

STATIC DIELECTRIC CONSTANT AS A TEXTURAL INDEX OF SNOW

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ABSTRACT

The dielectric constants of alpine snow samples with different stages of metamorphism and with different liquid water saturations have been measured in the frequency range of 10Hz to 50MHz using a plate condenser and network-analyzer. The limiting static dielectric constant, ϵ_s , has been derived from the measured frequency dependence of the complex permittivity of snow by a least-square-fit using the model of Cole-Cole. A strong dependence of ϵ_s on porosity, liquid water content and on the shape of the snow grains was found. Calculations of shape factors from the measured static permittivity based on the model of Polder and van Santen are given, and are compared to shape factors derived from an analysis of photographs of the snow samples.

INTRODUCTION

Structure and texture parameters like porosity, mean grain size and the shape of the grains are of particular importance for the interpretation of electrical (Kopp 1962, Evans 1965), optical (Warren 1981), mechanical (Bader and Kuroiwa 1962) and hydraulic (Denoth and Seidenbusch 1979) properties of a natural snow cover. Also, the microwave response - attenuation and scattering - depends distinctly on the type of snow (Hofer and Mätzler 1980). Texture parameters of dry snow samples can be derived from an analysis of thin sections (Good 1975); however, this method has not yet been applied to wet snow. Two pieces of information, porosity and liquid water content, can be obtained relatively easily with sufficient accuracy: snow porosity can be calculated from the measured density and the liquid content; liquid content can be measured with a freezing calorimeter or deduced from the high-frequency dielectric constant (Colbeck 1978, Denoth and others 1984, Tiuri and others 1984). Particle shape, however, is most difficult to specify quantitatively.

Under the condition that the inclusions of a mixture of dielectrics can be approximated by ellipsoids, particle shape is defined by the axial ratio and can be derived from an analysis of the low-frequency (static) dielectric constant (cf Hasted 1973). This method has been applied to snow by Bader and Kuroiwa (1962) and Keeler (1969). The static permittivity has been interpreted by the model of Wiener (1910); the form number u has been used to describe, in some detail, the structure of the snow. The form numbers derived from the static permittivity range from 10 to 25; corresponding numbers derived from the limiting high-frequency permittivity, however, are scattered between 2.5 and 10. Therefore, this form number is only of limited usefulness for snow structure characterization. In this paper the static permittivity is interpreted by the model of Polder and van Santen (1946), whereby the shape factor g_s (depolarization factor) of the solid inclusions (snowgrains) is uniquely related to their axial ratio; so this shape factor may be used as a textural index value. Measurements were made in the natural snow cover in the Stubai Alps (3 000 m a.s.l.), Austria.

INSTRUMENTATION

In the model of Polder and van Santen, the static dielectric constant of snow, ϵ_s , is related to porosity ϕ ,

liquid water content by volume W , the shape factor of the solid component g_s , and the shape factor of the liquid inclusions g_l , by:

$$\epsilon_s = \frac{1-\phi}{3} (\epsilon_i-1) \sum_n [1+(\epsilon_i/\epsilon_s-1)g_{s,n}]^{-1} + \frac{\epsilon_w-1}{3} W \sum_n [1+(\epsilon_w/\epsilon_s-1)g_{l,n}]^{-1} \quad (1)$$

ϵ_i , ϵ_w are the static dielectric constants of ice and water, respectively. Typical values of the relative sensitivity $\epsilon_s^{-1} \partial \epsilon_s / \partial x$ of ϵ_s to the parameters $x = (W, \phi, g_s, g_l)$ are given in Table 1, calculated for dry ($W = 0$) and moderate wet snow ($W = 0.05$) with a mean porosity

TABLE 1. SENSITIVITY OF THE STATIC DIELECTRIC CONSTANT ϵ_s TO LIQUID WATER CONTENT W , POROSITY ϕ , SHAPE OF THE SNOW GRAINS g_s AND SHAPE OF THE LIQUID INCLUSIONS g_l . VALUES OF $\epsilon_s^{-1} \partial \epsilon_s / \partial x$, $x = (W, \phi, g_s, g_l)$ ARE CALCULATED FOR A MEAN POROSITY $\phi = 0.5$, A MEAN SHAPE FACTOR $g_s = 0.2$ AND FOR TWO CASES OF WETNESS $W = 0$ AND $W = 0.05$.

snow wetness	$\frac{1}{\epsilon_s} \frac{\partial \epsilon_s}{\partial W}$	$\frac{1}{\epsilon_s} \frac{\partial \epsilon_s}{\partial \phi}$	$\frac{1}{\epsilon_s} \frac{\partial \epsilon_s}{\partial g_s}$	$\frac{1}{\epsilon_s} \frac{\partial \epsilon_s}{\partial g_l}$
$W = 0$	3.4	-3.5	-1.4	—
$W = 0.05$	2.8	-2.9	-1.0	-0.08

