COMMISSION 7: CELESTIAL MECHANICS (MÉCANIQUE CÉLESTE)

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The Vice-President assumed the duty of writing this report only one month before the deadline and this circumstance certainly affected its completeness. Because of the great diversity of research under way in the various fields of Celestial Mechanics, triennial reports have been usually divided into chapters critically describing in some detail the achievements in specific domains of Celestial Mechanics. The choice of the chapters reflects, this time, also the areas for which it has been possible to find a colleague willing to stop other duties and give priority to writing a report on the progress of the area. The Vice-President acknowledges that, without the kind cooperation of several colleagues and the members of the Organizing Committee, this task would not have been possible and we would have to regret the absence of a Celestial Mechanics report in this volume.

Since the last report, Commission 7 has sponsored IAU Symposium No. 152 "Chaos, Resonance and Collective Dynamical Phenomena in the Solar System" (Angra dos Reis, Brazil, July 1991) and co-sponsored IAU Symposium No. 160 "Asteroids, Comets, Meteors 1993" (Belgirate, Italy, June 1993). It has also co-sponsored two joint discussions during the XXIst General Assembly (Buenos Aires, 1991): "Origin of Stars and Planetary Systems" and "Reference Systems: What are they and what's the Problem".

Other international conferences of interest to Commission 7 were: "Hamiltonian Systems and Celestial Mechanics" (Guanajuato, Mexico, October 1991); "The dynamics and evolution of minor bodies with galactic and geological implications" (Kyoto, Japan, October 1991); 3rd Alexander von Humboldt Colloquium for Celestial Mechanics: "Qualitative and Quantitative Behaviour of Planetary Systems" (Ramsau, Austria, April 1992); "Interactions between Physics and Dynamics of Solar System Bodies" (Val André, France, June 1992); "Ergodic Concepts in Stellar Dynamics: Theory, Computer Experiments, Observations" (Geneva, Switzerland, March 1993); the International Conference on Hamiltonian Dynamics: "Integrability and chaotic behaviour", (Toruń, Poland, June 1993); the NATO Advanced Study Institute "From Newton to Chaos: Modern Techniques for Understanding and Coping with Chaos in N-Body Dynamical Systems" (Cortina d'Ampezzo, Italy, July 1993); "Dynamics and Astrometry of Natural and Artificial Celestial Bodies" (Poznań, Poland, September 1993); "Seventy-five years of Hirayama Collisional families: The role of collisions in the Solar System history", (Sagamihara, Japan, November 1993) and "Advances in Non-Linear Astrodynamics" (Minneapolis, USA, November 1993)

Several regional or national meetings were also held during the past three years: the 22nd, 23rd and 24th annual meetings of the Division of Dynamical Astronomy of the American Astronomical Society (Key Biscayne, May 1991; Chicago, June 1992 and Santa Barbara, May

J. Bergeron (ed.), Reports on Astronomy, Vol. XXIIA, 15–28.

1993); the 24th and 25th Japanese Symposiums on Celestial Mechanics (Tokyo, January 1991 and December 1992); the 5th and 6th Brazilian Colloquiums on Orbital Dynamics (Curitiba, November 1990 and Aguas de São Pedro, November 1992); the first Italian Conference on Celestial Mechanics (L'Aquila, May 1993); the 15th Spring School of Goutelas (France): "Interrelations between Physics and Dynamics of Minor Bodies in the Solar System" (April 1991); the Chamonix winter schools: "Dynamique relativiste appliquée à l'Astrometrie, à la Mécanique Céleste et aux modèles d'Univers" (February 1992) and "Integrabilité des Systèmes Differentiels, techniques symplectiques en physique et applications à la Mécanique Céleste" (February 1993) and the workshop "Artificial Satellite Theory" (Washington, November 1993).

Several books directly related to Celestial Mechanics were published since the last report:

Benest, C. & Froeschlé, C. (eds.): 1990, "Modern Methods in Celestial Mechanics", Ed.Frontières, Gif-sur-Yvette.

Benest, C. & Froeschlé, C. (eds.): 1992, "Interrelations between Physics and Dynamics for Minor Bodies in the Solar System", Ed.Frontières, Gif-sur-Yvette.

Bois, E., Oberti, P. & Henrard, J., (eds.): 1993, "Interactions between Physics and Dynamics of the Solar System Bodies", Kluwer A.P., Dordrecht.

Brumberg, V.A.: 1991, "Essential Relativistic Celestial Mechanics", Adam Hilger.

Capitaine, N., (ed.): 1991, "Journées 1991: Systèmes de reference spatio-temporels". Observatoire de Paris.

Capitaine, N., (ed.): 1992, "Journées 1992: Systèmes de reference spatio-temporels". Observatoire de Paris.

Clube, S.V.M., Yabushita, S. & Henrard, J., (eds.): 1992, "Dynamics and Evolution of Minor Bodies with Galactic and Geological Implications"

Dvorak, R. & Henrard. J., (eds.): 1992. "Qualitative and Quantitative Behaviour of Planetary Systems", Kluwer, Dordrecht.

Ehlers, J. & Schaefer, G. (eds.), "Relativistic Gravity Research with Emphasis on Experiments and Observations". Lecture Notes in Physics, 410, 1992, Springer-Verlag, Berlin.

Ferraz-Mello, S. (ed.): 1991, "Chaos, Resonance and Collective Dynamical Phenomena in the Solar System", Proceedings of IAU Symposium No. 152, Kluwer, Dordrecht.

Gerasimov, I.A.: 1990, "Weierstrass Functions and applications to Mechanics and Astronomy", in Russian, Moscow University.

Hughs, J.A., Smith, C.A. & Kaplan, G.H. (eds.): 1991: "Reference Systems", Proceedings of IAU Colloquium No. 127, U.S. Naval Observatory.

Kinoshita, H. & Yoshida, H. (eds.): 1991, Proceedings of the 24th Symposium on Celestial Mechanics, Tokyo.

Kinoshita, H. & Nakai, H. (eds.): 1992, Proceedings of the 25th Symposium on Celestial Mechanics, Tokyo.

Lacomba, E.A. & Llibre, J. (eds.): 1993, "Hamiltonian Systems and Celestial Mechanics", World Scientific, Singapore.

