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Tidal barrier and the asymptotic mass of proto gas-giant planets

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Abstract. Although late stage gap formation reduces the surface density in the vicinity of protoplanets, simulations suggest gas may continue to leak through the protoplanets tidal barrier, replenishing the gas supply and allowing protoplanets to acquire masses comparable to or larger than that of Jupiter. Global gas depletion is a possible explanation for gaseous planets with lower masses in weak-line T-Tauri disks and ice giants in our own solar system, but it is unlikely to have stalled the growth of multiple systems around nearby stars that contain relatively low-mass, close-in planets along with more massive and longer period companions. Here, we suggest a potential solution. We show that supersonic infall of surrounding gas onto a protoplanet is only possible interior to both its Bondi and Roche radii. Although the initial Bondi radius is much smaller than its Roche radius, the former overtakes the latter during its growth. Thereafter, a positive pressure gradient is required to induce the gas to enter the Roche lobe of the protoplanet and flow is significantly reduced. We present the results of analysis and numerical simulations to show that the accretion rate increases rapidly with the ratio of the protoplanets Roche to Bondi radii. Based on these results we suggest that in regions with low geometric aspect ratios gas accretion is quenched, resulting in relatively low protoplanetary masses.

Keywords. planet formation, accretion

1. Introduction

During the final stages of the sequential planet formation scenario, once a protoplanet acquires sufficient mass to allow gas to be freely accreted, it enters a dynamical gas accretion phase. During this stage, the cores' gravity dominates the flow out to the Bondi radius (R_B), where the local escape speed becomes comparable to the gas sound-speed in the disk. R_B increases linearly with mass, more rapidly then the associated Roche radius (R_R). Once the planet has grown to sufficiently high mass, its thought that the tidal perturbation is sufficient to induce the formation of a gap in the disk near the protoplanets' orbits (Goldreich & Tremaine 1979, 1980). The formation of this gap is generally thought to quench further gas accretion (Lin & Papaloizou 1979, 1985).

Many 2D and 3D global simulations of protoplanet-disk interaction see (Papaloizou & Terquem 2006) show that tidal torque from a sufficiently massive embedded protoplanet is indeed sufficient to produce a gap. However, several simulations indicate gas will continue to flow into the gap (Lubow *et al.* 1999; Kley *et al.* 2001; D'Angelo *et al.* 2002). The accretion rate determined from these simulations suggests that despite a significant reduction in Σ near the orbit of the protoplanet, the growth timescale for Jupiter-mass protoplanets is shorter than the global-disk gas depletion timescale ($\tau_{\rm dep} \sim 3 - 10$ Myr) inferred from the observations of young disks (Haisch *et al.* 2001). If the gap region is continually replenished by diffusion into the gap, the asymptotic mass of the protoplanets would well exceed the observed upper limit of the planetary mass function (Marcy *et al.* 2005).

Here, we explore the possibility that even if the gap is replenished with tenuous gas, only a small fraction of that gas may be accreted onto protoplanets with R_R greater than the disk thickness (H). The model parameters in most previous simulations are chosen to approximate those inferred from the minimum mass nebula model in which the ratio between the disk thickness and radius (H/a) is assumed to be around 0.07. With this assumption, R_B becomes comparable to R_R for a protoplanet with $M_p \sim M_J$. Self consistent models of disk thickness indicate that the H/a at the snow line can be significantly smaller (Garaud & Lin 2007). Taking this possibility into account, we suggest that, in the limit of $R_R > H$, a tidal barrier imposed by the protoplanet limits the gas flow from most regions inside the gap into the Roche lobe. Much of the mass that diffuses into the gap accumulates into the horseshoe orbits in the co-orbital region, but is not able to accrete onto the planet. To explore this, §2 contains an analysis to highlight the nature and importance of the tidal barrier. In §3 we present a set of high-resolution 2D hydrodynamic simulations to demonstrate that the flow into the Roche lobe is indeed quenched for protoplanets with $R_R > H$ despite diffusion of gas into the gap region. Finally, we summarise our results and discuss their implications in §4.