$\phi = 0.5$ and a mean shape factor $g_s = 0.2$. Compared to the effects of changes in liquid content, porosity or grain shape, the influence of the shape of the liquid inclusions on ϵ_s is negligibly small. The relative sensitivity of ϵ_s to W and ϕ is higher approximately by a factor of 3 than the sensitivity to g_s . This fact implies the need for precise measurements of the dielectric constant, snow porosity and wetness. Due to the effects of ionic conductivity and/or effects of interfacial polarization, the static permittivity can not, in general, be measured directly; it can be determined, however, from the measured frequency dependence of the dielectric properties.

To measure with high accuracy the complex dielectric constant in the frequency range of 10Hz up to 50MHz, a network analyzer with a high-impedance plug-in unit and a rf-generator are used. A plate condenser consisting of 7 stainless steel plates, 10 x 13 cm², with a spacing of 2.1 cm is used as dielectric sensor. A block diagram of this measuring system is shown in Figure 1. The built-in microprocessor allows measurement of the impedance of the snow-filled condenser directly; the voltage division ratio between the test item (condenser) and a known resistance R is

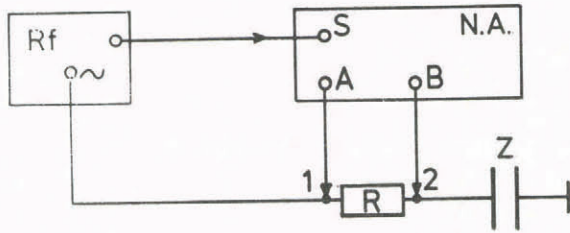


Fig.1. Block diagram of the measuring system. Rf = radio frequency generator; N.A. = network analyzer with two high-impedance inputs A and B, S = synchronization input; R = reference impedance; Z = dielectric sensor, plate condenser; 1 and 2 are probes.

evaluated. Results of measurements of the complex dielectric constant for two significantly different types of snow are given in Figure 2, whereby the real part ϵ' is plotted against the dielectric loss ϵ'' . Curve a in Figure 2 gives a continuous registration of the dielectric properties of a low-density ($\rho = 0.17 \text{ g/cm}^3$), recrystallized snow with some plate-like crystals; snow temperature was 0.4°C , snow wetness $W \sim 0.6\%$. A photograph of this snow sample is shown in Figure 3. Curve b in Figure 2 represents a medium-density ($\rho = 0.553 \text{ g/cm}^3$) wet sample ($W = 3.9\%$) of old snow with rounded grains. A photograph of this type of snow is shown in Figure 4. The frequencies of 1, 10, 100kHz and 1MHz are marked as circles in the plots a and b of Figure 2. Large increase of losses on the low frequency side is due to ionic conductivity and/or a possible secondary relaxation region.

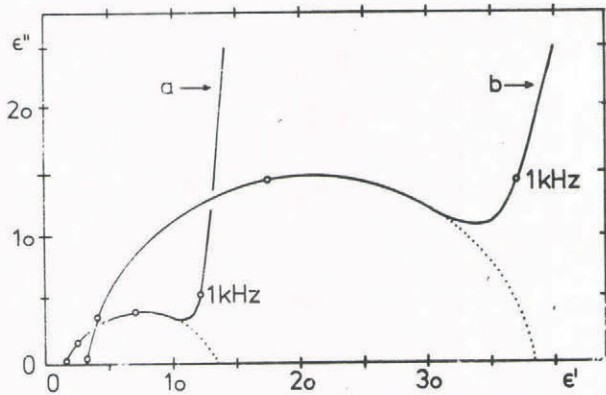


Fig.2. Continuous recording of the complex dielectric constant of a low-density recrystallized snow (a) and a medium-density old snow (b). Frequencies of 1, 10, 100 and 1 000 kHz are marked as circles.

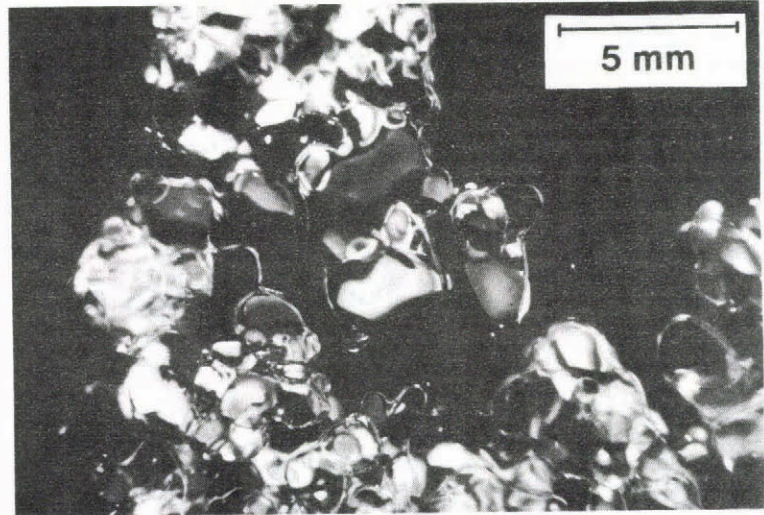


Fig.4. Photograph of the medium-density old snow; cf. Fig.2 (b).

