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Solar cycles reconstructed over the last millennium: Do Waldmeier and Gnevysev-Ohl rules work?

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Abstract. Using tree-ring radiocarbon ^{14}C data, solar cycles are now reconstructed for the last millennium, more than doubling the previously known statistic of direct solar observations and giving a new opportunity to validate basic empirical rules connecting solar cycle lengths, strengths and intensities. This includes the Waldmeier rule relating the cycle's strength to the length of its ascending phase, and the Gnevyshev-Ohl rule suggesting that cycles are paired so that the intensity of an odd solar cycle is higher than that of the preceding even cycle. Using the extended solar-cycle statistic, we found that the Waldmeier rule remains robust for the well-defined solar cycles implying that it is an intrinsic feature of the solar cycle. However, the validity of the Gnevyshev-Ohl rule is not confirmed at any reasonable statistical level, indicating that either the insufficient accuracy of the reconstructed solar cycles or that this rule is not a robust feature.

Keywords. Sunspot numbers, Solar cycle, Waldmeier rule, Gnevyshev-Ohl rule

1. Introduction

The Sun is an active star whose magnetic activity is dominated by quasi-periodic 11-year cyclicity (Hathaway 2015) which is a result of the action of the solar dynamo in the convection zone (Charbonneau 2020). However, neither period/length/shape nor the amplitude of solar cycles is constant as cycles vary in time ranging from Grand minima of solar activity when hardly any spots appear on the Sun to Grand maxima of very high activity cycles Usoskin (2017). Solar activity is traditionally studied using data from direct telescopic solar observations started in 1610. This sunspot-number series covers 36/37 solar cycles of which 24 are well defined, 8 are poorly resolved and four (during the Maunder minimum) are unresolved. One cycle might have been lost ca. 1800 due to sparse data (Usoskin et al. 2009). Well-defined cycles start in the middle of the 18th century and are standardly represented by the international sunspot number series (ISN) version 2.0 (Clette & Lefèvre 2016) available at the SILSO database (`https://www.sidc.be/silso/datafiles`).

There are several rules which relate different parameters of the solar cycles to each other and across the cycles. These rules were established empirically and form observational constraints on the solar dynamo theory (e.g., Hathaway 2015; Charbonneau 2020).

The most famous rule is the Waldmeier rule (Waldmeier 1935, 1939) which says, in its classical definition, that strong cycles rise fast, viz. the duration of the ascending phase of a solar cycle is inversely related to its strength. The Waldmeier rule is fairly stable across the well-established part of the sunspot-number record (Usoskin et al. 2021a). Sometimes the Waldmeier rule is incorrectly interpreted as an inverse relation between

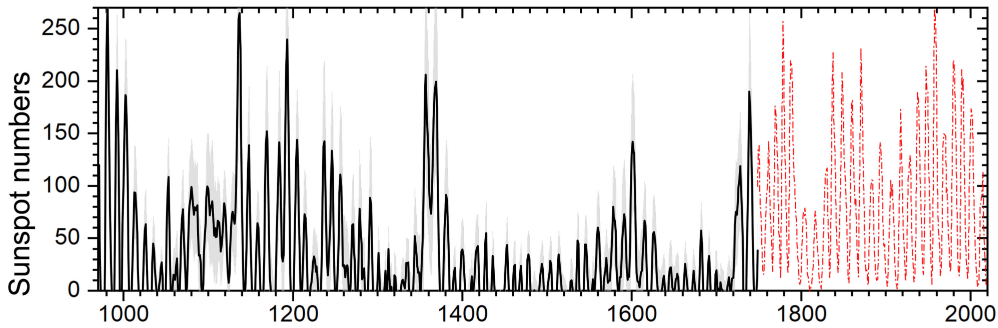

Figure 1. Annual sunspot numbers since 950 AD reconstructed from ^{14}C measurements (black curve) along with the 68% confidence intervals (Brehm et al. 2021; Usoskin et al. 2021b). ISN v.2.0 is shown with the red dashed curve (Clette & Lefèvre 2016).

the total length of a solar cycle and its amplitude. Here we use the classical definition of the Waldmeier rule.

