CORRESPONDENCE.

ON A TABLE OF MORTALITY DEDUCED FROM THE NEW EXPERIENCE OBSERVATIONS, H^{MF}.

To the Editor of the Assurance Magazine.

SIR,—Having recently constructed a Table of Mortality, based upon the Healthy Male and Female Observations published by the Institute of Actuaries, from which the number exposed to risk and the deaths during the first *three* years of Assurance—*i.e.*, years 0, 1, and 2—have been eliminated, I beg to place the same at your disposal for insertion in the *Journal* of the Institute, should you consider the Table sufficiently interesting and useful for publication. It has been very carefully prepared, and has been graduated by the "New Method of Adjusting Mortality Tables," proposed and explained by Mr. Woolhouse in the last volume of the *Journal*.

In explanation of my reason for excluding years of Assurance 0, 1, and 2 only, I may point out that the mortality amongst Male lives, which form 88 percent of the whole of my Table, during those three years taken together can be shown, by reference to Mr. Sprague's exhaustive paper "On the rate of Mortality amongst Assured Lives as influenced by the Duration of the Assurance" (vol. xv., p. 338), to be 68·87 percent of the expectation by the 17 Offices' Experience, and 72·34 percent of the expectation by the New H^M Experience itself, whereas the two succeeding years, 3 and 4, give 99·13 percent of actual to expected deaths by the 17 Offices' Table, and 102·84 percent by the New Experience Table. The Experience Committee have excluded years of Assurance 0 to 4 from the H^M Observations, and, I understand from Mr. Woolhouse's paper,

recommend the resulting table "for the general purposes of valuations," although the actual deaths during the two years 3 and 4 thus thrown out are quite cent percent of the expected mortality. In this way 166,166 years of risk and 1188 deaths are rejected; and therefore I have thought it worth while to construct a Table, based upon the H^{MF} Observations, and commencing at year of Assurance 3. Several eminent medical men express the opinion that the value of selection is practically lost after three years, and their opinion, though founded upon individual experience only, is confirmed by Mr. Sprague's investigations upon the subject, although there is no doubt that its effect is traceable for several years after. I venture however to suggest that the Table now produced may be assumed to indicate with considerable accuracy the mortality which may be expected to prevail amongst the assurers in a Life Office, such assurers consisting of male and female lives in fair average proportions, and from whom the effect of medical selection has passed away.

I have computed the probability of dying in a year at each age by the entire adjusted H^{MF} Table given by Mr. Woolhouse on page 396 of the last volume of the *Journal*, and have placed them side by side with the corresponding probabilities deduced from the partial H^{MF} Table to facilitate comparison. The diminished mortality at ages 10 to 18 by the partial experience, as compared with the total experience, is chiefly attributable to the small number of facts observed upon in both cases; but it is also due to the heavier mortality which prevailed at those ages in the two years immediately succeeding entry. It might therefore be advisable, in constructing monetary values, to disregard the probabilities of dying at ages 10 to 18 inclusive in favour of those obtained from the total experience. If this be done, the following would be the adjusted numbers-living and decrements at ages 10 to 24, to be substituted for those in the Table.

Age.	l_x	d_x	Age.	l_x	d_x	Age.	l_x	d_x
10	100,410	455	15	98,427	389	20	95,947	727
11	99,955	410	16	98,038	425	21	95,220	808
12	99,545	380	17	97,613	477	22	94,412	834
13	99,165	367	18	97,136	545	23	93,578	850
14	98,798	371	19	96,591	644	24	92,728	846

Taking however the two Tables as they stand, it will be noticed that the exclusion of the first three years of Assurance reveals an increased rate of mortality commencing at age 19, and that such increase progresses rapidly until it reaches a maximum at age 24, at which age the mortality is 32 percent more by the partial than by the total experience. The difference then begins to decrease with more or less regularity until the age of 80, after which a change occurs, the mortality at ages 81 to 85 being slightly more by the total experience than by the partial experience, which fact is due to the few admissions at those ages and to the superior vitality of the female lives included. The mortality by the partial experience then increases until the end of the Table. The following Table shows very clearly the comparative influence of medical selection upon the mortality at different ages.

H^{MF} (adjusted).—*Number of Deaths compared.*

Ages.	Total Experience.	Partial Experience.	Ages.	Total Experience.	Partial Experience.
10–19	100·00	93·54	60–69	100·00	102·89
20–29	100·00	124·21	70–79	100·00	100·72
30–39	100·00	112 23	80–89	100·00	100·39
40–49	100·00	107·09	90–99	100·00	103·12
50–59	100·00	104·15			

It will be observed that the adjusted partial experience Table presents a secondary maximum of mortality at the age of 24, the probability of dying at that age being greater than at the ages immediately preceding and succeeding it. Mr. Sprague has pointed out, in his paper above referred to, that this peculiarity occurs at the age of 22 in the adjusted H^{M} Table, and that the same fact is noticeable at the same age in Mr. Berridge's adjustment of the Peerage Table. If, however, we examine the complete unadjusted H^{M} Table, as well as the partial unadjusted H^{MF} Table, we shall find that this maximum of mortality occurs at age 23. The increased mortality at this particular age is still more clearly defined in the H^{M} Table, from which the first *five* years of Assurance have been excluded; and it is worthy of note that the observations of the Mortality of the Government Male Life Annuitants by the late and present Actuaries to the National Debt show the same increase at that age, and further, that in the original Peerage Tables this maximum of mortality is indicated at age 23 in both the Male and Female observations. In reference to this point I would call attention to the following figures, and would remark that the facts upon which the percentages are computed are, I think, sufficiently numerous to command confidence in the results.

