WR BINARIES : THEORETICAL ASPECTS.

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Abstract.

The different channels for the formation of WR-stars, as suggested in the literature, are investigated. The presently available tools, in terms of evolutionary recipies, are reviewed and the results investigated of the use of these tools, with respect to the WR-binaries.

Two of the three basic formation channels, mentioned in the literature, may serve as ways to obtain the presently observed group of WR-binaries : stellar wind mass loss in O-stars during the pre-WR phase and mass transfer in a massive close binary system. Discrepancies with observations necessitated the incorporation of stellar wind in the binary scenario.

Theoretical developments in the last years are reviewed and confronted with the observations. The necessity of mass transfer in the formation process is argued for a major fraction of the double-lined WR-binaries. Using specific evolutionary ingredients, the original parameters of the systems are calculated. We also discuss the appearance and disappearance of double-lined WR-systems as well as the existence of eccentric WR-systems after mass transfer. The evolution of a number of specific systems is discussed individually.

1. Introduction.

In what follows I will use the same ingredients for all model estimates, unless otherwise stated (in a very few cases). The ingredients are :

a) The models of Maeder and Meynet (1987) for main sequence stars. These models show an error in the calculation of the stellar wind mass loss, by using the (erroneous) formulae of de Jager and Nieuwenhuyzen (1986) instead of those of de Jager et al. (1988). We stick to them for reasons of consistency with the data in Smith and Maeder (1989), and because we couldn't recalculate the complete series in time at the Institute in Brussels.

b) The theoretical models for Wolf-Rayet stars, published by Langer (1989 a).

c) The formula for mass-dependent mass loss by Langer (1989 b).

d) The characteristics of double-lined WR binaries, as published by Smith and Maeder (1989).

I do not claim that these are the best, or even the right ingredients to model WR-stars, but making a choice has the advantage of avoiding endless discussions on right or wrong, and when largely applicated to observed systems, it can be tested for anomalies anyway. For the mass loss rate of Wolf-Rayet stars, Bandiera and Turolla (1990) propose a relation $M \sim M_{WR}^{2.3}$ using an analytical approach. Other (but

K. A. van der Hucht and B. Hidayat (eds.),
Wolf-Rayet Stars and Interrelations with Other Massive Stars in Galaxies, 213–228.

often comparable) values for the masses of the components in WR binaries can be found in the papers of van der Hucht et al. (1988), Schulte-Ladbeck (1989) and Moffat et al. (1990), to name but a few.

If we adopt the common idea that WR stars are helium stars, with eventually a small hydrogen envelope, then it is easy to understand why they originate from the more massive stars. If we take $X_{at} \leq 0.3$-0.4 as a necessary (but not sufficient) condition to have a WR star, then the X_c - q_c relation during core hydrogen burning (with $q_c = M_{cc}/M^*$) for stars with masses 15 to 40 $M_\odot$ shows that the smaller the mass of the star, the larger the fraction that has to be removed to get a low hydrogen abundance (figure 1).

On the other hand the stellar wind decreases strongly with lower initial stellar mass, so less mass is removed in smaller mass stars (the increase of main sequence lifetime is not able to compensate the decrease in M.

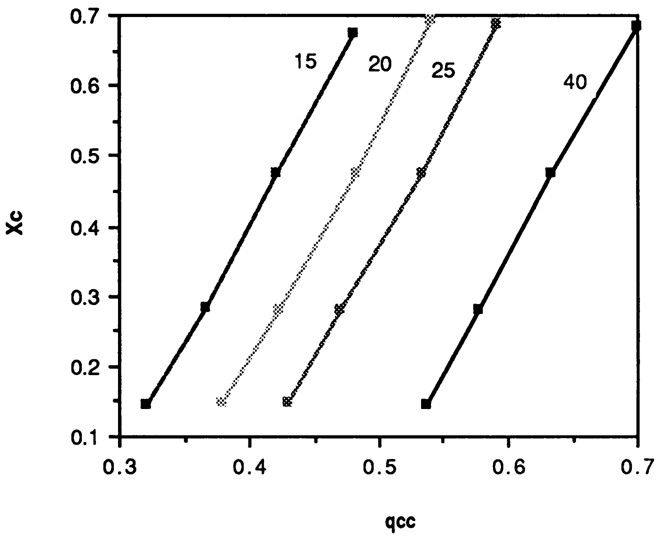

Figure 1. The central hydrogen content by mass (X_c) as a function of the relative convective core mass q_{cc}, for different initial masses of the star (given in $M_\odot$ at the curves).

However, mass transfer in binaries provides a natural and efficient way (externally provocated) to remove the excess of mass . Moreover, the smaller the mass of the loser, the larger the fraction of mass that is removed (typically 30 % for 40 $M_\odot$ to 60 % for 15 $M_\odot$). Therefore, this external instability can extend the mass range for WR stars downwards.

2. Observations of WR-binaries.

Because this subject is treated in more detail elsewhere, I will confine myself to a brief general description, with emphasis on quantities important for evolutionary models.

Approximately half of the WR stars are found in close binary systems. Although in many WR binaries the radial velocity variations of both components can be measured, the derivation of a spectroscopic orbit remains difficult. The WN4 binary HD90657 is an example of a system where the orbital solution depends on the line measurements.

Doom (1987) reviewed the masses of the WR (SB2) binaries, mostly using values derived by Hellings (1985, see also De Greve et al., 1987). He concluded that the average WN mass in these systems was 16.1 M_O, whereas for the WC stars the mass averages to 13.5 M_O. The respective average mass ratios are 0.52 and 0.42.