Lieske, J.H. & Abalakin, V.K., (eds.): 1990, "Inertial Coordinate Systems on the Sky", Proceedings of IAU Symposium 141", Kluwer, Dordrecht.

Liu, B.-L. & Fiala, A.D.: 1992, "Canon of Lunar Eclipses 1500 BC – AD 3000", Willmann-Bell, Richmond.

Neutsch, W. & Scherer, K.: 1992. "Celestial Mechanics. An Introduction to Classical and Contemporary Methods", BI, Mannheim.

Phillips, J.A., Thorsett, S.E. & Kulkarni, S.R. (eds.): 1992, "Planets around Pulsars", Astron. Soc. Pacific Conference Series, vol. 36.

Poincaré, H.: 1993, New Methods of Celestial Mechanics, American Institute of Physics, New York (History of Modern Physics and Astronomy, vols. 12–14).

Rastall, P.: 1991, "Postprincipia: Gravitation for Physicists and Astronomers", World Scientific, Singapore.

Roy, A.E., (ed.): 1991, " Predictability, Stability and Chaos in N-Body Dynamical Systems", Proceedings of the 1990 Cortina NATO ASI, Plenum, New York.

Schneider,M.: 1992, "Himmelsmechanik. BI, Mannheim.

Seidelmann, P.K., (ed.): 1992, "Explanatory Supplement to the Astronomical Almanac", University Science Books.

Wright, J.L.:1992, "Space Sailing" Gordon and Breach, Philadelphia.

PLANETARY COMPANIONS

Besides the specific progresses reported in the forthcoming sections, we have to report some new problems open by recent discoveries in Astrophysics concerning possible planetary companions of stars.

Two pulsars have shown residuals in the time of pulse arrivals leading to the suspect of the existence of planetary companions: PSR 1829-10 and PSR 1257+12 (Bailes *et al.*, *Nature* **352**, 311; Wolszcan & Frail, *Nature* **355**, 145). In the past, other similar cases (the Crab nebula pulsar PSR 0531+21 and PSR 0329+54) were announced but were not confirmed by subsequent observations and the residuals of PSR 1829-10 are now known to be of terrestrial origin. However, in the case of PSR 1257+12, the residuals fit a model consisting of two planets in almost circular orbits and no alternative hypothesis can so easily reproduce the observations. The minimum mass of these planets (corresponding to the case where the system would be edge on) is 3.4 and 2.8 Earth masses. The periods of 66.56 and 98.23 days are very close to a commensurability 2/3. The mutual perturbation of the orbits must reflect in additional variations in the pulse timings (Rasio *et al.*, *Nature* **355**, 325; Malhotra *et al.*, *Nature* **355**, 583). Peale (AJ **105**, 1562) and Malhotra (ApJ **407**, 266) indicate that 3 to 5 years of continuous observations will be enough to support or refute the planetary interpretation of the residuals. The current observations are already enough to exclude the possibility of planets larger than a dozen Earth masses.

The existence of planetary companions was also hypothesized by astrophysicists to explain several features of the gas cloud about the star *β Pic*: the inner disk gap at 20–30 AU, deduced from IR data, and the finite thickness of the cloud (30–50 AU at 200 AU from the star). The inner gap could be due to large bodies shepherding and clearing the fine dust and the finite thickness could be maintained by the gravitational perturbation of embedded bodies (see Norman & Paresce, in *The formation and evolution of Planetary Systems*, Cambridge, 1989). There are also some unusual redshift spectral absorption events attributed to the evaporation of infalling solid bodies (comets?). In this case, the presence of a planet has been invoked to perturb passing-by bodies and making some of the star-grazers detectable by spectroscopic means. A model showing planetary perturbations able to put numerous small bodies into star-grazing orbits has been established by Beust *et al.* (AA **247**, 505). The shepherding of the gas in the inner border has been studied by several authors (Scholl *et al.*, CM **56**, 381; Lazzaro *et al.*, CM **56**, 395 and *Icarus*, in press; Sicardy *et al.*, CM **57**, 373; Roques *et al.*, *Icarus*, in press and Beaugé & Ferraz-Mello, *Icarus*, submitted). The main result in these papers is the temporary capture of the particles in some exterior resonances with the planet. They are reviewed in section E of this report.

A. Relativistic Celestial Mechanics

(Victor A. Brumberg)

The starting point of many investigations on relativistic Celestial Mechanics during the period 1991-1993 was the adoption by the 21st General Assembly of IAU (Buenos-Aires, 1991) of the new recommendations on reference systems and time scales based, for the first time in the IAU history, on General Relativity. These recommendations aroused a lot of questions. Some authors tried to answer these questions and to discuss problems related with practical using of new recommendations in Celestial Mechanics, Astrometry and Geodesy (Brumberg, *Highlights of Astronomy*, **9**, 133; Seidelmann & Fukushima, AA **265**, 833; Soffel & Brumberg, CM **52**, 355). Main new results have been obtained in the following domains:

1. RELATIVISTIC THEORY OF REFERENCE SYSTEMS

In a series of papers, Damour, Soffel & Xu (*Phys.Rev* D, **43**, 3273; **45**, 1017; **47**, 3124) have elaborated a new theory of relativistic reference systems. Its main difference from the earlier theory by Brumberg & Kopejkin is the introduction of the physically more plausible Blanchet-Damour moments to describe the multipole structure of gravitational fields, instead of formally Newtonian multipole moments . This enables one to present the relativistic space-time transformations between different reference systems and the corresponding metric tensors in more compact form and gives an efficient tool to handle the problems of relativistic Celestial Mechanics on the post-Newtonian level of accuracy. Not too technical description of this (DSX) approach may be found in Damour *et al.* (IAU Coll 127, 50; Lect.Notes in Physics, 410, 46). Along with this, the earlier theory by Brumberg & Kopejkin has been re-formulated in the closed form (Brumberg, IAU Coll 127, 36; Klioner & Voinov, *Phys. Rev.* D **48**, 1451). Klioner (AA, in press) has investigated in detail the notion of relativistic kinematically non-rotating reference system (as prescribed by the IAU recommendations) and found that this notion involves ambiguities (being dependent on a place of a reference system in the whole hierarchy of reference systems) which may be removed only by specific conventions.

