energy transfer will result. For the Eddington-Sweet solution to be valid the effects of viscosity, inertia and Coriolis forces must be negligible, and Webbink (1977) has argued that to a first approximation they are. Because of this and the schematic flow patterns sketched by Webbink (1977) showing a flow rising in one component and falling in the other, some confusion exists in the literature on the role of the Eddington-Sweet currents. To clarify this we have numerically calculated the velocity component of the Eddington-Sweet circulation normal to equipotential surfaces for mass ratios $q = 1.0$, 0.4, and 0.1 (Smith & Smith, in preparation). From the topology of the flow we conclude that there

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are no tidally and rotationally induced currents linking the two stars. This result confirms that the classical Eddington-Sweet theory is inadequate for describing the energy transfer.

3. DIFFERENTIAL HEATING OF THE COMMON ENVELOPE

Shu et al. (1979) were the first to use Webbink's idea of differential heating as the driving force for circulation. But Hazlehurst & Refsdal (1978), Papaloizou & Pringle (1979) and Smith, Robertson & Smith (1979) have all shown that the discontinuity model of Shu et al. (1979) is unacceptable and we will not consider it here. Nonetheless we believe that non-uniform heating of the base of the common envelope is the best candidate for the driving mechanism. We have solved the hydrodynamic equations in a two-dimensional box of length a_1 and depth a_3 in a uniform gravitational field $\underline{g}$, containing a perfect gas with viscosity ν and thermal diffusivity κ. The full, compressible, hydrodynamic equations are written as follows for ease of numerical solution:

$$\partial\rho/\partial t + \nabla.(\rho\underline{u}) = 0, \qquad (1)$$

$$\partial(\rho\underline{u})/\partial t + \nabla.(\rho\underline{uu}) + \nabla p + \rho\underline{g} - \nu\nabla^2(\rho\underline{u}) = 0, \qquad (2)$$

$$\partial T/\partial t + \nabla.(T\underline{u}) + (\gamma-2)T\nabla.\underline{u} - \kappa\nabla^2 T = 0, \qquad (3)$$

and $$p = R_*\rho T \qquad (4)$$

where the symbols have their usual meanings. The boundary conditions used are:

$$T = T_o \text{ on top, } T = T_B + T_1 \cos(\pi x/a_1) \text{ on base} \qquad (5)$$

$$w = \partial u/\partial z = 0 \text{ on top and base.} \qquad (6)$$

The variables ρ, T, u and w are assumed to be periodic in x. It was found easier to evolve time-dependent equations to a steady state than to solve time-independent equations implicitly. The steady-state solutions obtained are characterized by 7 dimensionless parameters:

$$\gamma,\ P \equiv \nu/\kappa,\ R \equiv ga_3{}^3/\nu\kappa,\ C \equiv ga_3/R_*(T_B+T_1-T_o)\ ,$$

$$A \equiv a_1/a_3,\ \tau_B \equiv T_B/(T_B+T_1-T_o),\ \text{and}\ \tau_1 \equiv T_1/(T_B+T_1-T_o).$$

The range of values explored to date is:

$$\gamma = 5/3,\ 1/256 < P < 8,\ 46.875 < R < 48000,$$

$$1/2 < A < 3,\ C = 3.75 \text{ and } 5,\ \tau_B = 2 \text{ and } 0.05 < \tau_1 < 0.45.$$

A typical solution for $R < 10^4$ consists of a single cell with flow rising at the hot end, as would be expected. The stagnation point is