H^{M} (unadjusted).—*Mortality Percent.*

	Age 22.	Age 23.	Age 24.
Total Experience ..	·625	·773	·686
Excluding years of Assurance. 0	·951	·898	·779
0 and 1	·835	1·060	·764
0 to 2	·861	1·267	·765
0 ,, 3	1·055	1 660	·643
0 ,, 4	1·108	1·705	·460
0 ,, 5	1·044	1·728	·560

These figures are very remarkable, and, with one trifling exception which ascribes the maximum to age 22, tend to confirm the opinion that it is at age 23 that the climax is reached. The increase of mortality at this age appears to be very decided and uniformly progressive in proportion to the length of time elapsed since selection, whereas at ages 22 and 24 the chance of dying fluctuates considerably when similarly examined in reference to the period when selection took place.

In conclusion, I give a few specimens of annuities and premiums computed at 4 percent interest, and based upon the adjusted H^{MF} experience

excluding the first three years of Assurance; and beg to add, that I shall be happy to furnish you with the complete 4 percent Table if desired.

Age.	Value of Annuity of £1.	Value of Reversion of £1.	Annual Premium for Assurance of £1.
10	19 7416	·202244	·009751
15	18·9855	·231325	·011575
20	18·1930	·261806	·013641
25	17·5818	·285314	·015355
30	16·8711	·312650	·017495
35	16·0145	·345600	·020312
40	15·0099	·384234	·024000
45	13·8265	·429750	·028985
50	12·4985	·480828	·035621
55	11·0439	·536773	·044567
60	9·4777	·597010	·056978
65	7·9032	·657570	·073858
70	6·3508	·717274	·097578
75	4·9025	·772980	·130958
80	3 7397	·817700	·172522
85	2·7026	·857592	·231619
90	1·8861	·888996	·308027
95	1·1431	·917574	·428156

I am, Sir,

Your obedient servant,

Cleveland House,   WILFRED A. BOWSER.
Lower Clapton, 17*th October,* 1870.

New Mortality Experience. H^{MF}, *excluding the first Three Years of Assurance.*

Age.	UNADJUSTED.		ADJUSTED.			PROBABILITY OF DYING IN A YEAR.	
	Number-living.	Decre-ment.	Number-living.	Decre-ment.	Expectation.	Partial Experience Adjusted.	Total Experience Adjusted.
10	100000	702	100000	354	48·91	·00354	·00442
11	99298	0	99646	340	48·08	·00341	·00409
12	99298	218	99306	337	47·24	·00339	·00388
13	99080	190	98969	341	46·40	·00344	·00381
14	98890	658	98628	358	45·56	·00363	·00385
15	98232	602	98270	382	44·72	·00389	·00404
16	97630	252	97888	416	43 90	·00425	·00436
17	97378	314	97472	461	43·08	·00473	·00482
18	97064	356	97011	515	42 28	·00531	·00543
19	96708	831	96496	603	41·50	·00625	·00604
20	95877	570	95893	701	40·76	·00731	·00649
21	95307	854	95192	783	40 06	·00822	·00679
22	94453	820	94409	826	39·39	·00875	·00691
23	93633	1113	93583	850	38·73	·00908	·00695

New Mortality Experience. H^MF, &c.—(continued.)