The data for the SB2 systems are taken from table 1 of the paper of Smith and Maeder (1989) that lists 27 systems. Within the framework of binary evolution only 16 are of interest, because only for those an estimate of the mass of both components is available. In fact only V444 Cyg and CQ Cep may have reliable mass estimates because the value of their inclination results from core eclipse of the WR star. This table also shows the interaction between models and observations : the inclinations of HD 63099, HD 94546 and HD 190918 (and hence the mass of the WR star) depend on the mass of the O-star. The latter is assigned according to its spectral type.

Let us first recall how good such an approximation is (or how bad).

Using the simple assumption of a main sequence location leads to the following mass ranges for the O-stars of the systems mentioned, if we adopt the models of Maeder and Meynet (other models with different stellar wind mass loss will lead to different mass ranges !) :

WR 9 - HD 63099 :	25 - 90 M_O	(i = 66° - 37°)
WR 31- HD 94546 :	20 - 88 M_O	(i = 43° - 25°)
WR 133 - HD 190918 :	15 - 85 M_O	(i = 20° - 11°)

Can we narrow these ranges using constraints from the luminosity ?

WR 9 (HD 63099) and WR 133 (HD 190918) are both located in a cluster or association. However, the membership of WR 9 (to Pup b or Anon) is doubtful (Smith and Maeder, 1989).

WR 133 is a member of NGC6871 (Lundstrom and Stenholm, 1984), and an $M_V = -4.3$ is derived for the WR star. The M_V (O9Ib) = - 6.3 leads to $M_{bol} = -9.4$ (Conti et al., 1983), or log $L/L_O = 5.66$. Using the same theoretical models this results in

M (OB) = 40 M_O M(WR) = 17 M_O i = 14°

The values, determined by Schulte-Ladbeck, purely from observational elements, are M(WR) = 13 M_O and M(O) = 29 M_O, but she consideres it (correctly) as a less well known system.

In the past the WR-star was attributed masses in the range 4 to 9 M_O. Vanbeveren (1989) considers the large period and the late spectral type of the secondary as indications of a relatively late non-conservative case B mass transfer. According to this author the original primary had a mass of $\sim$40 M_O. He adopted the same mass value for the WR star, 9.1 M_O, as Massey (1982).

In any case, with the values resulting from the luminosity, the system is no longer among the binaries with small masses of the WR-star.

3. Is there a need...?

Do we need mass transfer to transform O + O type systems into WR + O systems (or WR + B systems) ?

Why is the question interesting in the first place? If mass transfer is the dominant formation process in binaries with moderate masses, we can examine the initial (and perhaps also the final) conditions of the WR stage in a more secure way, because the mass transfer process is fairly well understood (although I can give a 2 pages long list of unsolved or unsufficiently solved problems). And this can throw light on our knowledge of the other formation channels. On the other hand, if mass transfer is not really necessary, we then have to examine the conditions for the mass loss to form the presently known systems.

De Greve, Hellings and van den Heuvel (1988) argue that at least some of the known systems (WR 133, 139, 31, and possibly also systems like WR 21 and WR 9) were formed primarily through mass exchange. The basic argument is the pronounced mass ratio ($\sim$ 3) and the rather short period (4-15 days). The idea behind it is that a simultaneous evolution without rejuvenation by accretion leads to a too late spectral type for the secondary, at the time of formation of the WR star.

If the mass ratio is coupled to standard characteristics of stellar evolution of massive stars (main sequence mass loss rate by stellar wind of the order of 10^{-6} $M_{\odot}$/yr, helium burning lifetime equal to 10 % or less of the main sequence lifetime and WR mass loss rate an order of magnitude larger (or more) than the main sequence mass loss rate), then a lower limit can be derived for the main sequence lifetime when only stellar wind is involved. This condition is not fulfilled for a number of systems.

Moffat, Niemela and Marraco (1990) investigated the spectroscopic orbits of WC binaries in the Magellanic Clouds and the implications for WR-evolution. They argue that a mass-luminosity relation for massive stars of the type $L \sim M^3$ allows a greater spread in age and therefore easily explains the early-type main sequence companions in some WC + O binaries, without the necessity of mass transfer to rejuvenate the companion star.

Let us first look at the mass luminosity relation on the main sequence. Because both mass loss and overshooting determine the HRD-position of the star, Figueiredo et al. (1990, preprint) calculated a number of evolutionary series for different values of the stellar wind mass loss rate and the overshooting parameter (masses between 10 and 60 $M_{\odot}$). The central set was calculated with the mass loss rate given by de Jager et al. (1988) and the overshooting parameter a= 0.25. For the central set the relation is

$$\log L/L_{\odot} = 1.522 + 2.468 \log M/M_{\odot} \qquad (1)$$

implying an exponent of 2.5 in the theoretical models. With such models too late spectral types are encountered if mass transfer is not introduced. Other values for the mass loss rate and the overshooting lead to similar relations with only a small variation of the exponent.

But also if we make a very rough estimate of the masses of the progenitors of the WR stars in WR + O binaries, we derive conditions that unevitably lead to previous interaction between the components of several of the observed WR binaries. To show this, we proceed in the following simple way.