2. RELATIVISTIC THEORY OF TIME SCALES

This topic is in intimate relation with the previous one because the new time scales are considered as the coordinate times of the corresponding reference systems. Based on the planetary and lunar semi-analytical theories of motion of the Bureau des Longitudes, Brumberg, Bretagnon & Francou (Journées Obs. Paris 1991, p.141) deduced the analytical relationships between these time scales and the physically registered proper time of an observer. In order to actually implement such time scales, one needs to solve the problem of remote clock synchronization. The algorithms to realize several specific types of such relativistic synchronization have been analyzed by Klioner (CM **53**, 81) and Klioner & Fukushima (*Manuscr. Geodaetica*, in press). These algorithms have an accuracy of 10–100 ps, which should be sufficient for the decade to come.

3. RELATIVISTIC REDUCTION OF ASTRONOMICAL OBSERVATIONS

Increasing precision of modern astronomical observations demands the elaboration of highly accurate models of observations. Such models involve the second-order effects in light propagation. Such effects, including the coupling of rotational and translational motion of the gravitating masses and the contribution due to the quadrupole gravitational field, have been studied in detail by Klioner (*Sov.Astron.* *35*, 523; NOAA Tech.Rep. No. 137, 188), Klioner & Kopejkin (AJ **104**, 897) and Hartmann *et al.* (*Veroeff. Bayer. Akad Wiss.* A No.108). The accurate model for optical observations of the accuracy of 1 microarcsecond has been proposed by Klioner & Kopejkin (AJ **104**, 897). VLBI observations demands, even now, relativistic models at the level of 1 ps in time delay and 1 fs/s in delay rate. Several versions of such models have been constructed, for instance, by Klioner (NOAA Tech.Rep.No.137) and Soffel *et al.* (AJ **101**, 2306). A significant progress has been achieved in the relativistic model for analyzing LLR data (Mueller *et al.*, ApJ **382**, L101; Lect.Notes in Phys. 410, 87) and GPS data (Schwarze *et al.*, *Manuscr. Geodaetica* **18**, 306).

4. RELATIVISTIC EQUATIONS OF EARTH SATELLITE MOTION IN BARYCENTRIC AND GEOCENTRIC COORDINATE SYSTEMS

Post-Newtonian equations of Earth satellite motion in the closed form and taking into account all necessary perturbing factors (rotation and non-sphericity of the Earth, solar-lunar action, superposition of different effects) were obtained in different papers (Brumberg, AA **257**, 777; Klioner & Voinov, *Phys.Rev.* D **48**, 1451; Damour *et al.*, *Phys.Rev.* D, in press).

5. RELATIVISTIC DESCRIPTION OF THE EARTH ROTATION

The results obtained in this domain are of preliminary nature. Some expressions for the angular velocity of the Earth rotation in different relativistic coordinate systems were obtained by Brumberg *et al.* (AA **275**, 651). An approach to construct the relativistic model of the Earth rotation has been developed by Bizouard *et al.* (Journées Obs. Paris 1992, p.76).

6. POST-POST-NEWTONIAN EFFECTS IN CELESTIAL MECHANICS PROBLEMS

Investigation of the post-post-Newtonian effects in the two-body problem is of importance with respect to the binary pulsar motion. Detailed analysis of these effects including their relation with gravitational radiation has been performed in a series of papers written or co-written by Schaefer (Schaefer, *Ann.d.Physik* **48**, 601; Damour & Schaefer, *Phys.Rev.Lett.* **66**, 2549, *J. Math. Phys.* **32**, 127; Junker & Schaefer, MNRAS **254**, 146; Schaefer & Wex, *Phys. Lett.* A **174**, 196; **177**, 461; Blanchet & Schaefer, *Class.Quant.Grav.*, in press). Some of these effects have no analogies in the Newtonian N-body problem and are of interest both from mathematical and physical points of view. Post-post-Newtonian treatment of the two-body problem was also done in a different manner by Kopeikin & Potapov (*Astron.Zh.*, in press).

B. Planetary Obliquities

(Sylvio Ferraz-Mello)

A major breakthrough was recorded in our knowledge of the evolution of the obliquities of the planets. The most striking results concern the Earth whose obliquity would have been frozen to its current value by the capture of the Moon. Indeed, the dynamics of the rotation of an oblate planet under the action of secular planetary perturbations is chaotic. In the case of the Earth, the integration of the equations of the precession over 18 million years shows a large chaotic zone. In its present state, the Earth avoids this chaotic zone and its obliquity is essentially stable, exhibiting only small variations of ±1.3° around the mean value of 23.3°. If the Moon were not present, the chaotic zone would extend from nearly 0° to about 85°. Thus, even if the initial obliquity of the Earth was very small, this chaotic behaviour could have raised it to a large value in a few million years (Laskar *et al.*, *Nature* **361**, 615). Precise solutions for the obliquity and the precession of the Earth were published by Quinn *et al.* (AJ **101**, 2287) and by Laskar *et al.* (AA **270**, 522). These solutions provide the necessary data for the computation of the insolation at the surface of the Earth in the past. The insolation at the latitude 65°, whose long term variations is associated to major variations in the Earth climate, varied in the past million years between 380 and 500 W/m^2. Without the Moon, this variation range increases by a factor 2.5. Laskar *et al.* claim that the Moon is a climate regulator for the Earth and, if it were not present, the climate on Earth would suffer drastic changes. We may wonder if organized life would be possible in such circumstances.

The planetary obliquity of Mars has been studied by Ward & Rudy (*Icarus* **94**, 160), Touma & Wisdom (*Science* **259**, 294) and Laskar & Robutel (*Nature*, **361**, 608). The main results are as follows: There is a strong secular resonance due to near equality of the frequency of precession with s_2'. The obliquity of Mars is chaotic with possible variations from 0° to 60°.

In what concerns the other planets the results are the following (Laskar *et al.*, *Nature* **361**, 608): The present spin of Mercury is very low and apparently trapped in a 2 : 3 spin-orbit resonance; however, provided its primordial period was smaller than 300 h, it must have suffered large-scale chaotic behaviour during its history. In the same way, if the primordial rotation of Venus was prograde, it has suffered large-scale chaotic behaviour in the past, being able to diffuse from 0° to nearly 90° in 1 Myr. When the obliquity is $\sim 90°$, it can evade the chaotic zone as the planet slows down and dissipative effects can then drive the spin axis to its present position (178°). The situation with the outer planets is different: The spectrum of the planetary forcing terms consists only of well isolated lines. In general, it is difficult to destroy the stability of the obliquities of these planets until their precession constant reaches 26 arcsec/yr. The obliquities of the outer planets are essentially stable and should be considered as primordial.

