A New Method of Determining Absolute Azimuth and Latitude and Suggestion for a New Type of Meridian Circle

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Abstract

This article describes a new method by which azimuth and the instantaneous latitude of a meridian-prime vertical-transit instrument can be determined absolutely in mid-and low- latitude areas, and discusses some experimental observations obtained on a remodeled ZEISS transit instrument. Requirements for the development of a new type of transit circle along the lines of this new method are also presented.

1. The basic principle and formulae

If a transit instrument is designed such that it can easily and exactly be switched between meridian and prime vertical positions, one could measure during the same night the sidereal time of a star's passage through both the eastern and the western half of the prime vertical and through the meridian, as well as the meridian zenith distance of the star.

Such an instrument is easy to calibrate. The determination of its azimuth does not require an external azimuth mark, and it can be used for the determination of absolute coordinates of celestial objects as well as for an absolute determination of the geographic latitude of the observing site.

For this purpose, the instrument has two pairs of pillars, A and B, which would ideally support its horizontal axis at positions exactly 90° from each other. In the course of the observations, the telescope can be lifted from its pillars and either totally reversed or rotated by one 90° and dropped into the second set of pillars.

In addition, the whole base plate to which the four pillars are rigidly attached can be rotated by 90°. The two positions of the base plate define

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positions I and II, respectively. In position I, the instrument sweeps the meridian when supported by pillars A and the prime vertical when resting in pillars B. In position II, when the whole instrument assembly has been rotated by 90° on a vertical axis, the telescope (ideally) points to the meridian when supported by pillars B and sweeps the prime vertical when it rests in pillars A.

Let the configuration be such that with the movable base plate in position I, the actual azimuth of the southern half of the meridian to which the instrument points (rests on pillars A) is 180° and let the angle between the axes defined by pillars-pairs A and B be $90° + \varepsilon$ such that the telescope, when switched from the south meridian to the last prime vertical is directed toward a point on the vertical whose azimuth is $90° - a - \varepsilon$.

With the base plate in position II, switching the telescope from the south meridian now pillars B to the east prime vertical entails a rotation by $90° - \varepsilon$, so that the actual azimuth of the instrument pointing to the eastern half of the prime vertical will be $90° - a + \varepsilon$ if the azimuth of the instrument in the south meridian position is $180° - a$.

2. The determination of the azimuth correction a.

Let T_e, T_w, T_m, respectively, be the (sidereal) clock readings at one and the same star's passage through the eastern and the western halves of the prime vertical and the meridian, respectively, and assume that these three quantities have already been corrected for inclination and collimation errors, and that T_m was likewise corrected for diurnal aberration. In base plate position I, we obtain eventually from the well-known basic relationships (see also Mao et al. 1983).

$$
\begin{aligned}
& a \; 1-\sin(\phi-\delta)\sin\phi\sec\delta \; + \varepsilon + \tfrac{1}{2}(T_e+T_w-2T_m)\sin\phi \\
& + t\,\Delta\delta\cos\phi\sec\delta\cos z = 0
\end{aligned}
\tag{2}
$$

which may be rewritten in the short form

$$
Aa + \varepsilon + k = 0, \tag{2'}
$$

where the meaning of the symbols in Eq. (2') is obvious. In Eq. (2), t is the star's hour angle on the western half of prime vertical, equal to half the duration between the star's passage through the eastern and western parts of the prime vertical. It may be computed from $\cos t = \tan\delta\cot\phi$ or also as $\frac{1}{2}(T_w - T_e)$. z is the prime vertical zenith distance of the star and may be calculated from $\cos z = \sin\delta\csc\phi$, and $\Delta\delta$ is the hourly rate of the star's declination.

ϕ, δ, t, and z enter Eq. (2) only as coefficients of small quantities, and therefore are always known with the required accuracy of at least 0!2, say.

If ε were known, each set of T_e, T_w and T_m, observed for the same star during this star's transits through the two halves of the prime vertical and the meridian, respectively, would allow one to calculate a from Eq. (2). ε can be determined by direct measurements in the laboratory, but also from astronomical observations, as follows.

3. The determination of ε

The base plate position II, Eq. (2') takes the form

$$Aa_{II} - \varepsilon + k = 0 \tag{2a'}$$

Note that (ideally), ε is an instrument constant and k may be regarded as known. Combining runs on the same star in positions I and II, one could obtain $Aa_I + \varepsilon$ and $AA_{II} - \varepsilon$. Note that subtracting these would not yield ε since a_I and a_{II} cannot be presumed identical.