We use the list of masses of Smith and Maeder (1989). If we look at the evolutionary tracks in the mass range 25 to 120 $M_{\odot}$ (using the models of Maeder and Meynet, 1986), we find a difference in luminosity between the helium burning stage and the ZAMS. In logarithm this difference varies from

$\Delta \log L/L_{\odot} = 0.25$ (25 $M_{\odot}$), over 0.38 (40 $M_{\odot}$), to 0.25 (120 $M_{\odot}$). For the sake of simplicity we take

$$\log L/L_{\odot}(\text{ZAMS}) = \log L/L_{\odot}(\text{helium burning}) - 0.3$$

for the whole mass range.

The luminosities of the observed WR binaries can now readily be transformed into estimated initial masses. The results are given in table 1 for systems with known luminosity of the WR star.

According to this simple exercise the WR stars in O-type binaries originate from stars in the mass range 23 to 85 $M_{\odot}$. The star WR 127 has the smallest progenitor. From a detailed study of V444 Cyg, with theoretical models including mass transfer, De Greve and Doom (1988) found a progenitor mass of 24 $M_{\odot}$, which comes close to the present solution, taking into account their use of models with larger overshooting of the convective core.

If, for each specific mass, we now equal the radius at red point (= right boundary of the main sequence) to the critical Roche radius, we obtain the minimum period for the occurrence of a case B (introduced as P^{I} by Plavec, 1968). Again for the sake of simplicity, we adopted $q_i = 0.7$ in the calculation. The results are given in table 1, together with the observed periods.

Six out of eleven systems have actually a period smaller than the critical one, implying interactive (and probably nonconservative) processes in their past. But even for the systems CV Ser and HD 186943 a short evolution beyond the main sequence would lead to radii large enough to meet the present critical radius. Hence, unless rightward evolution is prevented (by sudden, enhanced mass loss), most of the observed WR + O systems will have encountered an interactive phase during the pre-WR evolution. However, the mass transfer may have been highly nonconservative, with angular momentum loss determining the outcoming period.

Table 1. Luminosities and corresponding masses of the ZAMS progenitors of WR stars in double-lined binaries, and the corresponding minimum period for a case B of mass transfer. The present periods of the systems are given in the last column for comparison.

WR	System	$\log L/L_O$	M_i/M_O	P^I(d)	P_{obs} (d)
22	HD 92740	6.3	85	7.2	80.35
155	CQ Cep	6.03	59	13.8	1.64
47	HDE 311884	6.03	59	13.8	6.34
79	HD 152270	5.7	41	17.7	8.89
11	γ^2Vel	5.7	41	17.7	78.5
138	HD 193077	5.63	38	17.0	2.32
153	GP Cep	5.59	37	16.5	6.69
113	CV Ser	5.54	35	15.0	29.71
133	HD 190918	5.43	32	12.5	112.8
139	V444 Cyg	5.26	27	8.5	4.21
127	HD 186943	5.1	23	7.0	9.55

A second (though weaker) argument comes from the internal structure and the observed HRD location. WR stars are located at the left side of the HRD (small radii) and are considered to have hydrogen poor surfaces.

In order to obtain a surface hydrogen by mass values of $X \sim 0.3$-0.4, accepted for the onset of the WR stage, enough mass must be removed. If stellar wind is the principal actor also for stars with $M_i < 40$ M_O, the removal must take place on the main sequence because of the small present periods.

Using figure 1, and the main sequences lifetimes given by Maeder and Meynet, we arrive at the necessary average mass loss rates given in table 2.

We also calculated the average mass loss rate from observations. This value was obtained by averaging the values, published by de Jager et al. (1988), that are closest to the various evolutionary tracks (using at least 5 values). The required mass loss rates are at least a factor 5 larger than the observed ones.

4. Evolution with mass transfer.

The present state of the art is illustrated by a recent computation by Run Huang and Ron Taam, in a paper entitled "On the nonconservative evolution of massive binary systems" (1990). The authors consider the evolution of a massive system of 40 M_O + 25 M_O through case A and case B, both in the conservative and the nonconservative mode. They take into account the enhancements of the stellar wind by tidal effects and irradiation. Only the former turns out to be important, and then only for case A evolution. They also show that the use of radiaton Roche lobes is inappropriate during the mass transfer stage. The importance of the effect of continuum radiation on the shape of the Roche lobe was previously forwarded by several authors (Schuerman, 1972 ; Kondo and Mc Clusky, 1976 ; Vanbeveren, 1976, 1978 ; Zhou and Leung, 1987). However, in practice, when one component fills its Roche lobe, the radiation field is isotropic at large optical depths within the envelope of the star and one cannot include the radiation pressure term together with gravity.

Table 2. Necessary (average) mass loss rates on the main sequence, to expose layers with X = 0.3-0.4 (log M_c, column 2), and averaged observed mass loss rates along the evolutionary tracks of Maeder and Meynet (1988).

M_i/M_o	log M_c	log M_{obs}
15	- 6.1	- 6.6
20	- 5.9	- 6.4
40	- 5.4	-6.3

Also, the optical depth of the material which is ejected through L_1, is high ($\sim$ 100 for a rate of 10^{-4} M_o/yr). The composition is X = 0.602, Z = 0.044, convection is treated using the Schwarzschild criterion, stellar wind mass loss according to the formula of Waldron (1984), slightly adapted to

$$\log M = 1.07 \log L/L_o + 1.77 \log R/R_o - 13 \qquad (2)$$

Effects of the companion's gravity, continuum radiation pressure and centrifugal force are taken into account by assuming that the rate of mass loss is inversely proportional to the average effective gravitational acceleration on the surface.