The primordial obliquities of the planets have been recently studied by Tremaine (*Icarus* **89**, 85), Lissauer & Safronov (*Icarus* **93**, 288), Tanikawa *et al.* (*Icarus* **94**, 112), Lissauer & Kary (*Icarus* **94**, 126) and Dones & Tremaine (*Science*, **259**, 350; *Icarus* **103**, 67). In general, the origin of nonzero primordial obliquities is unknown and even the sign of the rotation induced by accretion is difficult to determine. Planets which accrete from large planetesimals have, on the average, shorter spin periods and greater obliquities than those accreted exclusively from small bodies. Present values for the obliquity of the outer planets require implausible large impacting bodies; for instance, to produce Saturn's obliquity, that planet should have accreted at least one planetesimal several times as massive as the Earth.

The study of the obliquities of the planets required important methodological advances: (a) Touma & Wisdom extended the symplectic N-body integration algorithms of Wisdom & Holman (AJ, **102**, 1528; **104**, 2022) to include the rotational dynamics of extended bodies; (b) Laskar extended the frequency analysis technique used to detect layers of chaotic behaviour in the orbital motion of the planets to the study of global dynamics (Laskar, *Icarus* **88**, 266; Laskar *et al.*, *Physica* D **56**, 253; see also Dumas & Laskar, *Phys.Rev.Letters* **70**, 2975).

C. Dynamics of Asteroids

(Andrea Milani & Sylvio Ferraz-Mello)

Very significant progress has occurred, since the last report, on many subjects related to the dynamics of the asteroid belt; we are now beginning to have a global understanding of the long-term dynamics of a very large portion of the phase space, which includes the orbits of most real asteroids. Although the different problems are strictly related, and the same techniques can be used for several tasks, we shall separate for ease of presentation the following subjects: 1. Proper elements and secular resonances; 2. Chaos and stability; 3. Mean motion resonances; 4. The model problem; 5. Binary asteroids; 6. Discovery and identification.

1. PROPER ELEMENTS AND SECULAR RESONANCES

The state of the art, as reported at the last IAU Assembly, was characterized by the development, then in progress, of new techniques of analytical and semianalytical investigation, with the numerical ("synthetic") techniques more and more used for confirmation and/or extrapolation of the analytical results. These methods are now fully developed and many results have been published. The location of the main secular resonances was derived with a purely analytical technique by Knežević *et al.* (*Icarus* **93**, 316). The extension of this calculation to much higher inclinations and eccentricities was possible with the use of a semianalytic method by Morbidelli & Henrard (CM **51**, 131; CM **51**, 169). The problem which were formerly intractable, because of the overlapping of a mean motion and a secular resonance, can now use the formalism of successive elimination of harmonics by Morbidelli (CM **55**, 101) and Morbidelli & Giorgilli (CM **55**, 131); in this way the occurrence of secular resonances inside and besides the Kirkwood gaps was investigated by Morbidelli & Moons (*Icarus* **102**, 316), Morbidelli *et al.* (AA, in press). Of course, there are always dynamical behaviours too complicated to be investigated only with analytical and semianalytical methods, but it is now possible to use the theories as a guide for selective numerical experiments on the most complicated orbits, such as the meteorite delivery route investigated by Farinella *et al.* (*Icarus* **101**, 174; CM **56**, 287).

A more detailed analysis of the dynamical structure of the phase space is possible for the "core" region of the asteroid belt, the one at low to moderate inclination, bounded by the secular $g - g_6$ resonance and by the 2 : 1 Kirkwood gap; this is where most real asteroids are. In this region we can apply a fully analytical theory, expanded to degree 4 (and more) in the eccentricities and inclinations and to order 2 in the planetary masses, developed by Milani & Knežević (*Icarus* **98**, 211; *Icarus*, in press). In this way, it has been possible to explore the web of higher degree secular resonances (with divisors formed by combinations of 4, 6 and even 8 fundamental secular frequencies), and to assess the reliability of proper elements computed with this theory. Proper elements in the core belt are now stable enough to identify even small asteroid families, as confirmed by numerical experiments spanning several Myr. A recent development has been the development of reliable and reasonably accurate proper elements even outside the core belt: with semianalytical methods, Lemaître & Morbidelli (CM, in press) have computed proper elements (also expanded to order 2 in the masses) for high-inclination asteroids and Morbidelli (*Icarus*, in press) has computed proper elements for secularly resonant asteroids. Proper elements for Hildas have been computed by Schubart (AA **241**, 297) and for Trojans by Milani (CM, in press), with synthetic methods based upon the post-processing of the output of a numerical integration; although these methods are still based on computational brute force, they do allow to understand the dynamical structure and to locate the main resonances, as in Milani (IAU Symp 160, in press). In conclusion, both the location of the main resonances and the computation of proper elements stable over millions of years are essentially solved problems for almost the entire asteroid belt. Reviews on these topics can be found in Lemaître (CM **56**, 103), Knežević & Milani (IAU Symp 160, in press), Froeschlé & Morbidelli (IAU Symp. 160, in press).

