BULL. AUSTRAL. MATH. SOC. 08A62, 08B05, 05B30, 20N05
VOL. 60 (1999) [341–342]

Varieties of universal algebras generated by finite algebras associated with cycle systems

SARAH ZAHRAI

The content of this thesis falls into two areas, *combinatorial designs* and *varieties of algebras*. The connection between these two areas arises from methods of defining operations in the underlying sets of certain types of combinatorial designs and thereby obtaining algebras. Conversely, combinatorial designs may be constructed from, and classified by, their corresponding algebras. One of the well known combinatorial designs discussed are *cycle systems* and *quasigroups* are the main class of algebras examined.

In Chapter 1, we begin with the story of finite algebras generated by finite bases and then presented a review of the relevant literature.

Chapter 2 contains the definitions and some preliminary results that are used in later chapters. It includes three sections; universal algebraic preliminaries, combinatorial and graph theoretic preliminaries and the relationship between these two topics.

In Chapter 3, we introduce a binary operation called "$*_j$" on the cycle systems and search for the properties of this operation. This groupoid operation provides some interesting results when the elements of a Galois field are viewed as the vertices of a cycle. This chapter begins by looking at "$*_j$" where the underlying structure is an integral domain and then we investigate different cases for j from the view point of graph theory. We shall also look at the conditions under which we can obtain various algebraic laws. At the end of this chapter we give some interesting properties of these systems for future research.

Chapter 4 presents another definition of "$*_j$" and gives us some other properties of this binary operation. In this chapter we discuss under what conditions this groupoid operation on the cycle systems gives rise to a quasigroup. An important result of this chapter is the generalisation of the Steiner law $x(xy) = y$, which holds in 3-cycle systems, to laws of the form $x\Big(\cdots\Big(x(xy)\Big)\cdots\Big) = y$ and $\Big(\cdots\Big((yx)x\Big)\cdots\Big)x = y$ in m-cycle systems. We call these laws *left* and *right Engel laws* by analogy with the Engel laws which play such an important rôle in the study of Burnside groups.

Received 19th May, 1999
Thesis submitted to The University of Queensland, November 1998. Degree approved, May 1999. Supervisor: Associate Professor S. Oates-Williams.

Finding a collection of identities to define the variety of an algebra leads to an important question and that is whether we can find a single law, which is much simpler than those found before, to define the variety of that algebra. As m-cycle systems under certain conditions are associated with varieties, in Chapter 5, we seek the answer to this problem and we show that the varieties of quasigroups associated with 2-perfect m-cycle systems where m=3 and 5, strongly 2-perfect m-cycle systems and 2-perfect extended m-cycle systems can be defined by a single law. In this chapter, for some of these cases more than one single law will be given. We also introduce other single laws for 3-cycle systems different from those previousely shown, and give a minimal basis for the variety of 2-perfect 7-cycle systems.

Centre for Discrete Mathematics and Computing
The University of Queensland
Queensland 4072
Australia