Another empirical relation is the Gnevyshev-Ohl rule (Gnevyshev & Ohl 1948), which says that solar cycles are tied in even-odd pairs so that the integral intensity (the sum of sunspot numbers over a cycle) of the leading even-numbered cycle tends to be smaller than the intensity of the subsequent odd-numbered cycle. Sometimes it is called the even-odd effect in the literature. There are several potential breaks of the Gnevyshev-Ohl rule: one was around the Dalton minimum ca. 1800 and may be related to a lost solar cycle (e.g., Usoskin et al. 2001, 2009; Karoff et al. 2015); and another during the last few solar cycles as related to the end of the Modern grand maximum.

These empirical rules were in focus of many earlier studies (e.g., Usoskin et al. 2001; Hathaway et al. 2002; Solanki et al. 2002; Aparicio et al. 2012; Carrasco et al. 2016; Takalo & Mursula 2018) using different approaches and different versions of the sunspot series. Not much can be added to these rigorous analyses based on the sunspot number records for the period of about 300 years after the Maunder minimum.

However, a new opportunity to verify the validity of these empirical rules has appeared recently. A new dataset of radiocarbon Δ^{14}C in tree rings was conducted by Brehm et al. (2021) who used the acceleration mass spectrometry at ETH Zürich to measure radiocarbon with annual resolution since 950 AD with the high accuracy of 1.8 permil. Although the uncertainties are comparable with the amplitude of the 11-year cycle in Δ^{14}C that is greatly attenuated by the carbon cycle (e.g., Bard et al. 1997), the combination of the high cadence and accuracy made it possible to reconstruct the solar cycle over the last millennium (Usoskin et al. 2021b). The new reconstruction (shown in Figure 1 as the black curve with the 68% confidence interval) covers the period of 970 – 1900 AD and significantly enlarges the statistic of known solar cycles: 96 solar cycles are identified. The last millennium was a special period including a cluster of four Grand minima of solar activity – Oort, Wolf, Spörer and Maunder minima in the 11th, 14th, 15th and 17th centuries, respectively (Usoskin et al. 2007; Inceoglu et al. 2015). Solar cycles cannot be reliably defined during the Grand minima, leaving about 50 well and reasonably-well-defined cycles for the analysis. A preliminary analysis of the Waldmeier rule was performed by Usoskin et al. (2021b) suggesting that it remains statistically significant on the millennial time scale. However, as reported by Usoskin et al. (2022), solar cycles provided by Usoskin et al. (2021b) had erroneously large error bars that could affect the statistical study of the Waldmeier rule.

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Here we report the results of a thorough statistical analysis of the Waldmeier and Gnevyshev-Ohl rules using the millennial-long sunspot-number series for the period 970 through 1900 including 85 solar cycles.

2. Data set

For the analysis we used the revised list of the reconstructed solar cycles from Usoskin et al. (2022) for the period 970 – 1900. Usoskin et al. (2021b) ascribed a quality flag Q to each reconstructed cycle: 0 – the cycle cannot be reliably identified; 1 – the cycle is greatly distorted; 2 – the cycle can be approximately identified, but its shape is distorted; 3 – a reasonably defined cycle; 4 – a well-defined cycle with a somewhat unclear amplitude; 5 – a clear cycle in both shape and amplitude. In the framework of this study, we applied the analysis primarily to the well-defined cycles (25 cycles with $Q \geq 4$) but discuss also reasonably defined (36 cycles with $Q \geq 3$) cycles. All analyses were applied to the annually-resolved data so that the ascending phase was defined with an accuracy of up to one year. The length of the ascending phase of a cycle was defined as the difference in full years between the year of the cycle minimum and the subsequent maximum as reported in Usoskin et al. (2022). Following the approach of Usoskin et al. (2021b), we considered, as the index of the solar-cycle strength, the cycle-averaged sunspot number $\langle SN \rangle$ which is more robustly defined than the maximum annual sunspot number. Direct sunspot-number data (ISN v.2) were not used at this stage to check whether the relations work for the reconstructed data and avoid bias.