2. The Role of the Tidal Barrier

To examine the role of the tidal barrier in stopping mass accretion we utilise a novel approach introduced by Korycansky & Papaloizou (1996) to analyse inviscid flow in a steady-state. Under these conditions two integrals of motion, the potential vorticity (hereafter vortensity) $\frac{\omega+2\Omega}{\Sigma} = \varpi(\psi)$ and the Bernoulli energy, $v^2/2 + c_s^2 \ln\Sigma + \Phi_g = E_B(\psi)$, are conserved. In the above equations, we adopt a Hill's approximation for the potential in this frame, $\Phi_g = -\frac{3}{2}\Omega^2 x^2 - \frac{GM_p}{(x^2+y^2)^{1/2}}$. The minor deficiency of these simulations, and similar those of Tanigawa & Watanabe (2002), was the artificial Keplarian boundary condition at large azimuth from the protoplanet. We demonstrate here that the modification of the global disk structure will significantly alter the accretion rate onto the embedded protoplanets.

To demonstrate the importance of the boundary condition, we utilise the Bernoulli constant to express the surface density at the sonic point (Σ_s) in terms of the surface density at infinity (Σ_∞), as $\Sigma_s \equiv \Sigma(x_s, y_s) = \Sigma_\infty \exp\left[-\beta\frac{R_R^2}{H^2} - \frac{1}{2}\right]$. Following the standard Bondi approach, we can write the mass accretion rate as

$$\dot{M}_p \simeq 2\pi R_R c_s \Sigma_s = 2\pi R_R H \Omega \Sigma_\infty \exp\left[-\beta\left(\frac{R_R^2}{H^2}\right) - \frac{1}{2}\right]. \quad (2.1)$$

The quantity β in the above equations encompasses the conditions at ∞, and highlights their importance. If $\beta < 0$, then the accretion rate, $\dot{M}_p$, decreases with increasing scale-height, H. However, if $\beta > 0$ then $\dot{M}_p$ increases with H.

In a self-consistent treatment of the disk, the planet will modify the density distribution throughout the entire disk. In the limit of severe surface density depletion, the magnitude of v_y within the gap may be substantially reduced from the Keplerian approximation. In this limit $v_y^2(x_\infty, y_\infty) = (9/4)\Omega^2 x_\infty^2 (1 - f_{\rm sub})$, where $f_{\rm sub} = \frac{2}{3}\frac{H}{x}\frac{H}{a}\frac{\partial \ln\Sigma}{\partial \ln x}$, and thus $\beta \simeq 27 f_{\rm sub}/2 > 0$. For any value of $0 < f_{\rm sub} < 1$, $\beta > 0$ for all streamlines with $x_\infty > \left(\sqrt{12}/(1+3f_{\rm sub})\right) R_R$. Thus, in cold disks ($H < R_R$) the tidal barrier suppresses the flow in the azimuthal direction such that only a small fraction of the gas which is

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diffused into the gap can actually reach its Roche lobe. Consequently, the accretion rate derived from Equation (2.1) is substantially smaller than that obtained when neglecting the effect of the azimuthal tidal barrier.

We can apply the appropriate boundary condition in protostellar disks to determine the asymptotic mass of protoplanets. Using the 2D mass accretion rate given by Equation (2.1), together with the approximate global depletion formula $\Sigma = \Sigma_0 \exp(-t/\tau_{\rm dep})$ (Ida & Lin 2004), we deduce an asymptotic mass for a protoplanet of

$$\frac{M_p}{M_*} = \frac{3}{|\beta|^{3/2}} \left(\frac{H}{a}\right)^3 \left[\ln \frac{8\pi|\beta|}{9} \left(\frac{a M_{\rm gap} \tau_{\rm dep}}{H M_* P}\right)\right]^{3/2} \tag{2.2}$$

where $M_{\rm gap} = \pi \epsilon \Sigma_o a^2$ is the characteristic mass associate with the gap region. For $\beta \sim 1$ and $\epsilon \sim 10^{-3}$, the above asymptotic mass would yield $R_R \sim H$. To calculate the actual values of β and f_{sub}, we must utilise numerical simulations.

3. Numerical Results

The 2D numerical scheme we use is a fully parallel hydrodynamical code with which the continuity and momentum equations are solved on a fixed Eulerian grid in 2D cylindrical coordinates (r, ϕ). We solve the standard continuity and momentum equations for a global 2D inviscid disk. We have adopted an isothermal equation of state and solve the equations in a frame rotating at the same angular frequency Ω as the planet. Following Tanigawa & Watanabe (2002), a $1M_{jup}$ planet is held fixed at $1AU$ and mass is accreted from the region $r_{sink} \leqslant 0.5R_R$ at a rate given by $\Sigma\,(t + \Delta t) = \Sigma\,(t)\,[1 - \Delta t/\tau_{sink}]$.