The most important features of the 'standard' cases are given in Table 3.

Table 3 : Characteristics at the beginning and the end of mass transfer, for the system 40 M_o + 25 M_o (Huang and Taam, 1990, sequences 1 and 4)

	case A (seq. 1)				case B(seq. 4)			
	M_1	M_2	P X_{c2}	X_{c1}	M_1	M_2	P X_{c2}	X_{c1}
begin	37.0	24.5	3.82 0.34	0.18	34.4	24.3	20.45 0.24	0.00
end	15.4	41.9	5.2 0.32	0.00	16.5	42.0	35.17 0.39	0.00

Sybesma (1986) investigated mass transfer in very massive systems, following case A and including overshooting and stellar wind. He found that the extent and effect of the mass transfer depend on the point in the evolution where mass transfer starts.

This aspect was examined in more detail by De Greve and Doom (1989), using the same kind of models, but concentrating on masses between 20 and 40 M_o. They calculated models of the mass-losing component through case AB of mass transfer (overshooting : Doom (1985) ; stellar wind : Lamers (1981) ; conservative mass transfer), using the code of Prantzos et al. (1986), for various masses and periods (in the range 1.5 to 4 days). From the results and similar results in the literature, several relations are derived between initial and final state, for the mass range 20 to 30 M_o. In particular, they give the maximum value of the helium convective core, which can be considered as an estimate of the mass at the onset of the WC phase. From their calculations it follows that for smaller initial periods, smaller remnant masses of the primary are obtained. For a 20 M_o star, the difference can be as large as 5.3 M_o : from M_{if} = 5.8 M_o (P_i = 1.5 d) to M_{if} = 11.1 M_o (P_i = 10 d). Final periods as small as 2.8 d are found.

Schulte-Ladbeck (1989) considered the masses of WR binaries and compared them with results from evolutionary computations. The inclinations of the WR-binaries were derived previously from polarimetric observations (see Moffat, Niemela and Marraco, 1990), hence no model dependence is found in the derivation of the masses of the components. She compared two series of computations, those published by Vanbeveren (1987, classical Schwarzschild convective core, nonconservative mass transfer) and those of

Doom and De Greve (1983, large, parametrised overshooting and conservative mass transfer). She finds a good agreement between observations and nonconservative binary models with classical cores. The models with large convective overshooting fail to describe systems with WR stars with observed masses > 10 $M_\odot$.

Although I do not want to enter in a debate on overshooting, a remark is necessary on the foregoing conclusion. It depends on the fact that models with a large amount of overshooting do not exhibit (case B) mass transfer. However, that effect is largely dependent on the formalism adopted for the stellar wind mass loss rate. In the models of Doom and De Greve Lamers (1981) formula was adopted, resulting in an absence of case B for $M_i > 33\ M_\odot$.

This formula gives high mass loss rates for that mass range. If instead we adopt the formalism of de Jager et al. (1988), mass loss rates are obtained a factor 2 smaller (averaged over the main sequence).

With such rates, models with masses >33 $M_\odot$ will evolve to the right of the HRD, and encounter mass transfer. This will result in much larger secondaries, and remove much, if not all, of the anomaly obtained by Schulte-Ladbeck. Anyway, I fully agree with one of her other conclusions : binary evolution, especially in the large mass range, might eventually help to set a limit on the amount of required overshooting.

Overshooting of the convective core leads to larger luminosities, thus to larger mass loss rates and therefore to smaller masses at the end of the main sequence. Both effects, stellar wind and overshooting, lead to a lower fraction of mass removed from the loser during mass transfer. Therefore, the gainer will be somewhat less massive, the mass ratio less extreme and the period not so large as in the classic case. Additionally, overshooting leads to larger remnants, because the decreasing hydrogen profile is located further outside in the star. The larger remnants in turn bring down the mass of the secondary, the mass ratio and the period.

Summarizing, the additional elements (overshooting, nonconservative mass loss) are necessary to obtain larger mass ratios M(WR)/M(O) and small periods after mass transfer. With them, it is in principle possible to find a satisfying evolutionary solution for each specific WR-system. Of course, all the solutions together must show coherence in the use of the various parameters.

5. Estimating initial parameters through mass transfer.

In this section I determine the initial parameters of the systems with more or less well determined mass estimates, through the application of mass transfer, and taking into account the present status of the WR star. For the latter I considered the following possibilities :

a) For WN-stars :

1. $M(WN) = M_f$, corresponding to the onset of the WN phase
2. $M(WN) = 0.5(M_f + M(He))$, corresponding to a situation halfway the WN phase

with M_f the mass at the end of the mass transfer, derived from single star models by $M_f = M_{cc}(X_c = 0.3)$. M(He) is the mass of the maximum extent of the helium core. When its outer layers appear at the surface, we may observe the star as a WC star (De Greve and Doom, 1988 a).

b) For the WC stars, we assume that they are at the onset of that stage. However, as Moffat, Niemela and Marraco (1990) argue, a low mass ratio combined with an early WC type may reflect a more advanced WC state.

When severe uncertainties on the mass range are given by Smith and Maeder, we calculate the initial values and the characteristics for the extreme values. The initial mass ratio given is the one assuming conservative mass transfer. The mass loss rates are the values derived from the equation of Langer (1989 b). Where available, we also recall the masses derived earlier on the basis of the simple luminosity assumption of the WR star. The results are given in tables 4 and 5.