On the other hand, meridian transit observations depend in no way on ε, and a_I and a_{II} can therefore be determined separately from observing of only meridian transit times of specially selected (azimuth) stars. Even though the errors in the catalogue positions of these azimuth stars enter fully into the calculated azimuths, the errors thereby introduced in the calculation of ε will be completely neutralized because ε is obtained by subtracting $Aa_{II} - \varepsilon$ from $Aa_I + \varepsilon$ and depends therefore only on $a_I - a_{II}$ which is free from errors caused by the errors of the azimuth star's catalogue position, as long as the same azimuth star was used for determining a_I and a_{II}. The azimuth stars these serve basically as fiducial marks whose precise positions need not be known for the purpose at hand.

4. The determination of the instantaneous latitude ϕ

The zenith distance of a star at its meridian passage through the south of the zenith, satisfies the relationship $\delta = \phi - Z$, If this is inserted into the relationship $\cos t = \tan\delta \cot\phi$ which holds on the prime vertical, we obtain

$$\sin(2\phi - Z) = \sin Z \operatorname{ctg}^2 \frac{t_\alpha}{2}, \tag{3}$$

Where $t_\alpha = \frac{T_w - T_e}{2}$ or $T_\alpha = T_m - T_e = T_w - T_m$, is observed. In Eq. (3), T_e and T_w must have been corrected for $(a+\varepsilon)$, the approximate variations of the observed star's coordinates $\Delta\alpha$, and $\Delta\delta$ as well as for the changes of clock rate; in addition, if T_m is used for the calculation of t_α, it must be corrected for the influence of a.

Eq. (3) thus allows one to calculate the latitude ϕ absolutely from timing successive prime vertical transits of a star and measuring its meridian zeinith distance without knowing its declination even approximately.

5. Experimental Results

In order to test the principle as well as the accuracy and precision of this method, we have modified a ZEISS transit instrument by mounting four support pillars to the base of the instrument. Each of the two pairs of pillars can support and define the horizontal axis, such that the remodeled instrument can be pointed to both the meridian and the prime vertical, and can be reversed completely in the course of either a prime vertical or a meridian observation. (In addition, the whole instrument assembly can be rotated on a vertical axis by 90°. The remodelled instrument records the transit times of a star photoelectrically before and after reversing. Since our instrument has no accurate divided circle for reading the zenith distance we only determined the azimuth a and the instrument constant ε and not ϕ.)

Experimental observations were carried out in 1982 (Mao et. al., 1983), and again from February to April 1983. During a total of 23 nights, ten determinations of ε were obtained, the standard error of a single observation being 9.5ms; hence we obtained $|\varepsilon| = 0\overset{s}{.}098 \pm 0\overset{s}{.}0030$. During the same period, 100 observations were made altogether. From the nightly averages of the observations (each night two to eight observations) we found the residual v of a single observation. These numbers showed the standard error of a single observation to be 12.5ms. Although we did not make latitude observations, a precision of up to $0\overset{''}{.}25$ to $0\overset{''}{.}30$ for a single observation might be expected at a location of 25° latitude, considering both the obtainable precision of timing the prime vertical transit and that of a modern transit circle for determining zenith distances.

6. The Requirements For the New Type of Meridian Circle

A transit circle for applying this new method of observing all objects in the prime vertical as well as on the meridian sets the following requirements:

1). Observations can be made in both the meridian and the prime vertical.

2). The instrument must be reversible in the course of each observation.

3). To make it possible to determine ε, the entire base of the instrument can be rotated by 90°.

4). A dioptric or catadioptric telescope with long focal length and a short tube is required to facilitate the various manuipulations. Although the optical axes of dioptric telescopes are not very stable, the reversing during each observation largely compensates for the instability, so that the strength of the telescope tube and the reliability in the mounting of the optical parts are the most essential design characteristics.

5). A precisely divided declination circle and a reading device must be in place to read precise zenith distances during meridian transits.

6). The variations of the optical axis due to the flexure of the tube and the shift of the objective by gravity should be determined with the help of additional equipment during the observation to each star in order to increase the precision of declination observations from reversing the instrument.

7). A device which can automatically determine the inclination of the horizontal axis relative to a mercury level should be developed.

7. Conclusions

The new method allows one to make absolute determinations in low latitude and, in addition, it also has following advantages:

1). The time interval between an azimuth and a latitude determination is very short (about four to six hours, and can be controlled by the selection of stars).

2). There will be no systematic errors due to differences between day-time and night-time observations since all observations are made at night.

3). Relatively many stars can be observed during each night and it is, therefore possible to increase the accuracies of the estimated values of both azimuth and latitude.

4). The azimuth can be determined without depending on a mire, so that the errors in observing the mire and the difficulties in establishing a highly stable one are also avoided.

5). As the result of the small zenith distances (generally less than 10°) of stars crossing the meridian, the influences of the refraction errors and the flexure corrections on latitude observations will be reduced.

Reference

Mao, Wei, et. al., 1983, Acta Astronomia Sinica, **24**, 169. (English Translation in Chin. Astron. Astrophys. **8**, 8).)