3 Charlotte Square,
Edinburgh, 2.
14*th January* 1964.

The Editor,
T.F.A.
Dear Sir,

In a recent Actuarial Note (*T.F.A.* vol. 28, p. 99) Mr. D. W. A. Donald has drawn attention to the fact that a change in the valuation rate of interest can sometimes induce a greater proportionate change in the value of a redeemable stock than in that of a perpetuity and gives the necessary conditions for this to happen. Earlier (*T.F.A.* vol. 25, p. 388) Messrs. Lundie & Hancock derived in a different context the necessary and sufficient condition, which can be expressed as $n > \frac{1+i}{i-g}$ or $i > \frac{1+ng}{n-1}$, depending upon which variables are regarded as fixed.

These inequalities can be obtained by differentiating log A with respect to i giving $-\frac{1}{i}\left\{1+\frac{v^{n+1}}{A}[n(i-g)-(1+i)]\right\}$ and since $-\frac{1}{i}$ is the proportionate rate of change of a perpetuity the inequality follows.

Critical values of i, above which the event in question occurs, for various combinations of n and g can be tabulated as follows :—

n \ g	·025	·03	·035	·04	·045	·05	·055
10	·139	·144	·150	·156	·161	·167	·172
20	·079	·084	·089	·095	·100	·105	·111
30	·060	·066	·071	·076	·081	·086	·091
40	·051	·056	·062	·067	·072	·077	·082
50	·046	·051	·056	·061	·066	·071	·077
60	·042	·047	·053	·058	·063	·068	·073

It is interesting to pursue further the shape of the curve $R=1-v^n+\frac{i}{g}v^n$ *i.e.* the ratio of a dated stock to a perpetuity of the same coupon.

$$\frac{\partial R}{\partial i}=\frac{v^{n+1}}{g}\{(1+i)-n(i-g)\}$$

$$\frac{\partial^2 R}{\partial i^2}=-\frac{nv^{n+2}}{g}\{2(1+i)-(n+1)(i-g)\}$$

confirming that the critical value gives a maximum and showing that there is a point of inflexion at $i=(ng+g+2)/(n-1)$. Also $R=1$ is an asymptote and when $i=g$ $R=1$, when $i=0$ $R=0$. The shape of the general curve is now clear. A better picture of the dimensions involved can be seen by considering the example of 3½% Funding 1999/2004 which is finally redeemable in just under 41 years. Working in half years,

$$g=\cdot 0175$$

$$n=82$$

$$\text{critical } i=\frac{1+1\cdot 435}{81}=\cdot 03$$

$$\text{point of inflexion at } i=\frac{1\cdot 452+2}{81}=\cdot 0426$$

With the aid of the following values, the graph can be accurately drawn.

i	R
·005	·525
·0075	·690
·01	·810
·0125	·897
·0150	·958
·0175	1·000
·03	1·064
·0425	1·047
·05	1·034
·06	1·020
·07	1·012
·08	1·006

From these figures it can be seen that if the yields are always the same, the likely loss in preferring 3½% Funding 1999/2004 to a perpetuity will exceed the likely gain at all except very low rates of interest. When $i=\cdot 03$ per half year, the loss is certain. This may not be quite what is meant by the expression " protection of a date ".

R

At this date, the price of $3\frac{1}{2}\%$ Funding 1999/2004 is 66·75. The price of $2\frac{1}{2}\%$ Treasury, the nearest available approximation to a risk-free perpetuity, is 42·38, which, on adjustment to a $3\frac{1}{2}\%$ basis, becomes 59·33. The present ratio therefore exceeds 1·12, which suggests that the market takes some other factors into consideration in valuing these securities.

Yours faithfully,

J. B. MARSHALL.