DATA EVALUATION

The limiting static permittivity $\epsilon_s (\omega \rightarrow 0)$ and the limiting high frequency permittivity $\epsilon_\infty (\omega \rightarrow \infty)$ are calculated from the measured frequency dependence of the complex dielectric constant ϵ^* using the model of Cole-Cole which is most suitable for application to snow:

$$\epsilon^* = \epsilon_\infty + (\epsilon_s - \epsilon_\infty) [1 + (i\omega\tau)^{1-\alpha}]^{-1} \tag{2}$$

τ represents the characteristic relaxation time, α accounts for the width of the distribution function of relaxation times and is given by the angle of intersection of the Cole-plot with the ϵ' -axis. To determine the parameters involved by a least-square-fit to the experimental data, it is preferable to apply a straight-line representation of Equation (2) (Böttcher and Bordewijk 1978, Denoth 1982). Liquid water content was measured by a freezing calorimeter, snow porosity was calculated from the density ρ and liquid content W : $\phi = 1 - (\rho - \rho_w \cdot W) / \rho_i$; ρ_i and ρ_w are the densities of ice and water. Mean grain size and the shape factor g_s were derived from an analysis of up to 100 individual grains in photographs of the snow samples (Denoth 1982). A compilation of the characteristic parameters for the measurements represented in Figure 2 is given in Table 2. W_c means the snow wetness calculated from the high-frequency permittivity ϵ_∞ . Upper limits of the corresponding errors are also given.

STATIC PERMITTIVITY

The dependence of static permittivity on porosity ϕ of two different types of dry snow samples with temperatures between -5 and -8°C is shown in Figure 5. The dots represent samples of aged, coarse-grained snow with well rounded crystals and samples of Alpine firn; the crosses stand for samples of new snow measured immediately after deposition, or snow in which some crystalline features have been observed. The lines in the figure represent theoretical borderline cases calculated from Equation (1) for spherical ($g_s = 1/3$) and plate-like ($g_s = 0$) inclusions. Samples of new snow, or snow which has not undergone appreciable transformation, are characterized by a small shape factor corresponding to thin discs; aged, coarse-grained samples are described by shape factors $g_s \leq 1/3$ corresponding to oblate spheroids. In addition, at a certain porosity, the static permittivity of new snow is considerably higher than that of an old snow sample built up of spherical grains; so the experimental data also indicate ϵ_s to be highly sensitive to the shape of the snow grains (cf Table 1). Static permittivity can, therefore, be used as a textural index and shape factors can be calculated from ϵ_s according to Equation (1).

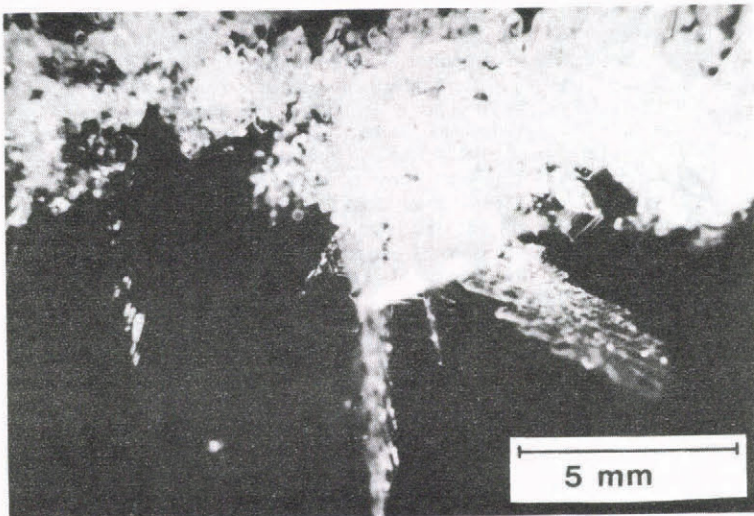


Fig.3. Photograph of the low-density, recrystallized snow; cf. Fig.2 (a).