2. CHAOS AND STABILITY

At the time of the previous IAU assembly, the main focus of the investigations of chaos and long-term stability was on the orbits of the major planets; more recently, the same problems have been raised for the asteroids. There are so many asteroids that an investigation of the stability of a significant proportion of their population amounts almost to a global exploration of the phase space. Moreover, the motion of the asteroids can be described within a simplified model, although some caution is required (see also 4.). The analytical, and fully rigorous, results on the stability of some asteroids are still scarce; however, the first results applicable for a time span as long as the age of our Solar System have been obtained for Trojan type orbits: see Celletti &

Giorgilli (CM **50**, 31); this is the continuation of earlier work by Giorgilli *et al.* (*J.Diff.Eq.* **77**, 167) and Simó (*Mem.R.Acad.Cienc.Ar.Barcelona* **48**, 303). Unfortunately, these results based upon the Nekoroshev method still apply only to the restricted 3D problem and to very small libration amplitudes. On the contrary, numerical experiments on larger and larger sets of initial conditions and for longer and longer times are done by many. Lecar *et al.* (*Icarus* **96**, 251) report that there is a correlation between the Lyapunov time and the crossing time obtained in a number of restricted 3-body problem experiments; they compute a best fitting power law, e.g., for the outer asteroid belt the crossing time is most likely to be proportional to the Lyapunov time raised to power 1.7. Holman & Wisdom (AJ **105**, 1987) and Levison & Duncan (ApJ **406**, L35) find different best fitting exponents (viz. 1.9 and 1.4) for experiments in the outer Solar System and point out that the dispersion around the best fitting curve amounts to two orders of magnitude in the time span for instability. The stability of orbits in the Trojan region, and also for hypothetical Trojans of other planets, has been investigated by Mikkola & Innanen (AJ **104**, 1641), Holman & Wisdom (AJ **105**, 1987) and De la Barre (PhD. UCLA, 1993): they all report a very complicated dynamical structure with gaps and peculiar behaviours still to be explained. On the other hand, Milani & Nobili (*Nature* **357**, 569) report that the asteroid *522 Helga* is chaotic but macroscopically stable for a time span of 1000 Lyapunov times; examples of this puzzling "stable chaos" are also reported by Mikkola & Innanen (AJ **104**, 1641), Milani (CM, in press) and Milani & Kneževič (*Icarus*, in press). In conclusion, the relationship between chaos and stability over time spans comparable to the age of the Solar System is far from clear and is currently the object of intense investigations. The long term instability for another class of orbits, more typical of comets, has been established by Bailey *et al.* (AA **257**, 315): a significant fraction of the very high inclination objects ends up in sungrazing orbits.

3. MEAN MOTION RESONANCES

The knowledge of the global dynamics of the mean motion resonances, from 3:1 to 4:3, recorded a rapid progress in the past triennium. The state of this knowledge, at the end of the past decade, was reviewed by Froeschlé & Greenberg (Asteroids II, 1989, 827). It consisted, mainly, of the explanation of the 3:1 and 5:2 gaps, by orbital intermitencies increasing the eccentricity to values high enough to allow the asteroid orbit to intersect the orbit of Mars (Wisdom, *Icarus* **72**, 241; Yoshikawa, *Icarus* **87**, 78) and the understanding of the low-eccentricity chaos of the resonances 2:1 and 3:1 as due to separatrix-crossing mechanisms associated to secondary resonances (Lemaître & Henrard, *Icarus* **83**, 391; Henrard & Caranicolas, CM **47**, 99).

The resonance 3:1 has been revisited by several authors. Ferraz-Mello & Klafke (Cortina 1990 NATO ASI, p.177) mapped the structure of the phase space up to $e = 0.9$ and showed that, besides the chaotic region found by Wisdom, there is another one at high eccentricities and these two chaotic regions are not always disjoint. Intermitencies in this region allow the asteroid to have close approaches to the Earth which, because its larger mass, may have played an important role in the scattering of the 3:1 resonant orbits. This structure was also found by Hadjidemetriou (CM **53**, 151) and orbits of this kind were computed by Saha (*Icarus* **100**, 434). The overlapping of the 3:1 resonance and the secular resonance ν_6 was studied by Ch.Froeschlé and Scholl (CM, **56**, 163).

The previous models were extended to the resonances 5:2 and 7:3 by Šidlichovský (CM **56**, 143; AA, in press), Klafke *et al.* (IAU Symp 152, 153), Yokoyama & Balthazar (IAU Symp 152, 159; *Icarus* **99**, 175) and Yoshikawa (*Icarus* **92**, 94). These studies show that the dynamics of the 5:2 resonance is very similar to that of the 3:1 resonance and the chaotic origin of the corresponding gap was confirmed by Ipatov (*Sov.Astron.Lett.* **15**, 324; *Icarus* **97**, 309). The 7:3 resonance presents confined low-eccentricity orbits (like the 2:1 resonance) and the action of Jupiter is not sufficient to explain the gap.

The existence of a gap at the 2:1 resonance and a group of asteroids at the 3:2 resonance was explained on the basis of different degrees of chaoticity under the perturbations of the outer planets (Ferraz Mello, IAU Symp 160, in press; Ferraz-Mello *et al.*, Cortina 1993 NATO ASI, in press). The Lyapunov times of the resonant orbits in the 2:1 and 3:2 resonances lie, respectively, in the ranges $10^{3.5} - 10^{5.5}$ and $10^6 - 10^7$. The averaged phase space of these two resonances, when the perturbations of Jupiter and Saturn are considered, has been studied by Morbidelli & Moons (*Icarus* **102**, 316). The results show sets of seemingly regular motions consistent with large Lyapunov times. Yoshikawa (*Icarus* **92**, 94) showed that the boundary of the 2:1 Kirkwood gap coincides with the region of resonant motions and Michtchenko (Dr. Thesis, USP) has shown that these limits coincide with the limits of the region of low-eccentricity chaotic orbits in the restricted three-dimensional problem. Simulations showing chaotic depletion in the 2:1 resonance were presented by Scholl in IAU Symposium 160. High Lyapunov times for the 3:2 resonant asteroids were also found by Franklin *et al.* (AJ **105**, 2336). Alfimova & Gerasimov (IAU Symp 152, 139) reconsidered the statistical hypothesis and associated the non-existence of low-eccentricity asteroids in the resonance 2:1 to their fast perihelion motion. Ferraz Mello (IAU Symp 160, in press) and Ferraz-Mello *et al.* (Cortina 1993 NATO ASI, in press) have also shown that the maximum value allowed for the eccentricity of an asteroid in the resonance 3:2 is ~ 0.4. Orbits with higher eccentricities are generally driven to close approaches to Jupiter and are scattered from this resonance.