Table 4. Characteristics of double-lined WC binaries, and initial parameters assuming case B mass transfer, the mass loss rate is derived from Langer's (1989 b) mass dependent formula ; $M_i(L)$ is the initial mass estimate obtained from the luminosity of the WR star. All masses are in M_O.

System WR	Type	P (d)	M_{WR}	q	M_{i1}	q_i	M_{WC} (10^{-5} Mo/yr)	$M_i(L)$
9	WC 5 + O.7	14.7	10	0.28	30	0.82	3.3	
42	WC 7 + O7V	7.9	8	0.59	25	0.02	1.5	
			14	0.59	38	0.31	7.4	
113	WC 8 + O8-9	29.7	12	0.48	34	0.37	5.6	35
11	WC 8 + O9 I	78.5	17	0.53	44	0.54	13.0	41
			26	0.53	60	0.97	46.0	41
79	WC 7 + O.5-8	8.9	4	0.36	15	0.08	0.4	41
			19	0.36	48	0.92	16.0	41

Table 5. The same as table 4, but for the WN-binaries.

System WR	Type	P (d)	M_{WR}	q	M_{i1}	q_i	M_{WC} (10^{-5} M_O/yr)	$M_i(L)$
139	WN 5 + O6	4.2	10	0.39	24	0.67	3.3	27
					26	0.26	3.2	27
21	WN 4 + O4-6	8.3	10	0.52	24	0.36	3.3	
					26	0.52	3.2	
			14	0.52	31	0.54	7.5	
					34	0.46	7.6	
31	WN 4 + O8V	4.8	7	0.44	18	0.34	1.3	
					20	0.21	1.4	
127	WN 4 + O9Ib	9.6	11	0.5	26	0.45	4.3	23
					29	0.37	4.2	23
			21	0.5	43	0.88	21.0	23
					47	0.83	21.0	23
133	WN 4 + O9Ib	112.8	15	0.43	33	0.83	9.2	32
					36	0.72	9.1	32
47	WN 6 + O	6.3	37	0.9	67	0.61	86.0	59
					72	0.64	92.0	59
			49	0.9	83	0.81	170.0	59
					90	0.91	200.0	59

Nonconservative mass transfer will result in larger q_i values for a given q_{WR}, or, for given q_i, nonconservative mass transfer leads to larger q_{WR} (with q_i and q_{WR} defined by inversed ratios, to obtain always values smaller than 1 : $q_i = \frac{M_{2i}}{M_{1i}}$, $q_{WR} = \frac{M_{WR}}{M_2}$).

Figure 2 shows the variation of the mass ratio during the WR phase, for an initial system of 30 $M_\odot$ + 24.6 $M_\odot$ (used previously for WR9 in the conservative case) for various fractions of accreted mass (expressed in transferred mass). In the conservative case ($\beta = 1$), the mass ratio evolves from 0.4 to 0.28 at the onset of the WC stage ($M_{WR} = 9.4\ M_\odot$, cfr. WR9), and reaches 0.14 at the end of the helium burning stage ($M_{WR} = 4.8\ M_\odot$). Only strong mass loss from the system (more than 50 % of the mass transferred) can change this range severely. In the extreme case that no mass is accreted ($\beta = 0$), the mass ratio varies from 0.63 over 0.44 to 0.22 (remark that this extreme case means a mass loss of 25 % of the total mass at the onset of mass transfer, and 22 % of the total initial mass).

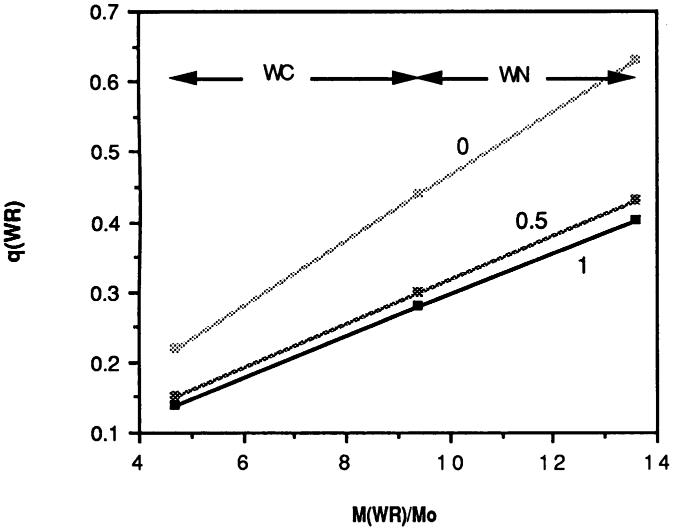

Figure 2. Evolution of the mass ratio during the WR phase, for a system of initially 30 $M_\odot$ + 24.6 $M_\odot$. The WN and WC phase are indicated. The curves are labeled with β, the fraction of transferred mass accreted by the secondary component during mass transfer.

Summarizing, for a system of 30 $M_\odot$ + 25 $M_\odot$, mass transfer (conservative and non-conservative) results in possible mass ratios for WN-binaries in the range 0.63-0.28, and in the range 0.43-0.14 for WC-binaries.