3. Results

3.1. *Waldmeier relation*

The ascending phase T_{A} of solar cycles takes a length between 4 and 7 years. Accordingly, we made distributions of the cycle-averaged sunspot numbers $\langle SN \rangle$ for these values of T_{A} considering the uncertainties of the sunspot number reconstruction. This was done by using a Monte Carlo approach as described below. First, all well-defined ($Q \geq 4$) cycles with the given T_{A} were selected with their $\langle SN \rangle$ and $\sigma_{\langle SN \rangle}$. Next, using a pseudo-random number R normally distributed with zero mean and unit variance, we ascribed the exact value of the cycle-averaged SN with the uncertainties taken into account as

$$\langle SN \rangle^* = \langle SN \rangle + R \cdot \sigma_{\langle SN \rangle}. \tag{3.1}$$

The last step was repeated 1000 times and distributions of the $\langle SN \rangle^*$ values were collected in histograms, one for each T_{A} and fitted with a Gaussian as shown in Figure 2. As seen, the Gaussian fit describes the distribution reasonably well.

Figure 3 shows the scatter plot of the Waldmeier relation using the distributions of the $\langle SN \rangle$ shown in Figure 2. A clear tendency is observed that cycles with longer ascending phases are weaker. This scatter was fit by the linear relationship as shown by the solid line with 90% (two-side) confidence intervals.

The best-fit linear regression appears highly significant (significance $p < 0.01$) as

$$\langle SN \rangle = (-26 \pm 8) \cdot T_{\mathrm{A}} + (203 \pm 47), \tag{3.2}$$

where T_{A} is the length of the ascending phase in years. This is consistent with the regression reported by Usoskin et al. (2021b), viz. $\langle SN \rangle$=(-26±16)$\cdot T_{\mathrm{A}}$+(197±90), but with smaller uncertainties, because of the corrected typo in the dataset. It is also fully consistent, within the uncertainties, with the Waldmeier rule based on the direct sunspot data (ISN v2.0, from SILSO) for the period since 1750 (Usoskin et al. 2021a), viz. $\langle SN \rangle$=(-20±9)$\cdot T_{\mathrm{A}}$+(179±41) for the cycle-averaged sunspot numbers.

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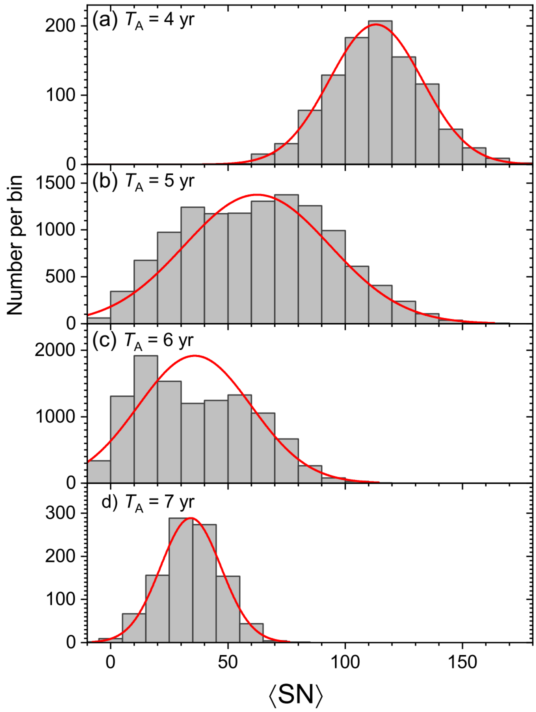

Figure 2. Distribution histograms of the $\langle SN \rangle$ values for the well-defined solar cycles ($Q \geq 4$) reconstructed from ^{14}C. Panels a through d correspond to the durations of the cycle ascending phase as $T_A = 4$, 5, 6 and 7 years, respectively. The red curves depict the best-fit Gaussians which are: a) 113 ± 20; b) 62 ± 31; c) 36 ± 24; d) 34 ± 13.