Some partial results were obtained concerning other resonances. The structure of the phase space of the 4:1 resonance has been mapped by Klafke *et al.* (IAU Symp 152, 153). It overlaps with the secular resonances ν_6 and ν_{16} in a very complex way; Scholl & Ch.Froeschlé (AA **245**, 316) have shown that this overlapping is responsible for the huge depletion in the whole region 2.07 – 2.13 AU. Ries (BAAS, in press) studied the resonance 4:3 in the frame of the planar restricted three body problem and found results pointing towards the survival of low eccentricity objects ($e < 0.075$). An extended numerical survey of all resonances was done by Dvorak *et al.* (AA, **274**, 627; see also IAU Symp 152, 145)

The regular solutions of the averaged problem were the subject of many papers (for a review see *Rev.Mex.Astr.Astrof.* **21**, 569). Ferraz-Mello *et al.* (IAU Symp 152, 167; CM **55**, 25) studied the solutions of the restricted elliptic problem where secular and mean-motion resonances happen simultaneously (corotations). Tsuchida (*Rev.Mex.Astr.Astrof.* **21**, 585), studied the regular librations of *279 Thule*. Moons & Morbidelli (CM, **56**, 273; CM, in press) studied the phase portrait of several resonances in the restricted circular problem up to very high eccentricities. They also included the secular resonances ν_5 and ν_6 and determined their positions in the plane of initial conditions. These papers use local representations of the disturbing functions valid for high eccentricities, to deal with the convergence problem (see Ferraz-Mello & Sato, AA **225**, 541; Morbidelli & Giorgilli, CM **47**, 145). In this respect, Ferraz-Mello (CM, in press) revisited Sundman's convergence criterion and extended it for the expansions in the inclination. According to Sundman's criterion the convergence radius at the main asteroidal resonances: 3:1, 2/1 and 3/2, is, respectively, ~ 0.35, ~ 0.20 and ~ 0.09 (in eccentricity). These limits put strong constraints to the domain of validity of analytical theories of the mean motion resonances.

Schubart (AA **241**, 297) and Michtchenko & Ferraz-Mello (CM **56**, 121) used synthetic theories to study the regular librational motion of the Hildas. Hadjidemetriou (CM **56**, 201) studied the families of periodic orbits of the first and second kind in the 2:1, 3:1 and 4:1 resonances. The stability of the solutions of the circular problem in the 2:1 and 3:1 resonances was studied by Dvorak & Kribbel (IAU Symp 152, 171).

4. THE MODEL PROBLEM

As the accuracy of the methods available to observe the asteroid orbits increases (see Ostro, AJ **102**, 1491; Yeomans et al. AJ **103**; Bec-Borsenberger AA **257**, 844; AA **258**, 94), the need arises not only for more and more sophisticated computational methods, but also to take into account more and more subtle perturbing forces. This process is analogous to what happened for the orbits of artificial satellites between the 60's and the 80's (for a review see Milani et al. *Non gravitational perturbations and satellite geodesy*, Hilger 1987). A controversy has arisen on the orbit of asteroid *1566 Icarus*, with Yeomans (AJ **101**, 1920) claiming that non gravitational forces were needed to fit the observations and Sitarski (AJ **104**, 1226) claiming that general relativistic effects were enough, as later acknowledged by Yeomans (AJ **104**, 1266). The use of mutual perturbations of asteroids as a way to estimate the mass of the largest asteroids has been continued (see Schubart AA **264**, 719; Kuzmanoski & Knežević, *Icarus* **102**, 93). It has even been proposed by Plastino & Vucetich (AA **262**, 321) and Orellana & Vucetich (IAU Symp 152, 185; AA **273**, 313) to use the orbits of some Trojan asteroids as a way to search for deviations from the known theories of gravitation. Another problem is the choice of a simplified model to be used to explain the occurrence of a given dynamical behaviour in the orbits of real objects. More care should be used in extrapolating, to the complexity of real orbits, the conclusions obtained from tests and/or theories based upon very simple models, such as the restricted 3-body problem. As an example, Schubart (CM **56**, 153) points out that the results on some chaotic orbits in the outer belt strongly depend upon the model and cannot be described in terms of the restricted problem.

5. BINARY ASTEROIDS

The interest for this subject was of course raised by possible observational evidence for contact binary asteroids and also by the need to assess the safe fly-by distance from an asteroid for space probes (Hamilton & Burns, *Icarus* **96**, 43). Some recent investigations focused on the long term stability of possible binary asteroids against the perturbations occurring at close approaches (Chauvineau *et al.*, *Icarus* **94**, 299; Farinella, *Icarus* **96**, 284). A method to identify former binary asteroids even after they split has been proposed for Trojans (Milani, IAU Symp. 160, in press). There are many open problems in this field.

6. DISCOVERY AND IDENTIFICATION

Although this subject concerns more Commission 20 and the WGNEO, it is important to point out that the renewal of interest on these topics has resulted in new investigations on the problems of accuracy and reliability of orbit determination (see e.g. Marsden, AJ **102**, 1539; Whipple & Hemenay, AJ **102**, 816; Kristensen, AA **262**, 606; Bowell & Muinonen, *Icarus*, in press). Some of the problems raised about the algorithms to be used to solve the problem of identification of newly observed asteroid orbits with some of the known ones, and to assess the reliability of these procedures, imply non trivial investigations in the fields of Celestial Mechanics, Probability Theory, Numerical Analysis and Image Processing; the specialists of Celestial Mechanics should follow more closely these discussions.

D. Dynamics of the Kuiper Belt

(Martin J. Duncan & Harold F. Levison)

In 1951, Kuiper pointed out that it seems unlikely that the disk of planetesimals that formed the planets would have abruptly ended at the current location of the planet Neptune. He suggested that originally there was a significant number of planetesimals in circular orbits about the Sun outside the planetary region. The objects that formed in this region presumably resemble present-day comets. This region is now called the *Kuiper belt*. Over the years, many researchers have attempted in vain to discover objects in the Kuiper belt (Levison & Duncan, AJ **100**, 1669). However, within the last year, Luu & Jewitt have discovered two large (radius on the order of 100 km) objects in this region of the solar system: 1992 QB1 (IAU Circular 5611) and 1993 FW (IAU Circular 5730).