The behaviour of the mass ratio during the WN and WC phase depends strongly on the mass loss rate applied for the WR star (here the mass dependent formula of Langer, 1990) and the definition of the onset of the WC phase. Those two parameters also determine the length of both phases. Figure 3 shows the ratio of the WC lifetime to the helium burning lifetime as a function of the initial WR mass. The smaller this mass is, the smaller the relative duration of the WC phase is. For masses $M_{WRi} \geq 25\ M_\odot$, the star will practically spend the whole helium burning phase as a WC star. Of course, it is possible that the WR phenomenon disappears before the end of the helium burning. This will reduce the WC phase. In figure 3 we assumed the WR phase to stop at 70 % of the helium burning lifetime, resulting in the lower curve.

Finally, we note that the mass loss rates given by the mass dependent equation of Langer, leads to results in agreement with the observations (within a factor 3), eccept for WR 133 (but what is well known for this system ?).

6. What about WR+WR systems and WR+c systems?

The existence of the former systems was predicted by Doom and De Greve (1981), as a result of the evolution of rather massive systems with mass ratios close to 1. Two candidate systems, showing spectral features of a WN and WC star, MR111 and GP Cep, were examined in more detail by Massey and Grove (1989). They found that the lines vary in phase, hence the WN and the WC characteristics are located in one star.

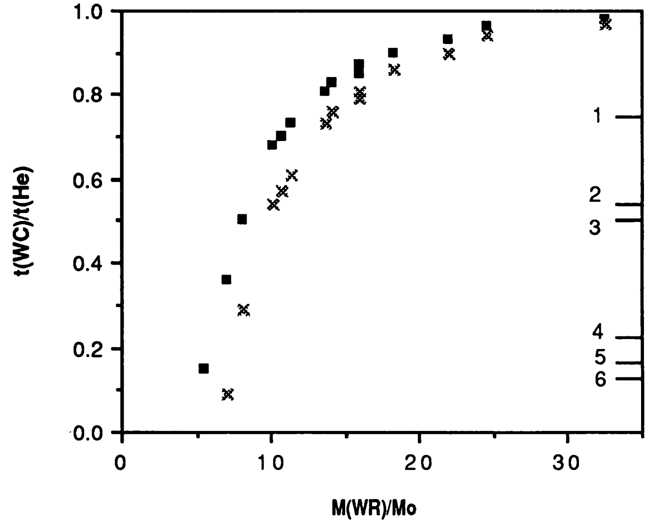

Figure 3. t(WC)/t(WR) as a function of initial WR mass M_{WRi} (upper curve, with t(WR) = t(He)). The lower curve is calculated assuming that t(WR) = 0.70 t(He). For comparison, similar ratios, given by Arnault et al. (1989) for different regions, are shown :

(1) SN 6-7.5 kpc ; (2) SN 7.5-9.5 kpc ; (3) SN 9.5-11 kpc ; (4) LMC ; (5) 30 Dor ; (6) SMC.

For MR111 the unseen companion should have a mass M > 15 M_0 and the WR-star is the less massive one. There is no sign of the companion spectroscopically because of the very large reddening of MR111, which makes it difficult to obtain an adequate signal-to-noise ratio in the blue. GP Cep turns out to be a double pair with P = 6.7 days for the O + WR.

The theoretical existence of the WR + WR phase in massive close binary evolution was forwarded by Doom and De Greve (1981). The essential requirements are that the remaining hydrogen burning lifetime of the mass accreting component is shorter than the WR lifetime of the primary, and that the following reversed mass transfer from secondary to primary is highly nonconservative.

We investigated the relevant timescales for $M_{1i} = 40\ M_O$ and $20\ M_O$, $q_i = 0.9$ and 0.95 and $\beta = 1$ (conservative) and 0.5.

We found that no WR + WR phase occurred for $q_i = 0.9$ (both values of β) and for $q_i = 0.95$, $\beta = 0.5$. Only in the conservative case both components become a WR star simultaneously. For the 20 M_O star, this phase lasts 73000 years, during which a WC + WN pair is seen with approximate masses 4 and 11 M_O. For the 40 M_O star, the phase lasts 27000 years and the masses are about 6 M_O and 20 M_O. In both cases, the magnitude difference between the two components is about 1. This problem is investigated in more detail elsewhere (De Greve, 1990).

Vanbeveren (1988b) presented a statistical study on the distribution of different groups of massive stars, and outlined the influence of mass transfer on the relative frequency of these groups. According to this study, the number of O- and WR-stars with compact companions is very low (< 5 %). He estimates that 5 to 20 % of the supernovae (from massive stars) results from WR-stars.

7. Conservative or non-conservative ?

Nonconservative mass transfer leads to lower masses for the resulting accretion star. The influence on the period depends strongly on the amount of the angular momentum the leaving gas is taking away.

The amount of mass lost from the system during mass transfer is still a free parameter. For massive stars external mass loss was introduced a long time ago to obtain better agreement between the required initial mass ratios for WR-binaries and the observed mass ratios for massive binaries (Abbott and Conti, 1987).

Recently, Meurs and van den Heuvel (1989) examined the number of evolved early-type close binaries in the Galaxy, under the assumptions of steady state star formation, case B of mass transfer, and stellar wind mass loss.

If for the helium star $M = 6 \pm 1\ M_O$ is adopted ($M_i \geq 20\text{-}25\ M_O$) and $\beta \leq 0.5$, then the observed and predicted numbers of WR binaries within 3 kpc from the sun match. In that case the number of WR + compact star is expected to be 0.05 to 0.1 times the number of normal WR binaries. They conclude that a value of $\beta = 0.5$ can readily be adopted, leading to a lower mass limit for the WR stars of 5 M_O.

8. Eccentric systems.

Some of the WR binaries have large eccentricities. How does this fit into the binary evolution scheme ?

