THE FOUR-ARMED RESPONSE NEAR THE LINDBLAD RESONANCES IN GALAXIES

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ABSTRACT. We show that there is an important four-armed term in the response of a flat galaxy to an imposed two-armed spiral field near the Inner and Outer Lindblad Resonances.

We will show how an important four-armed component arises near the Lindblad Resonances of a two-armed spiral, or a bar. By "important" we mean that it is of the same order as the two-armed component, although it may be numerically smaller.

We will describe the main steps leading to this result, while the detailed calculations are included in a forthcoming paper (Contopoulos 1978).

We assume that the potential of a (flat) galaxy is composed of an axisymmetric background and a two-armed spiral perturbation

$$V = V_o(r) + A(r)\cos[\Phi(r) - 2\theta]. \qquad (1)$$

The azimuth θ is calculated in a frame of reference rotating with angular velocity Ω_s. In the case of a bar the phase Φ is a constant.

We know now that the spiral perturbation can be analysed into components of the form (Kalnajs 1971)

$$V_{\ell m}(I_1, I_2) \begin{matrix}\cos\\ \sin\end{matrix} (\ell\theta_1 - m\theta_2), \qquad (2)$$

where $(I_1, I_2,\ \theta_1, \theta_2)$ are action-angle variables and ℓ,m integers. Namely I_1 is the radial action, I_2 the azimuthal action (essentially the angular momentum), θ_1 the epicyclic angle and θ_2 the azimuth of the epicyclic center; θ_2 is close to the azimuth of the star, θ.

The most important terms of the form (2) are (Contopoulos 1973; paper I):

W. B. Burton (ed.), The Large-Scale Characteristics of the Galaxy, 187–190.

https://doi.org/10.1017/S0074180900014443 Published online by Cambridge University Press

$$A_o \cos\,(2\theta_2 + \text{const.}), \tag{3}$$

$$A_1 \cos\,(\theta_1 - 2\theta_2 + \text{const.}), \tag{4}$$

and

$$A_2 \cos\,(\theta_1 + 2\theta_2 + \text{const.}), \tag{5}$$

where A_o, A_1, A_2 are of the same order as the amplitude A.

The orbit of each star in the potential (1) is an oscillation around a periodic orbit. In general the periodic orbits are almost circles, therefore the orbits of stars are slightly perturbed epicycles. However near the main resonances of the galaxy the periodic orbits change considerably. Near the Particle Resonance the most important term in the potential is the term (3), while near the Inner and Outer Lindblad Resonances the most important terms are (4) and (5) respectively.

In the lowest approximation the periodic orbits at a resonance are given by setting the argument of the corresponding term (3), (4), or (5) equal to zero, or to π. E.g. at the Inner Lindblad Resonance the periodic orbits are given approximately by

$$\theta_1 - 2\theta_2 + \text{const.} = 0, \pi. \tag{6}$$

These two orbits are nearly ellipses, perpendicular to each other. The non-periodic orbits form, in general, rings around one of the periodic orbits (6).

In order to find the density response we must superpose all the orbits oscillating around the periodic orbits (6).

However, the fact that we have two perpendicular populations of orbits is not sufficient to produce a four-armed component in the density distribution. If the density around each periodic orbit has a sinusoidal form, similar to (4), then the maxima of density of one population coincide with the minima of density of the other population, and thus the contributions of the two populations tend to cancel each other. If the amplitudes of the two populations are equal, only the axisymmetric background remains, but if one population is stronger, its amplitude is simply reduced.

On the other hand we may think of a situation where only the immediate neighborhoods of the periodic orbits are populated with stars. In such a case the four-armed component is clearly seen, but the distribution of orbits is not sinusoidal, but indudes several higher harmonics.