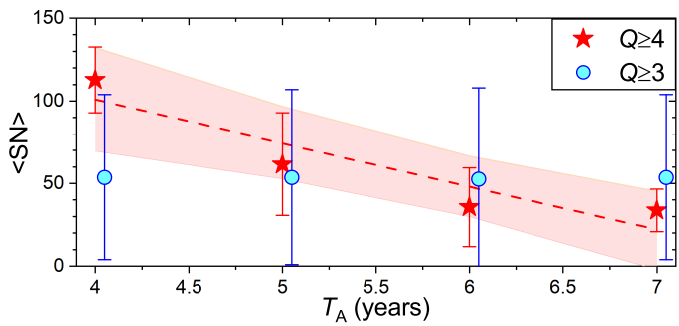

Figure 3. Relation between the cycle-average sunspot number $\langle SN \rangle$ and the length of the cycle ascending phase T_A for the millennium-long sunspot reconstruction. The red stars correspond to the distributions of well-defined cycles ($Q \geq 4$) shown in Figure 2, while the red dashed line with pink shading depicts the best-fit linear regression and its 90% (two-sided) confidence interval, respectively. The blue dots correspond to solar cycles with $Q \geq 3$.

We have also checked the validity of the Waldmeier rule using a larger statistic of poorly-defined cycles ($Q \geq 3$). The corresponding $\langle SN \rangle$ vs. T_A scatter is shown by blue dots in Figure 3. Even though the blue dots are consistent with the stars within uncertainties, there is no statistical relationship between the two variables, because of the too large error bars. Accordingly, the poorly-defined cycles spoil the empirical relation.

Thus, the Waldmeier rule is confirmed, using well-defined reconstructed cycles for the last millennium in the same quantitative shape as known for the last centuries from direct sunspot observations.

3.2. *Gnevyshev-Ohl rule*

We have also tried to check the validity of the Gnevushev-Ohl empirical rule using well-defined cycles ($Q \geq 4$), which form three clusters: (I) cycles $1-6$ corresponding to years $976-1040$ AD; (II) cycles $16-26$ ($1142-1262$), and (III) cycles $56-59$ ($1584-1632$). Since the absolute numbering of the cycles is unknown to define odd- and even-numbered ones, we have checked different options of cycle pairings within each cluster: the first cycle in the cluster is 'even' (pairing a) or 'odd' (pairing b). Thus, there are eight possible combinations of pairing cycles within the clusters, e.g., Ia + IIb + IIIa. We have checked all of them. As the cycle's intensity, we considered the product of the cycle-average sunspot number $\langle SN \rangle$ and the cycle length T from Table 1 of Usoskin et al. (2021b):

$$I = \langle SN \rangle \times T. \tag{3.3}$$

As the measure of the Gnevyshev-Ohl rule, we considered the difference in the intensities of the paired cycles, viz.

$$\Delta I_i = I_{i+1} - I_i. \tag{3.4}$$

The analysis was done also by applying a Monte-Carlo method as described in Section 3.1. We found that, for all possible combinations, ΔI appears indistinguishable from zero at the 1σ level ($p > 0.3$). This is likely because of the large uncertainties of the reconstructed sunspot numbers (of the order of a few hundred in units of ΔI) which are greater than the expected Gnevyshev-Ohl effect of about a hundred in units of ΔI as estimated from ISN (Usoskin et al. 2021a).

Thus, the Gnevyshev-Ohl rule cannot be confirmed using the new reconstructed solar cycle on the millennial timescale.

4. Conclusions

Here a quantitative analysis of the empirical solar-cycle relations has been performed using the updated solar-activity reconstruction based on ^{14}C data (Brehm et al. 2021; Usoskin et al. 2021b, 2022). This new dataset extends the known solar-cycle variability over the last millennium more than doubling the statistics of the cycles. However, not all cycles are reconstructed equally well. Here we mostly consider 25 well-defined cycles, with the quality flag $Q \geq 4$, and discuss ten more poorly-defined cycles with $Q = 3$. Non-defined cycles with $Q < 3$ are not considered.

We have shown that the Waldmeier rule saying that the cycle strength is inversely related to the length of the solar-cycle ascending phase, is fully confirmed on the millennial timescale using well-defined cycles. The use of poorly-defined cycles smears the relation. This implies that the Waldmeier rule is an intrinsic feature of the solar dynamo (e.g., Charbonneau 2020; Nandy 2021).

We have also checked the validity of the Gnevushev-Ohl rule, saying that solar cycles are tied in pairs of an even-numbered cycle followed by a slightly stronger odd-numbered cycle, on the millennial time scale. However, because of the relatively large uncertainties, the result is inconclusive, and the rule can be neither confirmed nor rebutted.

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