Perhaps the most intriguing argument for the existence of a populous Kuiper belt is one put forth by Fernández (MNRAS **192**, 481), who suggested that it is the source of the known Jupiter-family comets (with periods, $P \lesssim 20yrs$, hereafter JFCs). Indeed, the most comprehensive numerical integrations done to date, performed by Quinn, Tremaine, & Duncan (ApJ **355**, 667), show that the inclination distribution of JFCs is inconsistent with them being captured long-period comets, but is consistent with JFCs originating in the Kuiper belt. If so, there should be hundreds of millions of comet-sized (1–10 km radius) objects in the Kuiper belt, although most would be too faint to be discovered using conventional ground based surveys (Levison & Duncan, AJ, **100**, 1669). We should note, however, that parts of Quinn, Tremaine, & Duncan's argument are still contested (Stagg & Bailey, MNRAS **241**, 507).

One link in the theory of the Kuiper belt as the origin of JFCs has been missing. Neither Fernández (MNRAS **192**, 481) nor Quinn, Tremaine, & Duncan (ApJ **355**, 667) could directly model the mechanism responsible for injecting Kuiper belt objects into Neptune-crossing orbits. In 1990, Torbett & Smoluchowski (*Nature* **345**, 49) showed that most particles with initial values of perihelion distance less than $\sim 45AU$ and eccentricity greater than ~ 0.01 are chaotic due to the gravitational perturbations of the giant planets and therefore can in principle leave the Kuiper belt. However, since the expected depletion timescales at the current epoch are on the order of the age of the solar system, they could not follow this evolution in their 10 Myr simulations.

The missing link was filled within the last year with the publication of two papers, by Levison & Duncan (ApJ **406**, L35) and Holman & Wisdom (AJ **105**, 1987), on the long-term dynamical evolution of objects in the Kuiper belt. Both sets of authors integrated the orbits of more than a thousand massless test particles on initially low-inclination, low-eccentricity orbits within the Kuiper belt. Holman & Wisdom's integrations lasted for 10^8 years and focussed on initially circular test particle orbits. Levison & Duncan's lasted for 10^9 years, using an approximation for the planetary perturbations but studying a range of initial eccentricities for the Kuiper belt objects. Integrations of this magnitude have only recently become possible because of advances of computer hardware as well as the development of powerful new integration techniques by Wisdom & Holman (AJ **102**, 1528).

Both groups found that particles which eventually crossed Neptune's orbit often showed long periods of relatively low-eccentricity oscillations punctuated by a very rapid jump to Neptune-crossing eccentricity. This flux may be the ultimate source of present-day short-period comets. For initially near-circular orbits an intricate structure in the region between 35 and 45 AU is found at the end of the simulations. However, an inner edge of the Kuiper belt can be said to be at approximately 36AU. Also, there is a low density region between 40 and 42 AU due to the presence of several resonances in this region of the solar system. For higher initial eccentricities the depletion is much more extensive interior to 42 AU.

Both groups also found that there is a rough correlation between the Lyapunov time of a Kuiper belt object and the time it takes to become a Neptune-crosser (although there is a wide dispersion about the mean crossing time for a given Lyapunov time). Levison & Duncan (ApJ **406**, L35) found that the correlation is in good agreement with the results of Soper, Franklin & Lecar (*Icarus*, **87**, 265). The currently active regions are found to be those in which the Lyapunov timescales are between about 0.2 and 0.8 Myr.

The minor bodies 1992 QB1 and 1993 FW may well be the first of a large number of objects to be discovered in the Kuiper belt. The object 1992 QB1, which is currently estimated (IAU Circular 5855) to have a semi-major axis of 44AU, an eccentricity of 0.09 and an inclination of 2 degrees is located in a relatively stable region according to the simulations of Levison & Duncan. Similar comments apply to 1993 FW, for which the best available estimates give orbital elements very similar to 1992 QB1. Very recently, 4 more objects have been found beyond Neptune; 1993 RO, 1993 RP (IAU Circular 5867), 1993 SB and 1993 SC (IAU Circular 5869), but their orbits are very poorly constrained at this time. It is possible that some or all of these objects may be resonantly trapped in regions near Neptune's Trojan points.

E. Capture into Resonance

(Jacques Henrard)

The effect of small non-conservative forces in shaping the solar system as we see it now has been brought to the attention of dynamicists by Goldreich (MNRAS **130**, 159). This effect is particularly dramatic when a resonance is encountered. It is now well established that tidal dissipation can lead to capture into orbit-orbit resonance and is responsible for at least some of the commensurabilities among the satellites of Jupiter and Saturn (see the review by Peale in *Natural Satellites*, Univ. Arizona Press, 1986). Recently, the scenario of capture into resonance of three of the Galilean satellites (Yoder, *Nature* **279**, 767) has been rewritten by Malhotra (*Icarus* **94**, 399) to include episodes of high eccentricity for Ganymede which could be responsible for its resurfacing.

Tidal dissipation is also responsible for the spin-orbit resonances in which most of the regular satellites and Mercury are locked (see also the review by Peale). Recent contributions concern the stability of the spin-orbit resonance (Celletti & Falconi, CM **53**, 113; Celletti, CM **57**, 329) and the consideration of gas drag rather than tides to provide the "small" dissipation mechanism (Winter & Sessin CM **57**, 329).

The evidence of melting of the small satellites of Uranus revealed by Voyager II, has spurred interest in the tidal evolution of these satellites (Peale, *Icarus* **74**, 153) although they are not, at present, locked in resonance. Tittemore & Wisdom (*Icarus* **74**, 174) investigated numerically the passage of Ariel and Umbriel through a 5/3 resonance and found evidence of possible *temporary* resonance, something which had not been encountered in the studies concerning the satellites of Jupiter and Saturn. Dermott *et al.* (*Icarus* **76**, 295) found also, in their numerical investigation of the 3/1 Miranda-Umbriel resonance, evidence of temporary capture and they attributed to it the anomalously large inclination of Miranda and its post-accretional resurfacing. Tittemore & Wisdom (*Icarus* **78**, 63) identified the mechanism responsible for the disruption of the resonance: capture into *secondary* resonances.

Indeed, the small oblateness of Uranus (compared with the oblateness of Jupiter and Saturn) makes possible the apparition of secondary resonances inside the primary resonance. When Miranda evolves inside the 3/1 primary resonance, it encounters these secondary resonances and upon capture by one of them is dragged into the chaotic layer surrounding the primary resonance and then escapes. Malhotra & Dermott (*Icarus* **85**, 444) confirmed and extended these results. The extent of the chaotic layer and the location of secondary resonances were also confirmed by Henrard & Sato (CM **47**, 391) with a semi-analytical perturbative analysis. Two building blocks of the basic scenario, the probability of capture into secondary resonance and the mechanism of escape from the chaotic layer were further investigated by Malhotra (*Icarus* **87**, 249; **94**, 524), Henrard & Moons (*Icarus* **95**, 244), Henrard & Henrard (*Physica* D **54**, 135) and Henrard & Morbidelli (*Physica* D **68**, 187).