Tassoul (1987, 1988) discussed the effects of circularisation and synchronisation in early-type, detached close binaries. Synchronisation in early type stars is not only achieved by tidal friction, but also by large-scale meridional flows.

Furthermore, the hydrodynamical spin-down circularizes an eccentric orbit in a much more effective way than Zahn's (1977) radiative damping on the dynamical tide. The equation determining the circularisation takes the form

$$t_{cir}(yr) = 9.4\ 10^{4-N/4}\ \frac{(1+q)^{2/3}}{r_g^2}\ \left(\frac{L_O}{L}\right)^{1/4}\left(\frac{M}{M_O}\right)^{23/12}\left(\frac{R_O}{R}\right)^{5}(P_O(day))^{49/12} \qquad (3)$$

The factor $10^{-N/4}$ represents the effect of turbulence on the spin-down mechanism. Physically, it stands for the exchange of angular momentum between the inviscid interior and a thin Ekman-type boundary layer. For early-type binaries N is a small number and we may adopt N = 0.

Table 6. Circularisation times t_{circ} calculated with eq. (3), for massive main sequence binaries, compared to the main sequence lifetime t_{ms}. Masses are in M_o, time in years.

M_{1i}	q_i	r^2_g	t_{circ}	t_{ms}
60	0.9	0.085	1.2 E9	3.4 E 6
	0.3		9.5 E8	
40	0.9	0.082	8.0 E 8	4.8 E 6
	0.3		6.2 E 8	
20	0.9	0.074	4.2 E8	8.7 E 6
	0.3		3.3 E 8	

We calculated the circularisation time for ZAMS binaries with primary masses 20 M_o to 60 M_o and mass ratios in the range 0.9 to 0.3. The values for the radius of gyration were taken from De Greve (1976), with extrapolation of the 60 M_o value. The results, given in table 6, show that the proposed mechanism is not able to circularize the orbits of the massive O-binaries during the main sequence. The values were calculated for a period of 10 d. A period of 5 days leads to circularisation times some 40 % of those in table 5.

The circularisation times are always about two orders of magnitude larger than the main sequence lifetime. Therefore, massive binaries with a substantial eccentricity will enter the advanced stages with it. However, as shown by Tassoul $t_{circ} < t_{ms}$ is indeed obtained for smaller masses ($M_i < 10\ M_o$).

A study of the origin and evolution of the eccentric, large period system WR 140 (e = 0.8, P = 7.9 yr) demonstrates that such systems can readily be formed through mass transfer, without circularisation of the orbit, at least not through effects of stellar wind, mass transfer or tidal forces (De Greve, poster, this symposium).

9. Appearance and disappearance of double-lined WR binaries.

For a WR binary to be observable as a double-lined binary, the visual magnitudes of the components must have approximately the same value.

De Greve et al. (1988) describe the characteristics of a system of initially 40 M_o + 20 M_o, based on models of Doom (1985), at the beginning and end of core helium burning. Using a B.C. of -4.5 (Smith and Maeder, 1989) for the WR star and a B.C. of -3.0 for the 19 M_o main sequence companion, we can readily arrive at the visual magnitudes :

Begin WR : M(WR) = 27 M_o, M_{vis} = - 5.5 M(O) = 19 M_o, M_{vis} = - 4.4

End WR : M(WR) = 13 M_o, M_{vis} = - 4.7 M(O) = 19 M_o, M_{vis} = - 4.5

In this case, the companion star, a O9.5-BO star, will be hardly visible in the spectrum at the onset of the WR phase, but has about the same visual magnitude at the end.

Such a system might start its WR state as a single-lined WN binary ($f(M) = 3.24 \sin^3 i$), and show up as a double-lined WR+O system when the WR star progresses to the WC state. For a larger initial mass ratio, the reverse may take place (cfr. poster of de Loore and De Greve), i.e the system might first evolve as a double lined WN binary, and lateron, during the WC phase, turn into a single-lined O-type binary with a WC component. The same problem was also investigated by De Greve et al. (1988), when investigating the absence of observed WR + B binaries.

More generally, if we assume as before, BC -4.5 for the WR star, and -3.5 for the O type companion (- 3 is more appropriate in case of an early type B companion), we can relate the luminosities to the visual magnitudes M_V through

Table 7. Luminosity difference Δ log L = log L(WR) - log L(O) at the beginning of the WR phase and the WC phase (resp. Δ log L(i) and Δ log L(WC)), and at the end of the WR phase (Δ log L(e)), with t(WR) = t(He). The various solutions for each system correspond to the results in tables 3 and 4.

WR	Δ log L(i)	Δ log L(WC)	Δ log L(e)
9	0.07	- 0.13	- 0.90
42	0.99	0.58	0.12
	0.71	0.51	- 0.37
113	0.54	0.32	- 0.46
11	0.21	0.12	- 1.10
	0.47	0.30	- 0.69
79	0.49	0.13	- 0.17
	0.18	0.04	- 1.03
139	0.10	- 0.31	- 0.75
	0.59	0.20	- 0.30
21	0.40	- 0.01	- 0.45
	0.82	- 0.11	- 0.61
	0.34	0.01	- 0.64
	0.47	0.19	- 0.56
31	0.27	- 0.27	- 0.46
	0.50	0.0	- 0.27
127	0.35	- 0.04	- 0.54
	0.50	0.17	- 0.44
	0.17	0.0	- 0.98
	0.24	0.09	- 0.95
133	0.12	- 0.15	- 0.90
	0.24	0.0	- 0.82
47	0.41	0.41	- 1.04
	0.55	0.52	- 0.84
	0.56	0.51	- 0.80
	0.35	0.35	- 1.13

$$\Delta M_V = M_{V,WR} - M_{V,O} = 2.5(\log L/L_o(O) - \log L/L_o(WR)) + 1 \qquad (5)$$