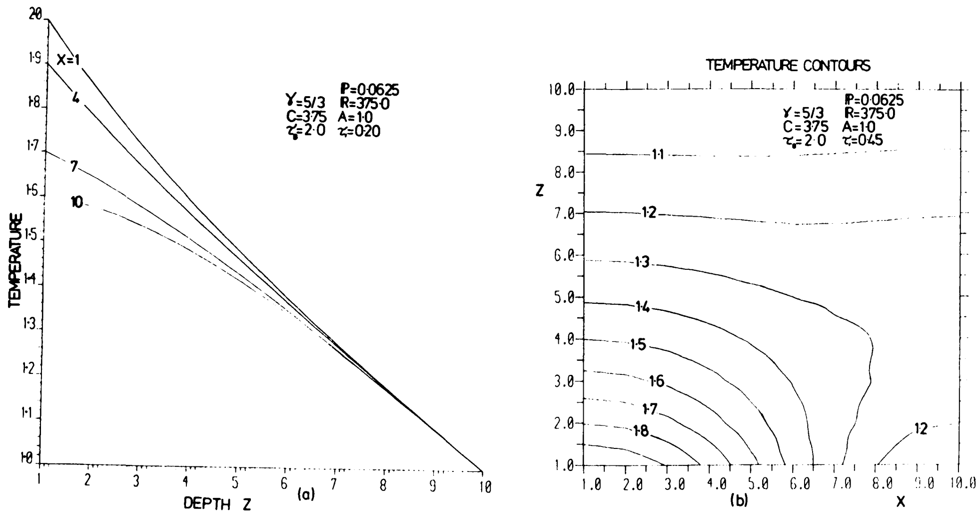

FIGURE 1.
(a) Temperature as a function of the vertical coordinate, z, for 4 different values of the horizontal coordinate, x; $\tau_1 = 0.20$.
(b) Temperature contours for the case $\tau_1 = 0.45$. Note the temperature inversion for x > 7 and the uniform temperature for z > 7.

located approximately one pressure scale-height, H_p, above the base and is nearer to the cool end for A > 1. The whole cell spans about $3H_p$ when C = 3.75. For $R > 10^4$ there is a second thin cell near the surface.

In general, decreasing P increases $|\underline{u}|$, and not surprisingly the maximum velocity is related to the temperature perturbation τ_1: $|\underline{u}|_{max} \approx \alpha\tau_1 c_s$ where $\alpha \approx 1$ for A = 1 and c_s is the sound speed. The temperature distribution shows several interesting effects. The horizontal temperature variations disappear after about one scale height (see fig.1). It is possible to form stable temperature inversions when the temperature perturbation is large (see fig.1(b)), i.e. $\tau_1 > 0.25$ for A = 1. But as the aspect ratio increases a larger value of τ_1 is needed to cause this. The size of the inversion increases with the ratio of the thermal to dynamical timescales, and with R, but is almost independent of P.

4. CONCLUSIONS

1) Differential heating will cause a circulation linking the two stars, which will operate throughout the common envelope, smoothing out horizontal temperature variations in about one pressure scale-height.

2) For realistic temperature perturbations, the horizontal velocity will be a considerable fraction of the sound speed. But the vertical

velocity will be much less due to the thin-layer geometry of the common envelope, i.e. high A.

3) Temperature inversions are unlikely to be formed in the common envelope, due to the high aspect ratio, A $\sim$ 100. If an inversion is formed then the system will not be in thermal equilibrium, since there will be a net flow of energy into the secondary.

Although these results are only preliminary, we believe that they will form a useful basis for further study of energy transfer in contact binaries.

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COMMENTS FOLLOWING SMITH, SMITH AND ROBERTSON

Nariai: I have presented the model of circulation a few years ago (Nariai, K.: Publ. Astron. Soc. Japan, 28. 587, 1976). Is your model essentially the same as mine?

D. Smith: Yes, many of the details are similar.

Shu: I would like to suggest that your calculation is more appropriate to Los Angeles than to the interiors of contact binaries. Our parameter δ is, in order of magnitude, given by your conbination $(C/PR)^{\frac{1}{2}}$, which is typically 0.1 or larger in your calculations. On the other hand, the true value of δ is four or more orders of magnitude smaller at the base of the common envelope of most contact binaries. Nevertheless, it is interesting that even with a relatively large value of δ, you get a temperature inversion.

D. Smith: The temperature inversions are a consequence of our lower boundary conditions and may not have any physical significance.

Shu: In reply to Webbink's comment, let me state that our definition of δ is $F/\rho h a_s$. Webbink's definition is different and associates the

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thermal timescale of a convection zone with the travel time of a convective blob. Webbink's definition is irrelevant to our discussion because it ignores the blobs have only a slight entropy excess or deficit with respect to their surroundings.

Anderson: In order to determine whether a temperature inversion over the cooler boundary can be maintained by circulation, you should extend a thermally non-conducting wall (representing the potental ridge between the two stars) up from the lower boundary. Then both flux and temperature should be specified at the two new lower boundaries.