Evolution through other possible resonances (2/1 Ariel-Umbriel and 4/1 Ariel-Titania) were numerically simulated by Tittemore & Wisdom (*Icarus* **85**, 394) and Tittemore (*Icarus* **87**, 110) as possible scenario leading to tidal heating of Ariel.

The effect of gas drag on a swarm of boulders or planetesimals orbiting in a protosolar nebula can lead them into capture in resonance with an embryo of planet. This is possible only for exterior resonances as shown by Weidenschilling & Davis (*Icarus* **62**, 12) and Patterson (*Icarus*

70, 319). Weidenschilling & Davis see in this mechanism a way to feed the embryo, Patterson see in it a way to form new embryos. The neoadiabatic theory (see Henrard, *Dynamics Reported*, Springer-Verlag 1993), the extension of the adiabatic theory to separatrix crossing used with success in the case of tidal dissipation, could be applied here also but only for particles 100 meters in radius or larger. Beaugé & Ferraz-Mello (IAU Symp. 152, 355; *Icarus* **103**, 301) consider the effect of Stokes drag dissipation upon smaller particles (10 meters in radius) and confirm the above mentioned results. They point out that the particles captured in resonance acquire eventually an equilibrium eccentricity which curiously enough depends only on the angular velocity of the gas and not on the drag coefficient or on the mass of the embryo. Nevertheless the probability of capture into resonance and the time scale of the capture do depend upon these other parameters.

A similar problem, connected with the evolution of dust grains around the star β-Pic under the influence of Poynting-Robertson drag, has been investigated by Scholl *et al.* (CM **56**, 381) and Sicardy *et al.* (CM **57**, 373). Very recently (*Icarus* submitted) Beaugé & Ferraz-Mello have applied the neoadiabatic theory to this situation and computed probability of capture in exterior resonances. As the equilibrium eccentricity is large in this case, the capture is only temporary; the dust grains collide with the protoplanet before reaching this equilibrium.

F. Symplectic Integrators

(Haruo Yoshida)

The exact time evolution (solution) of a Hamiltonian system is symplectic, i.e., the mapping from $(q(0), p(0))$ to $(q(t), p(t))$ conserves the symplectic 2-form $dp \wedge dq$. Traditional integration methods, such as the classical Runge-Kutta method, do not respect this fact and after a long-term integration, fictitious damping or excitation occurs which makes the result of integration unreliable. Recently there have been much interest in the integration methods which are designed to keep the symplectic property of the original Hamiltonian flow. These integration methods are in general called symplectic integrators (symplectic integration methods). With use of symplectic integrators, the error of energy does not grow monotonically. This comes from the existence of a conserved quantity (interpolating Hamiltonian) which is close to the original Hamiltonian. General reviews of this field of research are found in Sanz-Serna (*Acta Numerica* **1**, 243) and Yoshida (CM **56**, 27).

1. IMPLICIT SCHEME

For all Hamiltonian systems, implicit symplectic integration method is always possible. There are essentially two different kinds of implicit schemes. The first one is by the generating function (of Von Zeipel type). The generating function of mixed variable near the identity can be found to arbitrary order. These schemes are found by Feng & Qin (Lect.Notes in Math. 1297, 1), Channell & Scovel (*Nonlinearity* **3**, 231), etc. The second one is by the implicit Runge-Kutta method. Sanz-Serna (BIT **28**, 877) and Lasagni (ZAMP **39**, 952) found the condition of an implicit Runge-Kutta method to be symplectic when applied to a Hamiltonian system. The Gauss-Legendre type of Runge-Kutta methods fall in this category and the simplest 2^{nd} order one is the implicit midpoint rule.

2. EXPLICIT SCHEME

For a Hamiltonian which is a sum of two trivially integrable parts, explicit scheme are possible to design. The typical example is a Hamiltonian of the form $H = T(p) + V(q)$. The second-order one has been known as the leap-frog method. The third-order one is found by Ruth in 1983 and the fourth order one by Forest in 1987. Yoshida (*Phys.Lett.* A **150**, 262) and Suzuki (*J.Math.Phys.* **32**, 400; *Phys.Lett.* A **165**, 387) found the way to obtain the explicit scheme of higher orders. Other contributions in this direction are found in Forest (*J.Comp.Phys.* **99**, 209) and McLachlan & Atela (*Nonlinearity*, **5**, 541)

3. APPLICATION TO DYNAMICAL ASTRONOMY

Kinoshita, Yoshida & Nakai (CM **50**, 59) applied the 4th and 6th order explicit integrator to the Kepler problem and confirmed several advantages over the non-symplectic methods. Gladman *et al.* (CM **52**, 221) also tested symplectic integrators to the Kepler problem and they found that the introduction of variable steps decrease the advantage of symplectic methods. Gladman & Duncan (AJ **100**, 1680) simulated the evolution of test particles in the outer solar system. Wisdom & Holman (AJ **102**, 1528) divide the Hamiltonian of the solar system into two parts (pure Keplerian part and the perturbation) and applied the idea of 2^{nd} order explicit scheme to obtain a map which they call the N-body map. This N-body map has been discussed also in Wisdom & Holman (AJ **104**, 2022) and Holman & Wisdom (AJ **105**, 1987). See also Saha & Tremaine (AJ **104**, 1633).

4. APPLICATIONS IN OTHER FIELDS OF RESEARCH

Pullin & Saffman (*Proc.R.Soc.London* **432**, 481) applied the symplectic implicit Runge-Kutta method to the motion of point vortices successfully. Cary & Doxas (*J.Comp.Phys.* **107**, 98) used explicit scheme for plasma simulation. Some partial differential equations such as sine-Gordon and nonlinear Schrödinger equations are infinitely dimensional Hamiltonian systems. For the numerical integration of these partial differential equations, Herbst & Ablowitz (*J.Comp.Phys.* **105**, 122) tested symplectic schemes and found their advantages.

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