Applying this to the calculations for the individual systems in section 5, using the mass-luminosity law given in section 3, allows to verify the spectral visibility. We assume that the O-type companion becomes unobservable when the magnitude difference is about 1 magnitude or larger (Hynek, 1961; Massey, 1982). The systems will then be seen as a single-lined WR binary if $\Delta M_V \leq - 1$, or

$$\log L/L_o(WR) \geq \log L/L_o(O) + 0.8. \qquad (6)$$

Using relation (6), we calculated the luminosity difference for the systems in table 4 and 5, at the onset of the WR phase, the onset of the WC phase and at the end of the WR phase (= end of the helium burning phase). The results are given in table 7. for the first two phases, the difference Δ log L = log L(WR) - log L(O) remains positive for the majority of the solution (mostly between 0.5 and 0.2, except for WR 42, with values of 0.99 and 0.71 at the onset), whereas this difference is negative at the end of the WR phase (and exceeding - 0.8 for the systems WR 133, WR 47, and partly for WR 11 and WR 79). One can make the same reasoning in the case that the O-star becomes visually more luminous than the WR star (though the WR star may remain visible for a larger magnitude difference through its emission lines). In that case, the WR star will be unobservable during the last part of the WC phase.

The fraction of time spent as a single-lined O-type binary (with mass function typically 0.1 $\sin^3 i$) increases for larger masses of the WR star, larger initial mass ratios, or small mass loss from the system. For further details I refer to the contribution of de Loore and De Greve (this volume).

Acknowledgement.

This work has been supported by the National Fund of Scientific Research (NFWO - Belgium) under grant No. S 2/5 - LV. E96.

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DISCUSSION

Conti: It appears that some mass exchange, even in eccentric systems, should lead to matter appearing on the O star companion. For example, γ Vel, a WC star, should have reduced nitrogen rich material on the surface of its O supergiant companion. This is not observed. Do you have an idea as to why we do not see any anomalies?
De Greve: During accretion various mixing processes (due to rotation or a Rayleigh-Taylor instability resulting from the μ-inversion) may lead to almost normal abundances (*cf.* De Greve and Cugier, 1989).

Cherepashchuk: What can you say about the formation of WO+O binary systems?
De Greve: My study did not look into that. However, from the present results WO+O are formed from the most massive systems.

Vanbeveren: I would like to comment on the effect of radiation pressure on the critical Roche lobe. This effect does not affect the Roche lobe of the loser, but it becomes important once the mass transfer sets in.
De Greve: Huang and Taam argue just the contrary. Once the primary fills its Roche volume, the radiation is isotropic to a large optical depth in the envelope, so one cannot include the radiation pressure term together with gravity. On the other hand, the stream surrounding the secondary has a large optical depth due to electron scattering (typical 100 for $\dot{M} \simeq 10^{-4} M_{\odot} yr^{-1}$), preventing radiation from this star to exert a substantial force on a test particle outside the two components.

Underhill: If you accept the proposal that WR stars are young stars just coming into the ZAMS band, then any combination of O/B absorption line massive star with a less massive WR star can easily occur. Once the WR spectrum star loses its magnetic field then you will have two absorption line stars in a binary system. Because the more massive component evolves faster than the other star it will always be past a possible WR stage before the less massive star.

Maeder: I am always surprised that people in binary evolution consider, apart from mass transfer, that the components in a binary system evolve like single stars. Long before mass transfer occurs, binary interaction could drive internal mixing by tidal interaction and the resulting baroclinic instabilities.
De Greve: I agree that if such effect works efficiently on a timescale comparable to the main sequence, it should be taken into account.

Schulte-Ladbeck: I have a question about the circularization of binaries. EZ CMa has a short period of about 3.8 days, a rather high excentricity of about 0.3, and is also supposed to have a compact companion. Could you comment on its evolution in the framework of your models, please?
De Greve: The short-period eccentric binaries remain a problem, because for short periods (and small eccentricity) tidal circularization should work quite efficiently. But it is still possible, in view of the presence of a compact companion, that the preceding supernova explosion that produced the latter, provoked such a large eccentricity that it is still not reduced to zero.

Smith, Lindsey: To deduce the initial mass, you assume the WC stars have the mass of the He-burning convective core. However, the WC star evolves rapidly by mass loss from a starting value as high as $40M_\odot$ down to 5 or $10M_\odot$ where it becomes a supernova. You will therefore have underestimated the initial masses.
De Greve: I agree with the argument, and I said so in my talk. Adopting an advance WC stage will indeed increase the upper limit to the initial masses.

Moffat: I think the only way to really answer the question of mass transfer is to do hydrodynamics. Benz is starting this, and others are too, in Belgium I believe, with the 3-D SPH code and for V444 Cyg there are very preliminary calculations: density plots with velocity vectors. In the present V444 Cyg system the mass transfer is zero. I find it difficult to believe that in its predecessor, in which the wind strengths will be even more similar so the interface will be more intermediate between the two stars, that there mass transfer actually took place. That is something I do not understand.