REFLEXION AND TRANSMISSION OF ALFVEN WAVES IN AN ATMOSPHERE.

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If we want to study the transfer of energy by waves through a stellar atmosphere, we first need to know its reflectivity with respect to each of the three MHD modes. Here, we will consider linear Alfvén waves only. The problem is set as follows. An energy flux being given at the bottom (z=0) of the atmosphere, we ask for how much of it is recorded at an altitude z.

The atmosphere is assumed to be isothermal and permeated by a uniform, vertical magnetic field B. (The vertical is taken as the z-axis of a Cartesian system of reference.) The equations of motion can be put into a matrix form which exhibits the linear couplings between ascending and descending "modes" (Leroy, 1979). It is the so-called coupled modes description (Budden, 1961). They then write:

$$W' = \begin{vmatrix} ik(z) & -1/4H \\ -1/4H & -ik(z) \end{vmatrix} W, \qquad (1)$$

where the prime (') denotes differentiation with respect to z, $k(z) := \omega \exp(-z/2H)/v_A(0)$ ($v_A(0)$ is the Alfvén velocity at z=0 and ω is the frequency), and H is the scale height. A time dependence $\exp(i\omega t)$ is assumed throughout. In the limit $H \to \infty$, W_2 turns out to be an ascending wave and W_1 a descending one. In the case of a finite scale height it has been shown (Leroy, 1979) that W_2 (W_1) can still be regarded as an ascending (a descending) wave.

From Eq.(1) it can be shown that the components W_1 and W_2 each satisfy a linear second-order differential equation; it follows that W_1 and W_2 may each be expressed as a linear combination of two particular solutions, say F,f and G,g, respectively. These particular solutions cannot be arbitrary; they must be coupled by Eq.(1). A convenient choice of initial conditions is

$$F(0) = g(0) = 1 \text{ and } f(0) = G(0) = 0.$$

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Then, the solutions such that

$$W_1(0) = W_{10}\ , \qquad W_2(0) = W_{20}$$

may be expressed in matrix notation in the form:

$$\begin{vmatrix} W_1 \\ W_2 \end{vmatrix} = \begin{vmatrix} F & f \\ G & g \end{vmatrix} \begin{vmatrix} W_{10} \\ W_{20} \end{vmatrix},$$

or, inverting this relation:

$$\begin{vmatrix} W_{10} \\ W_{20} \end{vmatrix} = \begin{vmatrix} g & -f \\ -G & F \end{vmatrix} \begin{vmatrix} W_1 \\ W_2 \end{vmatrix} =: M(z) \begin{vmatrix} W_1 \\ W_2 \end{vmatrix} . \qquad (2)$$

Such a formulation is well known in the Optics of stratified media (Abelès,1950), where the matrix M(z) is called the characteristic matrix of the stratified medium.

Now, we calculate the reflectivity and transmissivity of a slab of atmosphere extending from z=0 to an arbitrary height z. To do this, it must be prescribed that no wave is descending into this slab from above. If we denote the amplitudes of the incident (ascending) and reflected waves at the bottom of the slab by A and R, respectively, and the amplitude of the transmitted wave by T, Eq.(2) yields:

$$\begin{vmatrix} R \\ A \end{vmatrix} = M(z) \begin{vmatrix} 0 \\ T \end{vmatrix} . \qquad (3)$$

By definition, the reflexion and transmission coefficients are r =R/A and t=T/A, respectively. It is easy to see that, in our case:

$$r = -f/F \qquad \text{and} \qquad t = 1/F.$$

We have shown that the energy flux density associated to a wave of amplitude W_i (i=1,2) is

$$B^2\ \omega^2/8\pi\ v_A^2(0))\ |W_i|^2;$$

the reflectivity and transmissivity are therefore given by $|r|^2$ and $|t|^2$, respectively.

We apply this to the particular case of the solar atmosphere. This is described as a sequence of two isothermal (lossless) layers; the first one represents the photosphere and chromosphere, the second one the corona. The discontinuity in temperature simulates the transition region. In order that the dynamical equilibrium be preserved the continuity of pressure is written at the interface between the two layers (z=h). As in Optics, each layer is described by its characteristic matrix, and the two-layer sequence by the product of these matrices (Abelès,1950). In Eq.(3) the matrix M(z) is then:

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$$M(z) = M_1(h)\ M_2(z-h), \tag{4}$$

where indices 1 and 2 refer to the first and second layer, respectively.

The numerical results for the solar case are presented in Table 1. We adopted the following figures:

H_1 = 200 km, T_1 = 5000 °K,
H_2 = 1000 km, $T_2 = 10^6$ °K,
h = 2000 km, $\rho_1(0) = 10^{-7}$ g.cm^{-3}.

The results are presented for two values of the magnetic field intensity, B = 1 G and B = 10 G, as well as for two oscillation periods, $2\pi/\omega$ = 300s and $2\pi/\omega$ = 20 h.

Whatever the strength of the magnetic field the Alfvén waves of shorter frequency (possibly generated by supergranulation motions) are almost totally reflected downwards at an altitude z= 5000 km. As for the waves of higher frequency, the weaker the magnetic field, the lesser reflected they are.

Table 1. Reflectivity of a slab of solar atmosphere extending from the bottom of the photosphere to an altitude z= 5000 km.

	$2\pi/\omega$ = 300 s	$2\pi/\omega$ = 20 h
B = 1 G	.03	.98
B = 10 G	.45	.99

<u>References</u>:

Abelès, F.: 1950, Annales de Phys. 5, pp. 598-640.
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