TABLE II. CHARACTERISTIC PARAMETERS FOR THE MEASUREMENTS REPRESENTED IN FIGURE 2.

snow type	ϕ	W_c (%)	g_s	ϵ_∞	$E(\epsilon_\infty)$	W_d (%)	ϵ_s	$E(\epsilon_s)$	τ 10^{-5}	$E(\tau)$
low-density, recrystallized snow	0.44±0.01	3.7±0.6	0.29±0.05	3.15	0.013	3.9±0.3	38.5	0.04	2.03	0.03
medium-density old snow, rounded crystals	0.78±0.03	0.6±0.5	<0.08	1.51	0.014	0.3±0.2	13.5	0.09	1.82	0.09

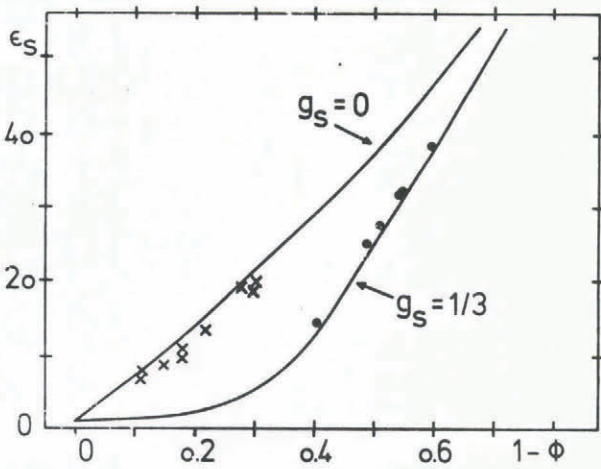


Fig.5. Dependence on porosity ϕ of the static permittivity ϵ_s of dry snow samples: x new snow; o, old, coarse-grained snow. Calculations of borderline cases for plate-like ($g_s = 0$) and spherical ($g_s = 1/3$) grains are shown as solid lines.

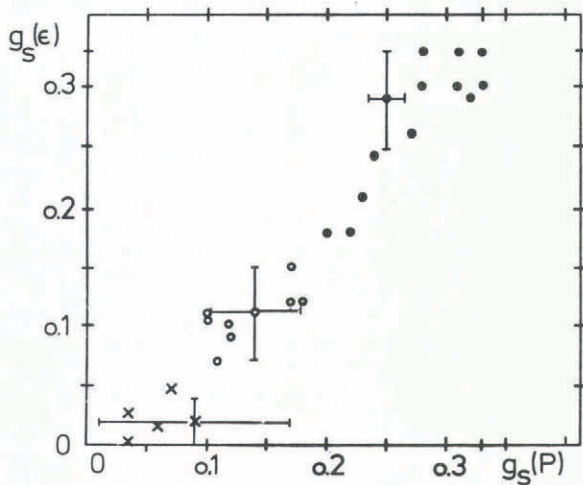


Fig.6. Comparison of the electrical shape factor $g_s(\epsilon)$ to the actual shape factor of the individual snow grains $g_s(P)$ for different types of snow: x = new snow, o = medium-grained, aged snow, • = old coarse-grained snow and Alpine firn. Error bars are also given.

THE SHAPE FACTOR

For snow characterization a correlation is needed between the electrical shape factor (depolarization factor) deduced from the measured static permittivity $g_s(\epsilon)$ and the actual shape factor of the snow grains derived from an analysis of photographs of snow samples, $g_s(P)$. This is shown in Figure 6, where the electrical shape factor $g_s(\epsilon)$ is compared to the mean shape factor $g_s(P)$ of the grains. Crosses stand for new, fine-grained snow, circles represent medium-grained (0.5 - 1mm) aged snow and dots represent old, coarse-grained (>1mm) snow and Alpine firn. Typical error bars are also given. With respect to relatively large errors, data points show a satisfactory correlation. At small shape factors $g_s < 0.2$, however, the photographic analysis gives higher values compared to those derived from ϵ_s , though problems inherent in the photographic technique for these types of snow give this small significance. At larger shape factors $0.2 < g_s < 1/3$, corresponding to nearly spherical grains, determination of g_s by the dielectric method is generally less accurate than determination by the photographic method.

CONCLUSION

Under the condition that static permittivity can be derived with sufficient accuracy $E(\epsilon_s) < 0.1$ from measured frequency dependence of dielectric constant, a shape factor which reflects the mean axial ratio of snow grains can be calculated and used as a textural index. A comparison of shape factors calculated from the static permittivities, with actual shape factors of grains derived from photographic analysis of snow samples, shows a satisfactory agreement. For nearly spherical grains, ie in the case of old, coarse-grained snow and alpine firn, the photographic method shows more or less the same accuracy as the dielectric method. In the case of low-density new snow or snow which has not undergone appreciable metamorphosis, the dielectric method is clearly preferable. In addition, the dielectric method is more applicable to dry and moderate wet snow samples.

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