One may argue that such a distribution contains higher order terms in A, therefore it is not a first order effect. However we will prove that even if we limit our discussion to first order terms in A we find the same effect. In order to do that we examine the perturbations of the orbits near the Inner Lindblad Resonance due to the term (5). If we introduce into (5) the values (6) (appropriate for the periodic orbits) we find terms of the form

$$A_2 \cos (4\theta_2 + \text{const.}). \tag{7}$$

Thus the perturbation is close to a four-armed term

$$A_2 \cos (4\theta + \phi), \tag{8}$$

where A_2 and ϕ are functions of r.

If we follow now the effects of the term (8) we find that the response contains a four-armed component, which is strongest near the Inner Lindblad Resonance (Contopoulos 1978). The 4θ-response is, in fact, of the same order as the 2θ-response, namely of $O(\sqrt{A})$. The amplitude of the four-armed component as a function of r in the models of paper I is given in Figure 1, both for a bar and for a somewhat tight spiral of inclination $\sim$ 16°.

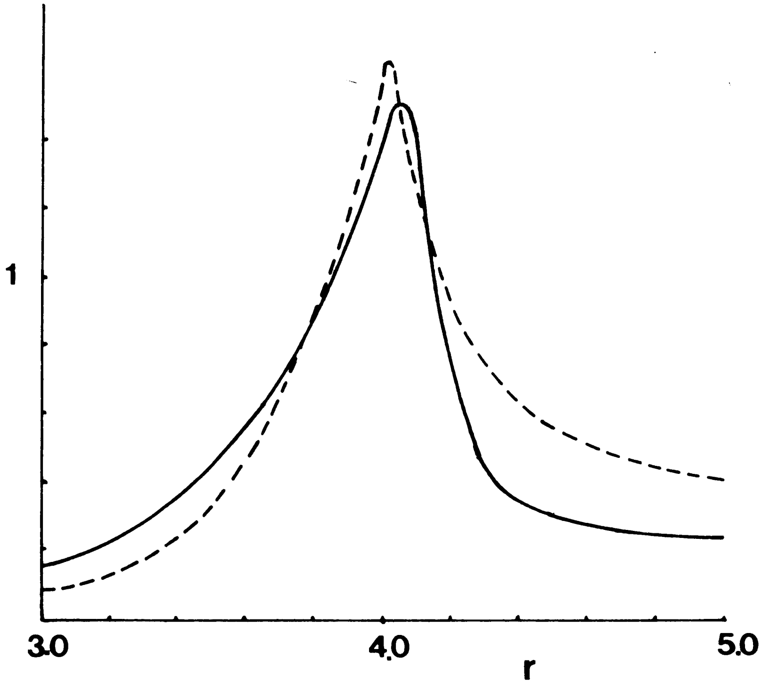

Figure 1. The amplitude of the four-armed term in the models of Paper I. (——) Spiral with k=-10/(r+4); inclination angle $\simeq$ 16°. (---) Bar. Resonance at r=3.73kpc.

https://doi.org/10.1017/S0074180900014443 Published online by Cambridge University Press

This form of the response seems to be quite general. Namely the four-armed term is strongest near the inner Lindblad resonance, but it is small inside and outside it.

This result is consistent with the empirically found 4θ-term in the case of the SB0 galaxy NGC 2950 (Crane 1975). Crane found that the amplitude of the 4θ-term in this galaxy has a localized maximum, as in Figure 1, while the 2θ-component has a broader maximum in the same general region. This can be explained if both maxima are associated with the Inner Lindblad Resonance (Contopoulos and Mertzanides 1977).

Another example of a conspicuous four-armed component in the inner parts of a galaxy was found by P.O. Lindblad (Figure 2).

Figure 2. An anonymous southern galaxy that shows one strong and one weak bar (ESO photograph).

A similar four-armed component should be expected near the Outer Lindblad Resonance. It remains to be seen if the various models that introduce a four-armed component in the outer parts of our Galaxy can be explained by such a mechanism, or whether it is necessary to consider a separate four-armed mode, which forms a new grand design of its own.

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