has worked his way through English mathematical problems, will realize the value of such an appendix to the students.

I found the book not only an excellent introduction to analytic geometry, but an ideal preparation for a study of analytic projective geometry. In this connection, I should mention that in the latter half of the book, homogeneous coordinates, line coordinates, circular points at infinity, cross ratio, and homographic correspondences are all defined and developed. The author also deals with the principle of duality; pencils of lines and conics; ranges of points; polars; the triangle of reference; the quadrangle; and many other basic ideas in projective geometry.

The first half of the book consists of a standard exposition of analytic geometry but done in much more subtle manner and with more finesse than one finds in the average American text. As a challenging introduction to analytic geometry, I would recommend this book to an honours mathematics under-graduate class.

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Sur les Fonctions Méromorphes et les Fonctions Algébroides. (Extensions d'un Théorème de M.R. Nevanlinna.) Par King-Laï Hiong. Mémorial des Sciences Mathématiques. CXXXIX, Gauthier-Villars, Paris, 1957. 104 pages.

A few words of advice for the prospective reader: If you recognize the following inequality relating to a meromorphic function f,

$$m(r, f'/f) < 16 + \log^+\log^+(1/f(0)) + \log(1/\rho)$$
$$+ 2\log(\rho/r) + 3\log[\rho/(\rho - r)] + 4\log^+ T(\rho, f)$$

and are familiar with the functions m and T, you are undoubtedly qualified to proceed at once with the reading of this volume. If not, as was the case with the reviewer, you should first consult one or more of the references listed in the bibliography as you will not find in this book definitions of the three fundamental functions involved, m, T and N.

To cover this gap R. Nevanlinna's monograph: Le Théorème de Picard-Borel et la Théorie des Fonctions Méromorphes, Gauthier-Villars, Paris, 1929, can be recommended for the clarity and completeness with which it treats the basic subject matter required. From this authority the following summary has been extracted.

Let $f(x)$ be a function of a complex variable x, meromorphic in $|x| < R \leqslant \infty$. For a given complex number z let $n(r, z)$ denote the number of roots of the equation $f(x) - z = 0$ contained in $|x| \leqslant r$, each root being counted according to its multiplicity. Let functions N and m be defined by

$$N(r, 1/(f-z)) = \int_0^r [n(t,z) - n(0,z)]t^{-1}dt + n(0,z)\log r,$$

$$m(r, 1/(f-z)) = (2\pi)^{-1}\int_0^{2\pi} \log^+ |f(re^{i\varphi}) - z|^{-1} \, d\varphi,$$

where $\log^+ |u|$ denotes $\log |u|$ when $|u| \geqslant 1$ and 0 when $|u| < 1$. These definitions apply also when $z = \infty$, but in this case the functions are denoted by $N(r,f)$ and $m(r,f)$. The logic of this notation lies in the fact that the roots of the equation $f(x) - z = 0$ satisfy $1/(f-z) = \infty$ when z is finite and $f = \infty$ when $z = \infty$.

The function N is determined by the distribution of the moduli of the roots of $f-z = 0$ and serves as a measure of the "density" of the roots as $r \to \infty$. The function m gives the mean value of $\log^+ | 1/(f-z)|$ on the circle $|z| = r$, and so indicates the "intensity" of the mean convergence of $f(x)$ to z as $r \to \infty$.

The sum m + N has a fundamental role to play. It is an increasing function with r, being convex in $\log r$, and this is also true of N but not of m. Nevanlinna's First Fundamental Theorem tells how m + N behaves with changing z. It asserts that m+N, minus a term which remains bounded as $r \to \infty$, is the same for all values of z, including $z = \infty$. The value of m + N when $z = \infty$ defines the function T, so that $T(r,f) = m(r,f) + N(r,f)$. According to the theorem then

$$m(r,1/(f-z)) + N(r,1/(f-z)) = T(r,f) + O(1).$$

The sum m + N represents the "total affinity" of $f(x)$ for z, and the theorem indicates that this is invariant with changing z except for the term $O(1)$.