We present the results of two models in which $H/a = 0.04$ and 0.07 respectively. Figure (1) shows both the total mass and accretion timescale as a function of orbital period for these models. The accretion rates approach a steady state on the time scale of 100 orbital periods. Scaled to $5AU$, we see that the growth timescale becomes comparable to the gas depletion timescale. The results in Figure (1) clearly indicate that the mass of a planet embedded in a disk with a lower aspect ratio levels off at significantly lower mass than one embedded in a disk with larger (H/a). Based on these results, we compute, from Equation (2.1), the value of

$$\beta \simeq \left(\frac{R_R^2}{H_2^2} - \frac{R_R^2}{H_1^2}\right)^{-1} \ln\left(\frac{\dot{M}_1 H_2}{\dot{M}_2 H_1}\right) \tag{3.1}$$

where the subscripts 1, 2 refer to the values for the two models. We find the value of β increases from a very small value to the order of unity after ~ 50 orbits, in agreement with our analytic predictions.

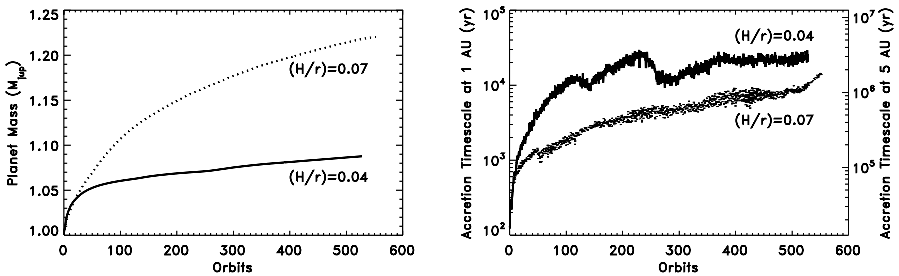

Figure 1. The total mass of the planet as a function of time (left). The solid line shows results for $\left(\frac{H}{a}\right) = 0.04$, while the dotted line shows $\left(\frac{H}{a}\right) = 0.07$. The right-hand panel shows the accretion timescale for both simulations. The left-hand ordinate shows the timescale for a planet at 1 AU, while the right-hand ordinate shows the timescale for a planet located at 5 AU.

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4. Conclusion

In this paper, we present evidence to suggest that the asymptotic mass of protoplanets is determined by the structure of their nascent disk. Although the tidal interaction between the disk and massive protoplanet leads to the formation of a gap, disk gas may continue to diffuse into the gap. The main physical process highlighted here is that the tidal potential of the central star provides a barrier to the gas flow in the vicinity of the protoplanet, even if gas is allowed to diffuse into the gap. Only gas that is able to overcome this barrier is accreted. In the limit that the protoplanets' mass is small and its Roche radius is smaller than the disk thickness, the tidal potential barrier is shallow and modest surface density variation would provide adequate pressure gradient to overcome the tidal potential barrier while preserving vortensity along the stream line. However, in cold disks with thicknesses less than the protoplanets' Roche radius, a very large surface density gradient is required to create the critical pressure gradient force needed to overcome the tidal barrier along the stream line. Consequently, mass supply from the feeding zone into the vicinity of the protoplanet is quenched.

Based on this consideration, we suggest that the asymptotic mass of the protoplanet is determined by the condition $R_R \sim H$. This condition is similar to the thermal criterion for gap formation (Lin & Papaloizou 1986). The co-existence of multiple gas-giant planets around the same host star also suggests that their asymptotic mass is related to the local tidal truncation (through gap formation) rather than the global depletion of the disk. The asymptotic mass distribution within such systems is determined by both the planets' formation location and epoch. In a steady-state disk, the value of the aspect ratio H/a determined from the disk mid-plane structure generally increases with a (Garaud & Lin 2007). In a viscous evolving disk, Σ in the outer regions of the disk decreases with a much more rapidly than in a steady disk. In general, H/a attains a maximum at a radial location $a_{\max}$ which is an increasing function of the the accretion rate through the disk. During the depletion of the disk, $a_{\max}$ decreases. Outside $a_{\max}$, the disk is shielded from the stellar irradiation and the density scale decline rapidly with radius. This process may determine the mass distribution within multiple gas giant planet systems. It may also limit the domain of gas-giant formation.

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