A KRAMERS-KRONIG ANALYSIS OF INTERSTELLAR EXTINCTION*

R. Y. CHIAO**, M. J. FELDMAN, and P. T. PARRISH

Dept. of Physics, University of California, Berkeley 94720, Calif., U.S.A.

Abstract. Integrals over the observed spectrum of interstellar extinction to 9 μm^{-1} are related to known laboratory properties of specific compounds. Using Lα hydrogen densities and cosmic abundance ratios, many suggested grain materials can be eliminated.

Little is known about the identity and density of the interstellar grains. It would be desirable then to develop a rigorous analysis which would lead to strong quantitative conclusions. We discuss an approach involving integrals of the extinction over the infrared, optical and ultraviolet portions of the spectrum. Using cosmic abundance ratios and recently determined Lα hydrogen column densities, several suggested grain candidates are evaluated.

We employ the Kramers-Kronig relation connecting the real part of the dielectric susceptibility $\chi'(\omega)$ to an integral over frequencies of the imaginary part $\chi''(\omega)$

$$\chi'(\omega_0) = \frac{2}{\pi}\int_0^\infty \frac{\omega\chi''(\omega)}{\omega^2 - \omega_0^2}\,\mathrm{d}\omega\,. \tag{1}$$

This is applied to the grain component of the interstellar medium as a whole.† $\chi''(\omega)$ is given by

$$0.921\,A(\omega) = \kappa(\omega) = \frac{4\pi}{c}\omega\chi''(\omega)\,,$$

where A is the extinction measured in units of (magnitude cm^{-1}) and κ in units of (cm^{-1}). Assuming a specific grain species, $\chi'(\omega)$ is simply related to its column density and optical and solid state properties.

We take two limits of Equation (1), $\omega_0 = 0$ and $\omega_0 \to \infty$, which yield two independent sums $\sum_0$ and $\sum_\infty$. These two sums (explicitly, integrated extinction) are then used to evaluate various grain candidates. For each sum a quantity Δ is derived which is the ratio of the amount of a given material expected on the basis of Lα hydrogen column density, $N(\mathrm{L}\alpha)$, and cosmic abundances (CARs), to the amount of material required to account for the integrated extinction.

* Work partially supported by the National Aeronautics and Space Administration Grant NGL 05-003-272 and the National Science Foundation Grant GP30424X.

** Alfred P. Sloan Foundation Fellow.

† For the first application of the Kramers-Kronig relations to the interstellar medium, see Purcell (1969).

Greenberg and Van de Hulst (eds.), Interstellar Dust and Related Topics, 59–61.

$$\Delta_0 = 3.21\times 10^5 \left[\frac{N(\mathrm{L}\alpha)}{\sum_0}\right]\frac{\mathrm{CAR}\times F}{n_g} \tag{2}$$

$$\Delta_\infty = 9.68\times 10^{-17}\left[\frac{N(\mathrm{L}\alpha)}{\sum_\infty}\right]\mathrm{CAR}\times Z_{\mathrm{eff}}^{\mathrm{lab}}. \tag{3}$$

F is a function of the assumed grain shape and dielectric function (Purcell, 1969), n_g is the number density of molecules in the grain, and $Z_{\mathrm{eff}}^{\mathrm{lab}}$ is essentially the number of electrons per atom participating in the extinction. (An extention of this work, to be published later, will include a detailed discussion of the assumptions made in deriving these equations).

The data sample consists of those stars with published ultraviolet extinction measurements by Bless and Savage (1971), supplemented by optical and infrared extinction information. The infinite sum is taken out to two frequencies: a full sum involving extinction out to 9.0 μm^{-1} and a partial sum out to 5.4 μm^{-1} which serves to isolate the effect of the ubiquitous 4.6 μm^{-1}e xtinction peak.

Table I is an evaluation of Equations (2) and (3) for individual compounds. Each entry represents an upper bound to the proportional contribution of a specific compound to the integrated extinction. The grains are assumed to be extreme prolate spheroids aligned parallel to the electric acis. This geometry maximizes the extinction efficiency of the grain. Each compound is ascribed a CAR on the basis of the elemental CARs of Aller (1961). The second iron entries use a recent value determined by Foy (1972). These should represent lower and upper limits to the actual CAR for iron. CARs are the major source of uncertainty in this analysis (the entries in Table I scale with the actual CARs). Although Lα densities are at best measured to $\pm 20\%$ (Savage and Jenkins, 1972), by measuring the ratio $N(\mathrm{L}\alpha)/\sum$ for each star and averaging over our sample a best fit value gives one sigma uncertainties of 10% for these ratios.

Little information is provided by Δ_∞. The integrated extinction in the spectral region 5.4 μm^{-1} to 9 μm^{-1} is strongly dependent upon particle size and shape. We expect then that our generous assumptions regarding geometry might be a gross overestimate in this case. In view of the strong results provided by $\Delta_{\infty/p}$, we emphasize

TABLE I

Ratios of actual to required material abundances

Compound	Δ_0	$\Delta_{\infty/p}$	Δ_∞
SiO_2	0.13	0.06	0.40
Fe	0.02	0.09	0.06
	0.19	1.02	0.73
(MgFe)Sio_4	0.04	0.01	0.13
	0.12	0.02	0.41
C (Diamond)	0.46	<0.01	1.47
C (Graphite)	0.61	5.02	3.22
SiC	0.11	0.43	1.01
CH_4	0.94	<0.01	3.29
NH_3	0.44	<0.01	1.17
H_2O	1.87	<0.01	3.04

that these numbers are strict upper limits for the respective compounds, assuming only that ascribed CARs hold.

In specific we point out that (1) compounds involving iron or silicon (often suggested for the low frequency extinction (Wickramasinghe and Nandy, 1971; Gilra, 1971)) seem underabundant to account for the zero-frequency sum; (2) consequently, more attention should be paid to the lighter compounds such as graphite and the frozen gases, CH_4, NH_3, H_2O; and (3) only graphite, SiC, and iron may substantially contribute to the infinite-frequency partial sum.

References

Aller, L. H.: 1961, in *The Abundances of the Elements*, Interscience Publishers, New York, p. 178.
Bless, R. C. and Savage, B. D.: 1971, *Astrophys. J.* **171**, 293.
Foy, R.: 1972, *Astron. Astrophys.* **18**, 26.
Gilra, D. P.: 1971, *Nature* **229**, 237.
Purcell, E. M.: 1969, *Astrophys. J.* **158**, 433.
Savage, B. D. and Jenkins, E. P.: 1972, *Astrophys. J.* **172**, 491.
Wickramasinghe, N. C. and Nandy, K.: 1971, *Monthly Notices Roy. Astron. Soc.* **153**, 205.