Nevanlinna's Second Fundamental Theorem, in combination with the first, throws light on the relative behaviour of m and N. In its original form it states that, for distinct values $z_1, z_2, \ldots, z_q$, with $q \geqslant 3$,

$$(q-2)T(r,f) < \sum_{\gamma=1}^{q} N(r,1/(f-z_\gamma))+N(r,f') - N(r,1/f')$$

$$- 2N(r,f) + S(r,f),$$

where $S(r,f)$ can be assigned appropriate upper bounds. For example, if $f(x)$ is of finite order, $S(r,f) = O(\log r)$. One of the applications is to the study of "exceptional" points. For any value z, $\lim_{r\to\infty}(m/T + N/T) = 1$ and hence $\delta(z) = \underline{\lim}_{r\to\infty} m/T = 1 - \overline{\lim}_{r\to\infty} N/T$, and so $0 \leqslant \delta(z) \leqslant 1$. At a lacunary point (where $f-z = 0$ has no roots) we find that $\delta(z) = 1$. At most other points, however, $\delta(z) = 0$; in fact there are at most a denumerable set of points z where $\delta(z) > 0$. These are called "exceptional" points. At such points the density of the roots is less than at ordinary points whereas the intensity of convergence of $f(x)$ to z is correspondingly greater.

Nevanlinna's second theorem serves as a starting point for Hiong's study. The monograph consists of five chapters. The first three are concerned in the main with bringing together various extensions and generalizations of the second theorem, comprising results obtained by Milloux, Ullrich and Hiong himself. The introduction of the derivative $f^k(x)$ proves very fruitfull. A sample theorem is the one which uses a function $f = \alpha_0 f + \alpha_1 f' + \ldots + \alpha_\ell f^{(\ell)}$, in which the coefficients are holomorphic functions, and yields an inequality for $T(r,f)$ which includes in its right hand side the terms N associated with a point a for f and a point b for f_ℓ. The applications of the new theorems serve to expand considerably the theory of exceptional points.

Chapter IV extends the two fundamental theorems to algebroid functions. $u(x)$ is called algebroid if it is defined by an equation $A_n(x)u^n + A_{n-1}(x)u^{n-1} + \ldots + A_0(x) = 0$ in which the coefficients are holomorphic and not all simultaneously zero at any point. These extensions were first obtained by Selberg and by Valiron about twenty years ago, but, according to the author, no well developed account of them has hitherto been given.

Chapter V contains an exposition of work initiated by Nevanlinna himself on systems of meromorphic functions, and also of H. Cartan's theory of linear combinations of p holomorphic functions. From the latter theory the author deduces an important inequality for a class of algebroid functions which is more precise than any previously obtained.

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Integral Equations by S.G. Mikhlin, translated by A.H. Armstrong. Pergamon Press, 1957. $12.50.

The full title of this book is "Integral Equations and their Applications to Certain problems in Mechanics, Mathematical Physics, and Technology" and it describes the book quite well. The contents divide into two parts; these deal with the theory and applications respectively. The theoretical part contains the usual account of the linear theory of Fredholm, Volterra and Hilbert-Schmidt, and in addition, a brief and elegant introduction to singular equations. A proof of the existence of an eigenvalue and eigenfunction (of a completely continuous operator) is given, although the author avoids any terminology of functional analysis and the various properties of operators are, so to say, smuggled in. In the above proof use is made of Ascoli's theorem, without calling it by its name.

Throughout the theoretical part the emphasis is intensely practical and it is a pleasure to see numerical solutions of linear (algebraic) equations rubbing shoulders with the high matters of L^2 convergence, etc. A purist might say that the book cannot make up its mind whether to be practical, heuristic and useful or rigorous, elegant and theoretical in scope. One other minor comment on the first part: the readers on this continent may be somewhat perturbed by the "Bunyakovsky-Schwarz" inequality but this wears off after a few minutes of reading.

It is on the merits of the second, and much longer, part that the book stands, and these are undoubted and extensive. The author states his preferences clearly and the applications are drawn almost exclusively from elasticity and hydrodynamics. In view of numerous other applications (electromagnetic theory, wave-guide and antenna problems, probability, Weyl's theory of singular differential equations, etc.), it is rather a pity but 'De gustibus...'. There are six massive chapters in the second part: Dirichlet (and Neumann) Problems, the Biharmonic Equation (Applications of Green's Function), the Generalized Method of