Age.	UNADJUSTED.		ADJUSTED.			PROBABILITY OF DYING IN A YEAR.	
	Number-living.	Decre-ment.	Number-living.	Decre-ment.	Expectation	Partial Experience Adjusted	Total Experience Adjusted.
24	92520	783	92733	851	38·08	·00918	·00695
25	91737	610	91882	840	37 43	·00914	·00700
26	91127	863	91042	802	36·77	·00881	·00709
27	90264	826	90240	811	36·10	·00899	·00733
28	89438	775	89429	830	35·42	·00928	·00758
29	88663	898	88599	828	34·75	·00934	·00783
30	87765	888	87771	830	34·07	·00946	·00806
31	86877	761	86941	839	33·39	·00965	·00823
32	86116	866	86102	830	32·71	·00964	·00837
33	85250	812	85272	813	32·02	·00953	·00855
34	84438	838	84459	825	31·33	·00977	·00875
35	83600	740	83634	833	30 63	·00996	·00900
36	82860	870	82801	850	29 93	·01026	·00933
37	81990	885	81951	870	29·24	·01062	·00967
38	81105	921	81081	893	28·55	·01101	·01000
39	80184	935	80188	901	27 86	·01135	·01027
40	79249	898	79287	905	27·17	·01141	·01050
41	78351	893	78382	907	26·48	·01157	·01068
42	77458	903	77475	911	25·78	·01176	·01091
43	76555	899	76564	920	25·08	·01202	·01127
44	75656	939	75644	938	24·38	·01240	·01173
45	74717	981	74706	969	23 68	·01297	·01232
46	73736	979	73737	1007	22·99	·01366	·01301
47	72757	1084	72730	1050	22·30	·01444	·01372
48	71673	1061	71680	1087	21·62	·01516	·01442
49	70612	1112	70593	1128	20·94	·01598	·01511
50	69500	1224	69465	1163	20 27	·01674	·01577
51	68276	1218	68302	1189	19·61	·01741	·01651
52	67058	1241	67113	1227	19·38	·01828	·01732
53	65817	1142	65886	1268	18·29	·01924	·01831
54	64675	1265	64618	1314	17·64	·02033	·01945
55	63410	1483	63304	1362	16·99	·02151	·02065
56	61927	1457	61942	1413	16·36	·02281	·02196
57	60470	1436	60529	1465	15·72	·02420	·02336
58	59034	1540	59064	1523	15·10	·02578	·02489
59	57494	1449	57541	1583	14·49	·02751	·02669
60	56045	1750	55958	1661	13 88	·02968	·02873
61	54295	1693	54297	1737	13·29	·03199	·03104
62	52502	1841	52560	1824	12·71	·03470	·03366
63	50761	1952	50736	1911	12·15	·03766	·03647
64	48809	1949	48825	1992	11·61	·04080	·03937
65	46860	2094	46833	2047	11·08	·04371	·04233
66	44766	2065	44786	2100	10·57	·04689	·04543
67	42701	2124	42686	2137	10·06	·05006	·04866
68	40577	2257	40549	2157	9·566	·05319	·05204
69	38320	2304	38392	2189	9·076	·05702	·05599
70	36016	2003	36203	2229	8·594	·06157	·06095
71	34013	2087	33974	2281	8·125	·06714	·06686
72	31926	2466	31693	2354	7·674	·07427	·07368
73	29470	2393	29339	2404	7·250	·08194	·08154
74	27077	2802	26935	2438	6·852	·09051	·09004
75	24275	2222	24497	2404	6·484	·09813	·09799
76	22053	2366	22093	2365	6·135	·10705	·10581

New Mortality Experience. H^MF, &c.—(continued).

Age.	Unadjusted.		Adjusted.			Probability of Dying in a Year	
	Number-living.	Decrement.	Number-living.	Decrement.	Expectation	Partial Experience Adjusted.	Total Experience Adjusted.
77	19687	2113	19728	2257	5·811	·11441	·11322
78	17574	2302	17471	2131	5·497	·12197	·12110
79	15272	2017	15340	2001	5·191	·13044	·12938
80	13255	1776	13339	1863	4·895	·13966	·13868
81	11479	1762	11476	1710	4·609	·14901	·14907
82	9717	1432	9766	1568	4·328	·16055	·16068
83	8285	1536	8198	1426	4·060	·17394	·17426
84	6749	1214	6772	1270	3·810	·18753	·18857
85	5535	1211	5502	1115	3 574	·20265	·20267
86	4324	952	4387	955	3 355	·21768	·21732
87	3372	751	3432	805	3·150	·23455	·23248
88	2621	691	2627	651	2·962	·24781	·24581
89	1930	454	1976	520	2·773	·26316	·25923
90	1476	394	1456	411	2·585	·28228	·27778
91	1082	309	1045	319	2·405	·30526	·29708
92	773	304	726	233	2·242	·32093	·31069
93	469	235	493	168	2·066	·34077	·33029
94	234	0	325	124	1·875	·38154	·35694
95	234	26	201	79	1·724	·38806	·36441
96	208	130	122	48	1·516	·39837	·37334
97	78	39	74	36	1·207	·48648	·46809
98	39	0	38	26	·815	·68420	·65999
99	0	0	12	12	·500	1·00000	1·00000
100	0	0	0	0			

ON HERR LAZARUS'S PAPER ON THE THEORY OF PROBABILITIES.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In the July number of the *Journal* you inserted a letter from me, having for its object the elucidation of a passage in Herr Lazarus's paper " On some problems in the Theory of Probabilities." I have since received a very courteous communication from Herr Lazarus in reference to the subject of my letter; and I beg to send you the substance of that communication out of fairness to Herr Lazarus, at the same time feeling confident that it will greatly interest some of your readers.

He says, in explanation of the passage upon which my remarks were based, " The simplest way to find the sum $\Omega_0 + \Omega_1 + \Omega_2$ would be to extend " one of the equations (28) or (29), so as to include Ω_0. I think it is self-" evident from (28) that

$$\text{`` } \Omega_0 + \Omega_1 = \frac{\int_0^p x^{m-1}(1-x)^n dx}{\int_0^1 x^{m-1}(1-x)^n dx} - \frac{\int_0^p x^{m+z}(1-x)^{n-z-1} dx}{\int_0^1 x^{m+z}(1-x)^{n-z-1} dx}$$